News of kaons from the lattice

Tom Blum (UConn/RBRC)

FPCP 2022, Ole Miss

May 23, 2022

outline

- Direct CP violation (ϵ') in $K \to \pi\pi$ decays (main part of talk)
- Electromagnetic corrections to (semi-) leptonic π / kaon decays
- Kaon mass difference Δm_K
- Long distance contribution to indirect *CP* violation
- Rare decay $K^+ \to \pi^+ \nu\bar{\nu}$

Direct CP violation in the SM (kaons)

- Final $\pi\pi$ states can have I = 0 or 2.

$$\langle \pi\pi(I=2)|H_w|K^0\rangle = A_2 e^{i\delta_2} \qquad \Delta I = 3/2$$

$$\langle \pi\pi(I=0)|H_w|K^0\rangle = A_0 e^{i\delta_0} \qquad \Delta I = 1/2$$

- CP symmetry requires A_0 and A_2 be real.
- Direct CP violation in this decay is characterized by:

$$\epsilon' = \frac{i e^{\delta_2-\delta_0}}{\sqrt{2}} \left|\frac{A_2}{A_0}\right| \left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$

Direct CP violation

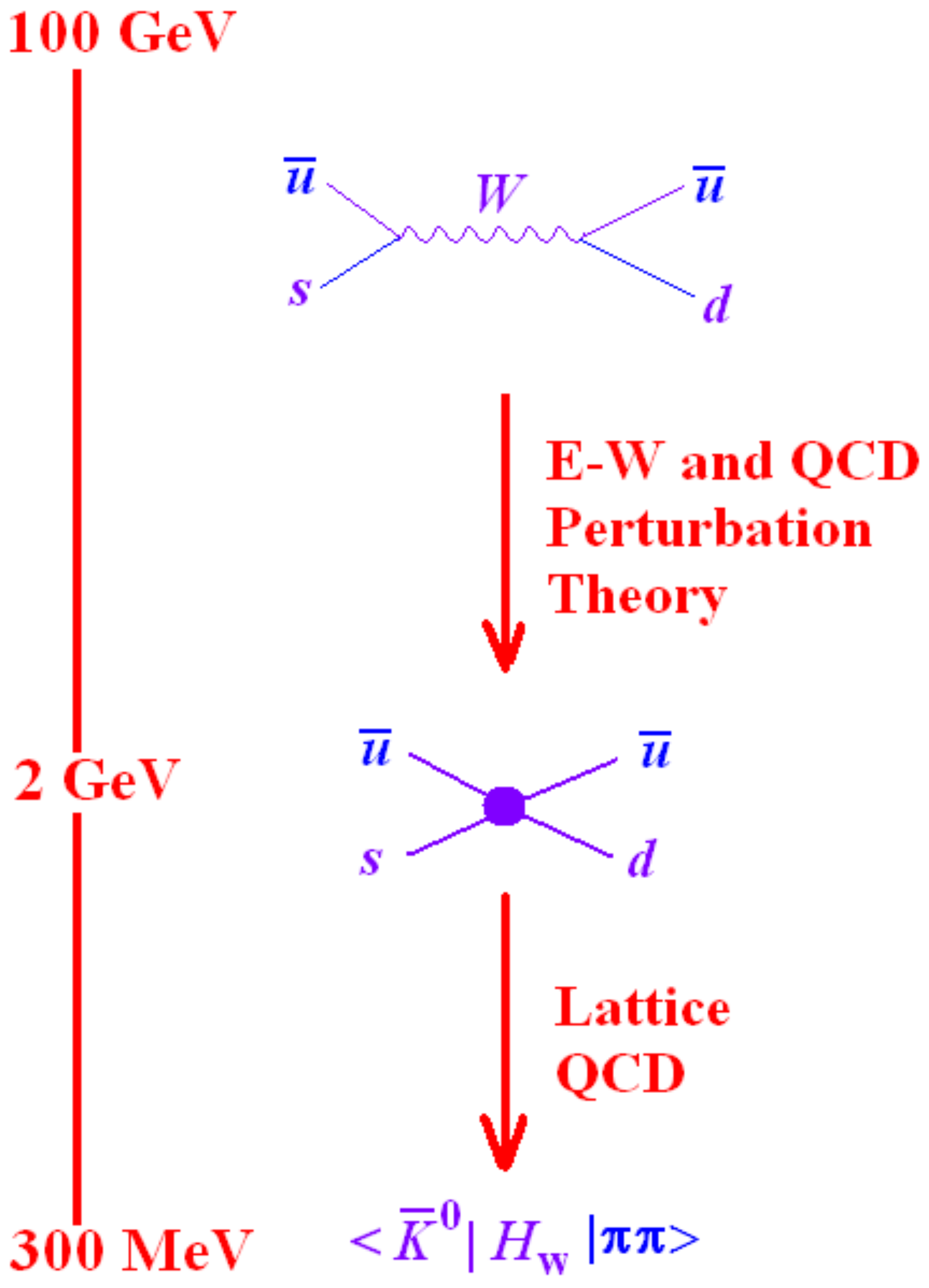
(slide: Norman Christ)

Direct CP violation in the SM (kaons)

- Represent weak interactions by local four-quark Lagrangian

$$\mathcal{H}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} \left[z_i(\mu) + \tau y_i(\mu) \right] Q_i \right\}$$

- $\tau = -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*} = (1.543 + 0.635i) \times 10^{-3}$
- $V_{qq'}$ – CKM matrix elements
- z_i and y_i – Wilson Coefficients
- Q_i – four-quark operators



100 GeV

E-W and QCD Perturbation Theory

2 GeV

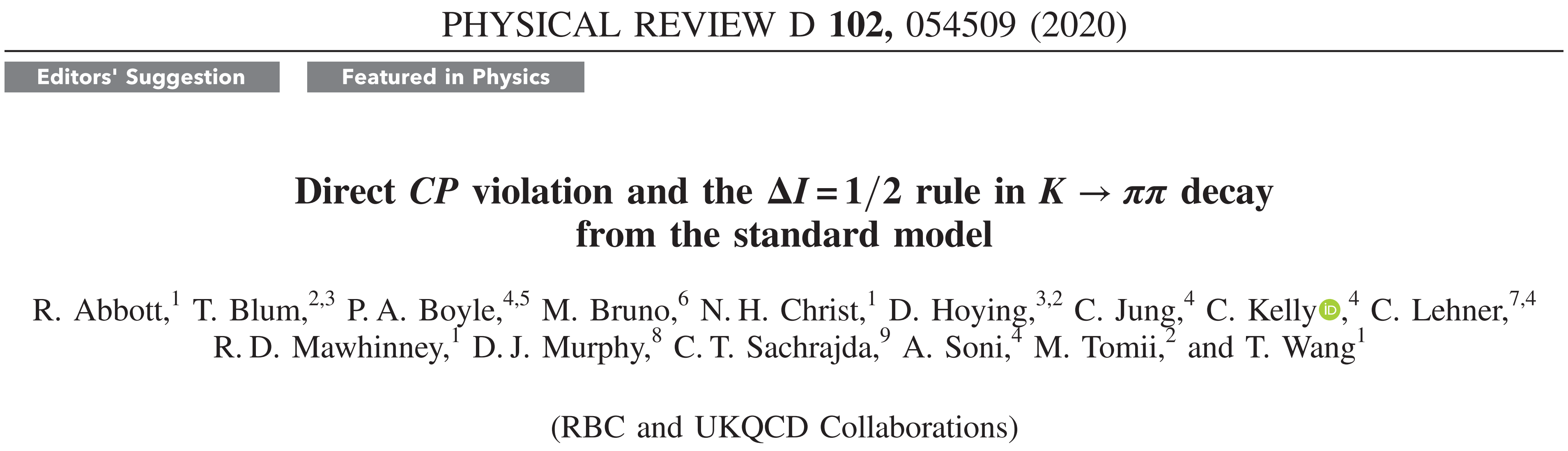
Lattice QCD

300 MeV

$<\bar{K}^0| H_W |\pi\pi>$

(slide: Norman Christ)

Direct CP violation in the SM (ϵ')



PHYSICAL REVIEW D **102,** 054509 (2020)

Editors' Suggestion **Featured in Physics**

Direct *CP* violation and the $\Delta I = 1/2$ rule in $K \to \pi\pi$ decay from the standard model

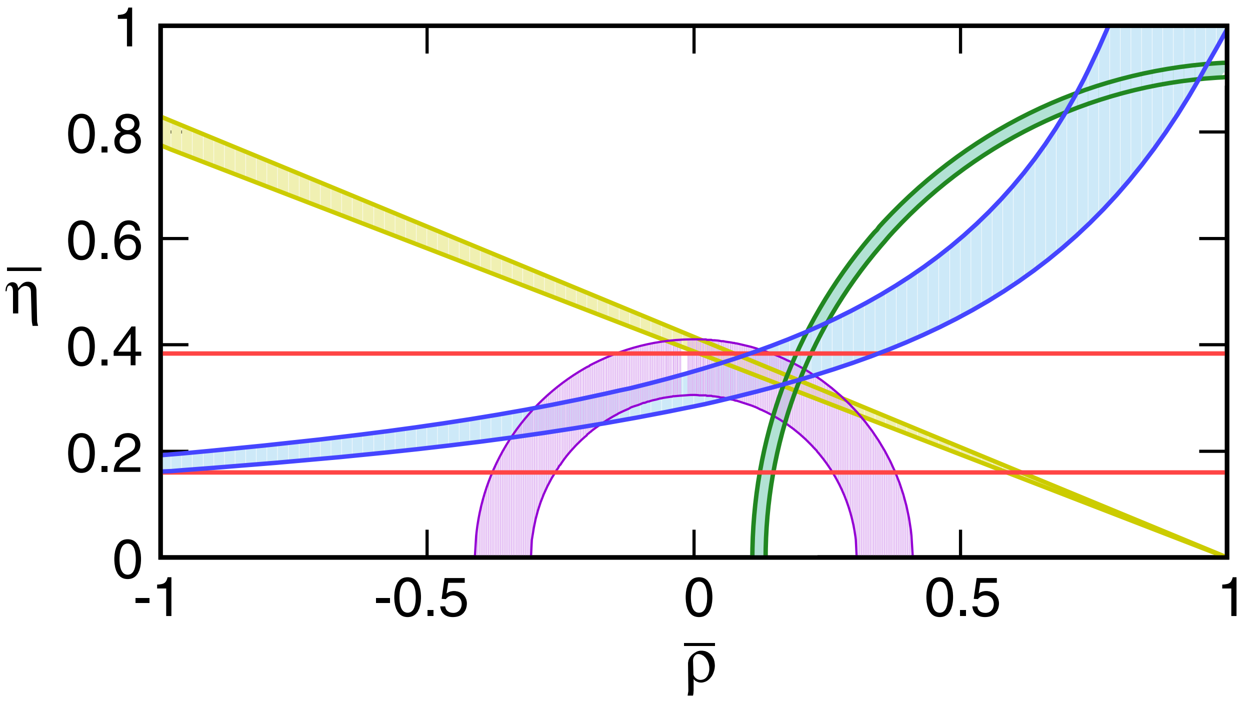
R. Abbott,[1] T. Blum,[2,3] P. A. Boyle,[4,5] M. Bruno,[6] N. H. Christ,[1] D. Hoying,[3,2] C. Jung,[4] C. Kelly,[4] C. Lehner,[7,4]
R. D. Mawhinney,[1] D. J. Murphy,[8] C. T. Sachrajda,[9] A. Soni,[4] M. Tomii,[2] and T. Wang[1]

(RBC and UKQCD Collaborations)

$$\mathrm{Re}(\varepsilon'/\varepsilon)_{\mathrm{expt}} = 0.00166(23). \qquad (115)$$

[NA48 2002, KTEV 2011]

$$\mathrm{Re}(\varepsilon'/\varepsilon)_{\mathrm{lattice}} = 0.00217(26)(62)(50). \qquad (114)$$



- G-parity boundary conditions (pions anti-periodic)
- Multiple $\pi\pi$ operators for control of excited states

Errors in the lattice calculation

$$\mathrm{Re}(\varepsilon'/\varepsilon)_{\mathrm{lattice}} = 0.00217(26)(62)(50). \qquad (114)$$

statistical, systematic, systematic IB

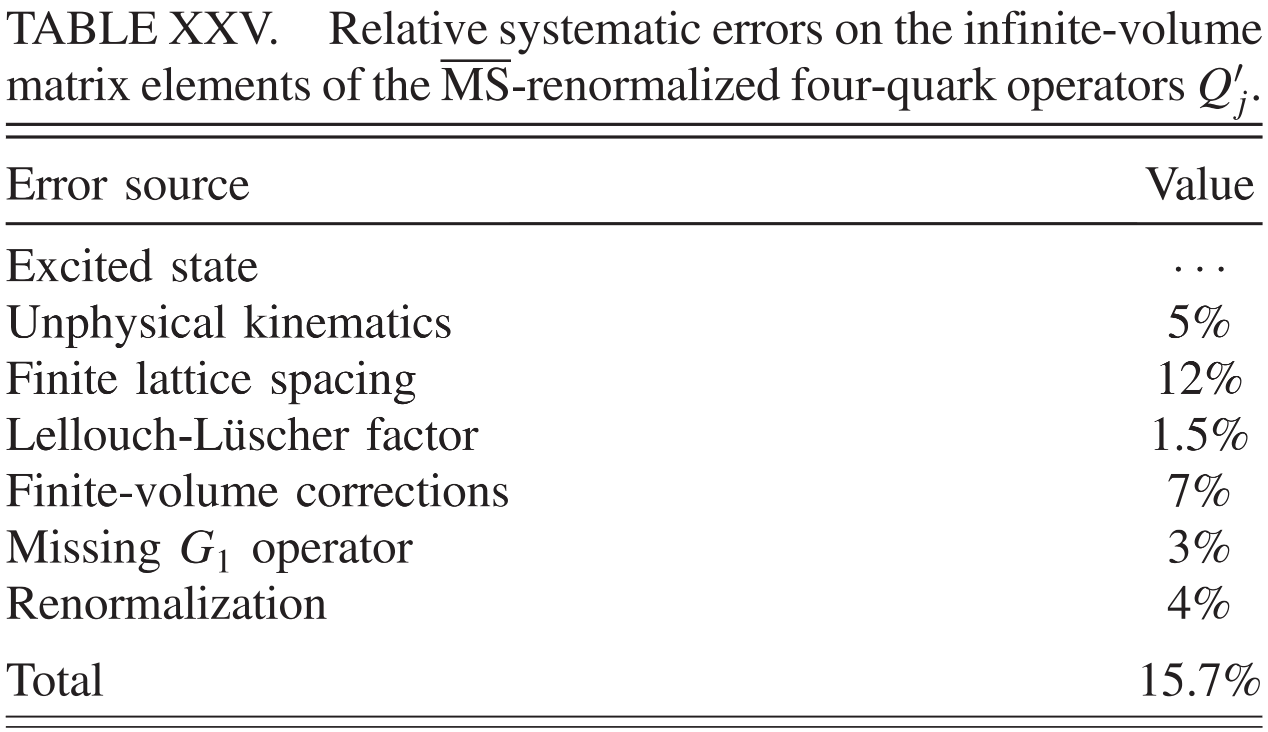


TABLE XXV. Relative systematic errors on the infinite-volume matrix elements of the $\overline{\mathrm{MS}}$-renormalized four-quark operators Q'_j.

Error source	Value
Excited state	...
Unphysical kinematics	5%
Finite lattice spacing	12%
Lellouch-Lüscher factor	1.5%
Finite-volume corrections	7%
Missing G_1 operator	3%
Renormalization	4%
Total	15.7%

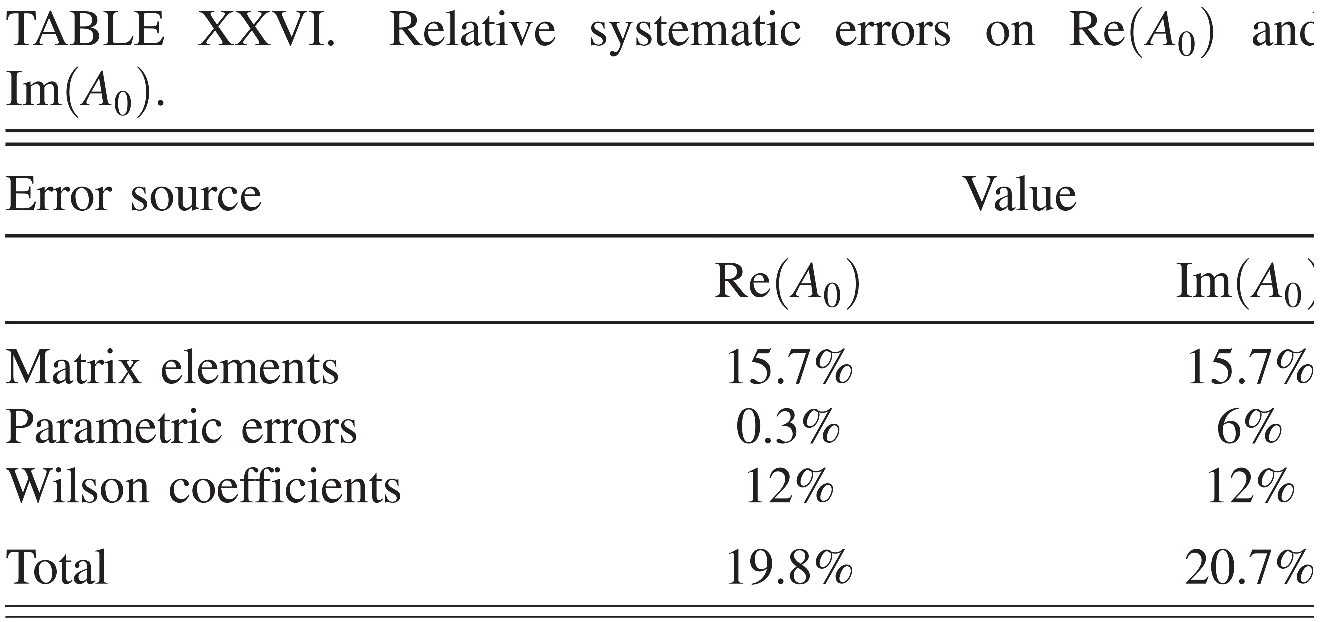


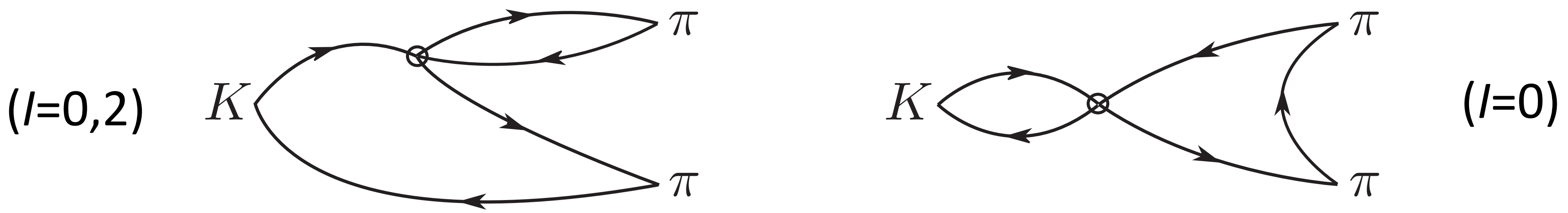
TABLE XXVI. Relative systematic errors on Re(A_0) an Im(A_0).

Error source	Value	
	Re(A_0)	Im(A_0
Matrix elements	15.7%	15.7%
Parametric errors	0.3%	6%
Wilson coefficients	12%	12%
Total	19.8%	20.7%

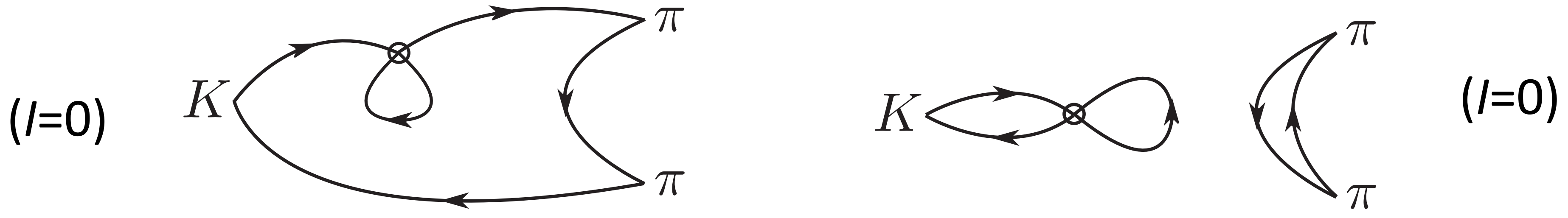
- Statistical (12%), systematic (29%), IB corrections (23%) [Cirigliano, *et al.*, 2020]
- Continuum PT at NNLO needed for Wilson coefficients (12%)
- New G-parity BC simulations at smaller lattice spacing underway
- EM/isospin corrections can be calculated in lattice QCD+QED, but incompatible with G-parity BC's

Periodic Boundary Conditions (PBCs)

- Every lattice calculation starts with a correlation function in Euclidean spacetime



(a) *type1* (b) *type2*



(c) *type3* $t \rightarrow$ (d) *type4*

- $C(t)$ is sum of exponentials, $e^{-E_n t}$, pull operators apart, $t_K \ll\ t_{\text{op}} \ll t_{\pi\pi}$ to get ground state contribution
- G-parity BC's: enforces non-zero momenta for the pions, so 2π ground state corresponds to physical kinematics
- For PBC's an excited state corresponds to physical kinematics

Why make life (seemingly) harder?

- G-parity more expensive ($\sim 2\times$) and need special ensembles
- G-parity conserves isospin: incompatible with QED corrections
- Ordinary PBC ensembles of gauge configurations exist already with many different lattice spacings, volumes
- Generalized eigenvalue problem (GEVP) method effective for resolving ground and excited states

GEVP method

(Blossier, *et al*., *JHEP* 04 (2009) 094)

- Matrix of correlation functions, *N* operators, *N* states

$$C_{ij}(t) = < O_i(t) O_j(0) >, \quad O_i = \pi\pi, \quad \pi = \bar{\psi}\gamma_5\psi$$

- Solve generalized eigenvalue problem,

$$C(t)v_n(t,t_0) = \lambda_n(t,t_0)C(t_0)v_n(t,t_0) \quad 1 \leq n \leq N, t_0 \leq t\,,$$

- Obtain (eigen-) energies of states, eigenvectors

$$\lambda_n(t,t_0) = e^{-E_n(t-t_0)}, \qquad v_n \propto u_n\,, \quad A_n = \sum_1^N O_i u_{ni}^*,$$

- Systematic error: *N*+1st state contaminates lower *N* states, energies

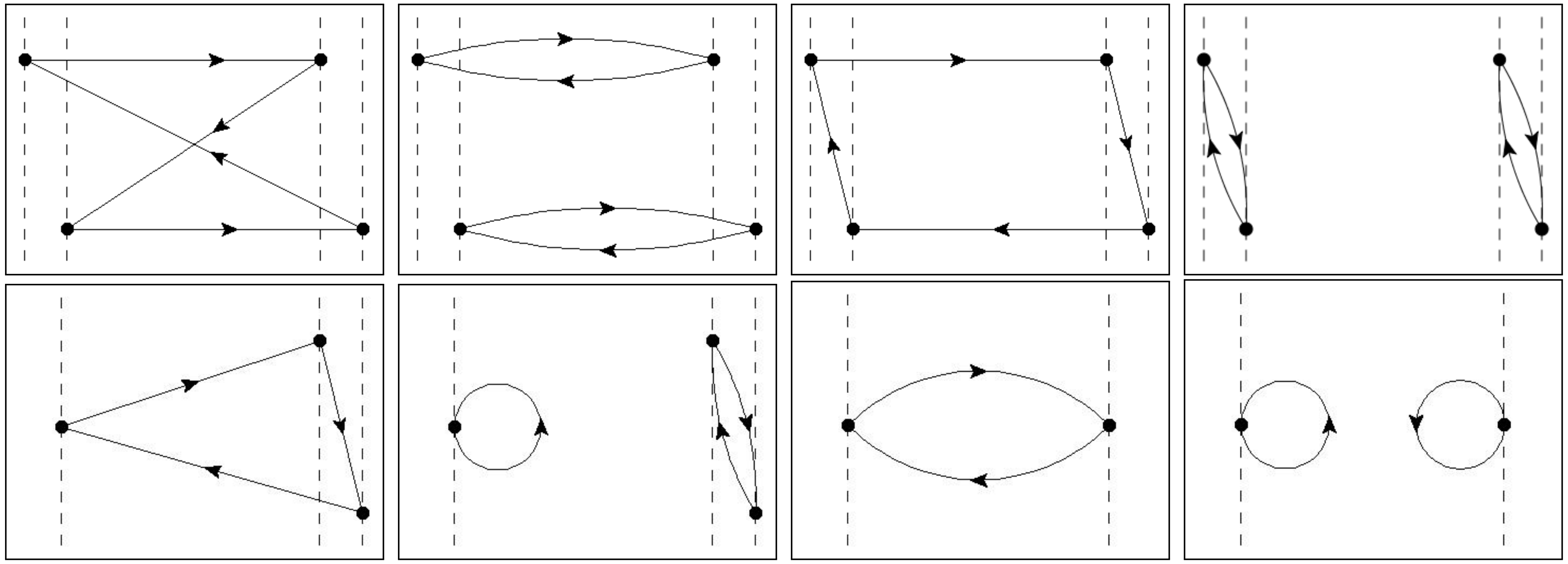
  (but the contamination is exponentially suppressed)

$\pi\pi \rightarrow \pi\pi$ (Dan Hoying)

- Construct $C_{ij}(t)$, two-pion two-point correlation function

$$C_{ij} = < O_i(t) O_j(0) >, \; O_i = \sigma \text{ and } \pi\pi; \quad \sigma = \bar{\psi}\,\psi, \qquad \pi = \bar{\psi}\,\gamma_5\psi$$

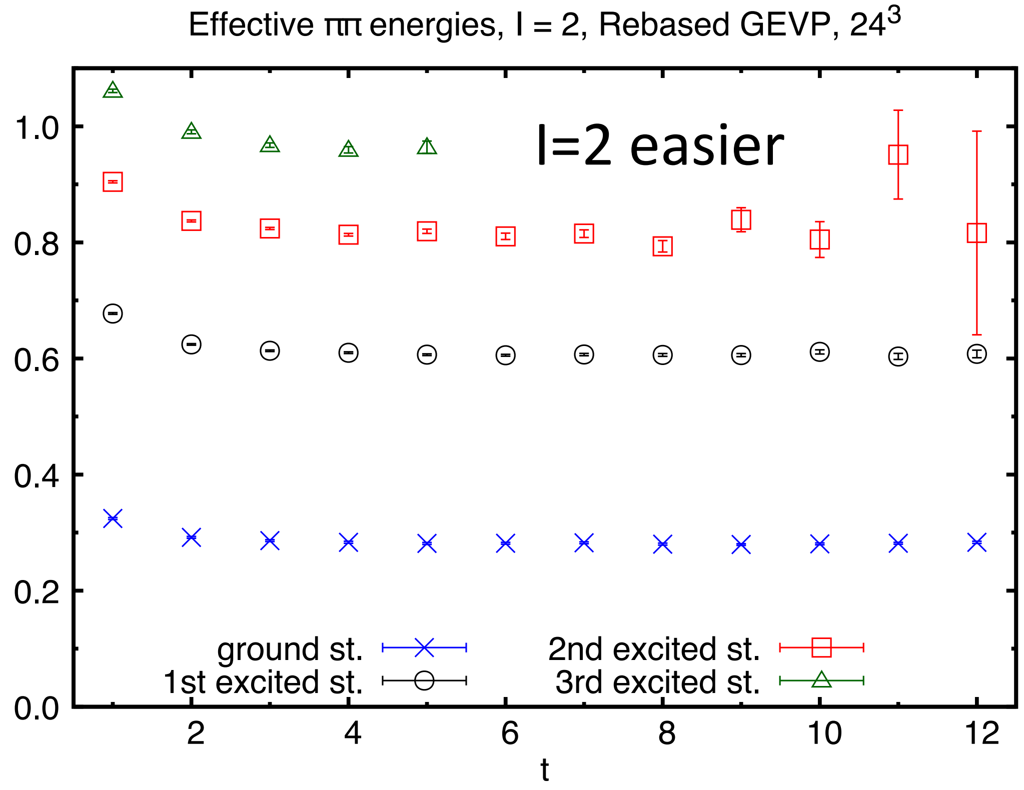
- Use two pion operators with relative momentum up to $\frac{2\pi}{L}(1,1,1)$ and scalar, or *N*=5
- Subtract vacuum state for *I*=0
- *I*=0, 2 states from linear combinations of Wick contractions



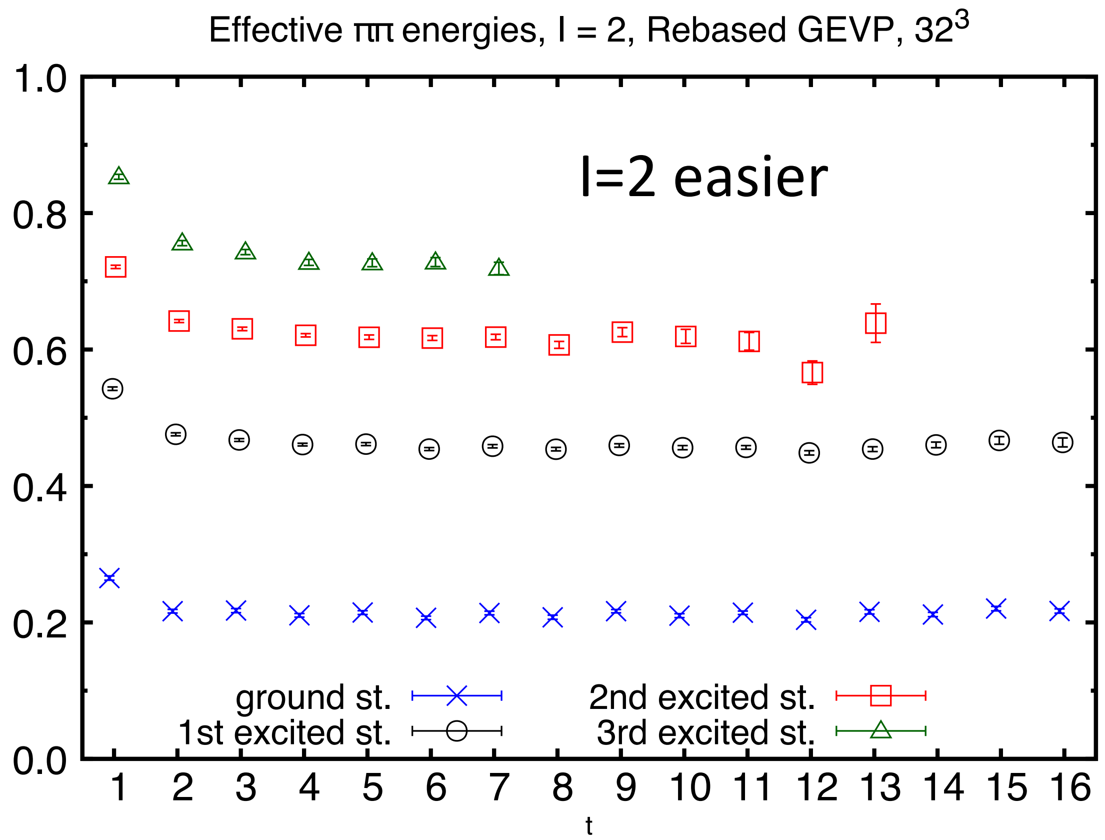
Ensemble details

- 2+1 flavors of Möbius domain-wall fermions (MDWF) with physical masses (m_π=140 MeV, *etc*)
- Iwasaki+DSDR gauge action for the gluons
- Two ensembles, 24^3 and 32^3 both with N_t=64
- (inverse) lattice spacing a^{-1} = 1 and 1.4 GeV (relatively coarse)
- Statistics: 258 and 107 configs for 24^3 and 32^3, respectively
- 32^3 ensemble matches G-parity BC ensemble (741 configs)

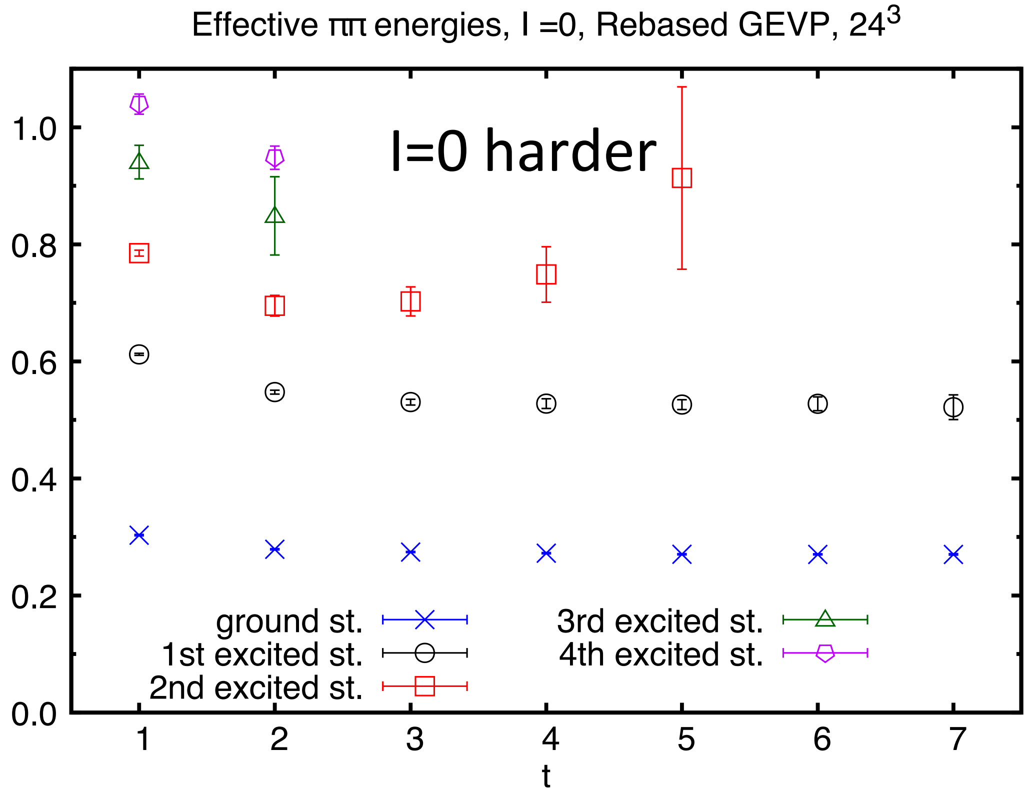
$\pi\pi \to \pi\pi$ energies


Effective ππ energies, I = 2, Rebased GEVP, 24^3

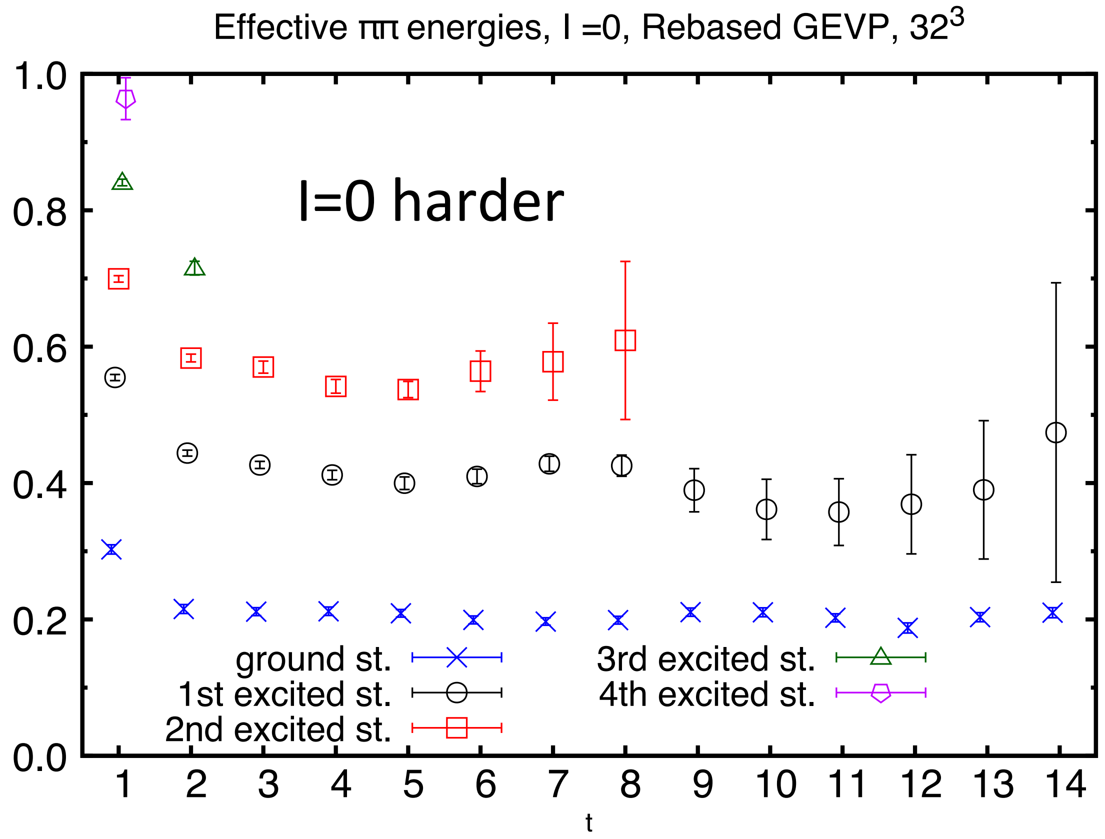
I=2 easier


Effective ππ energies, I = 2, Rebased GEVP, 32^3

I=2 easier

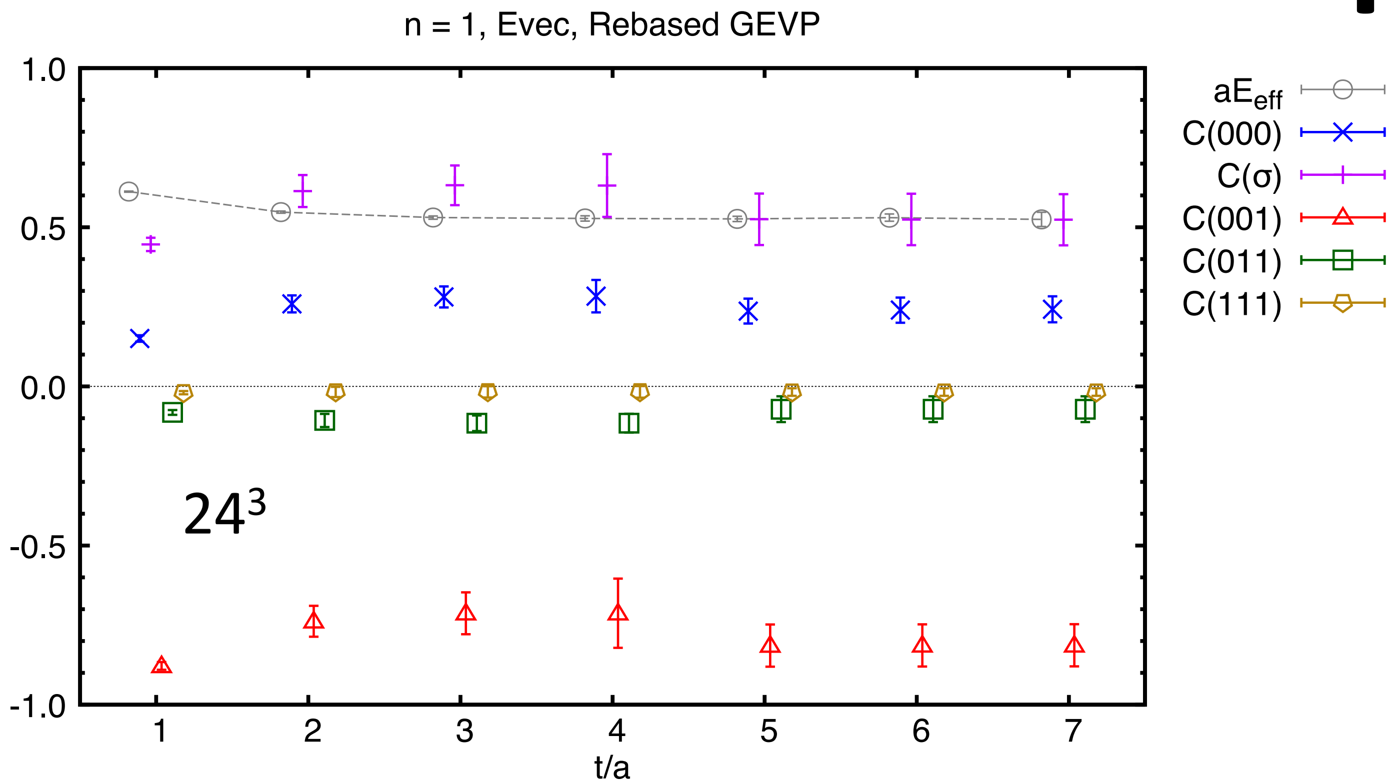

Effective ππ energies, I =0, Rebased GEVP, 24^3

I=0 harder


Effective ππ energies, I =0, Rebased GEVP, 32^3

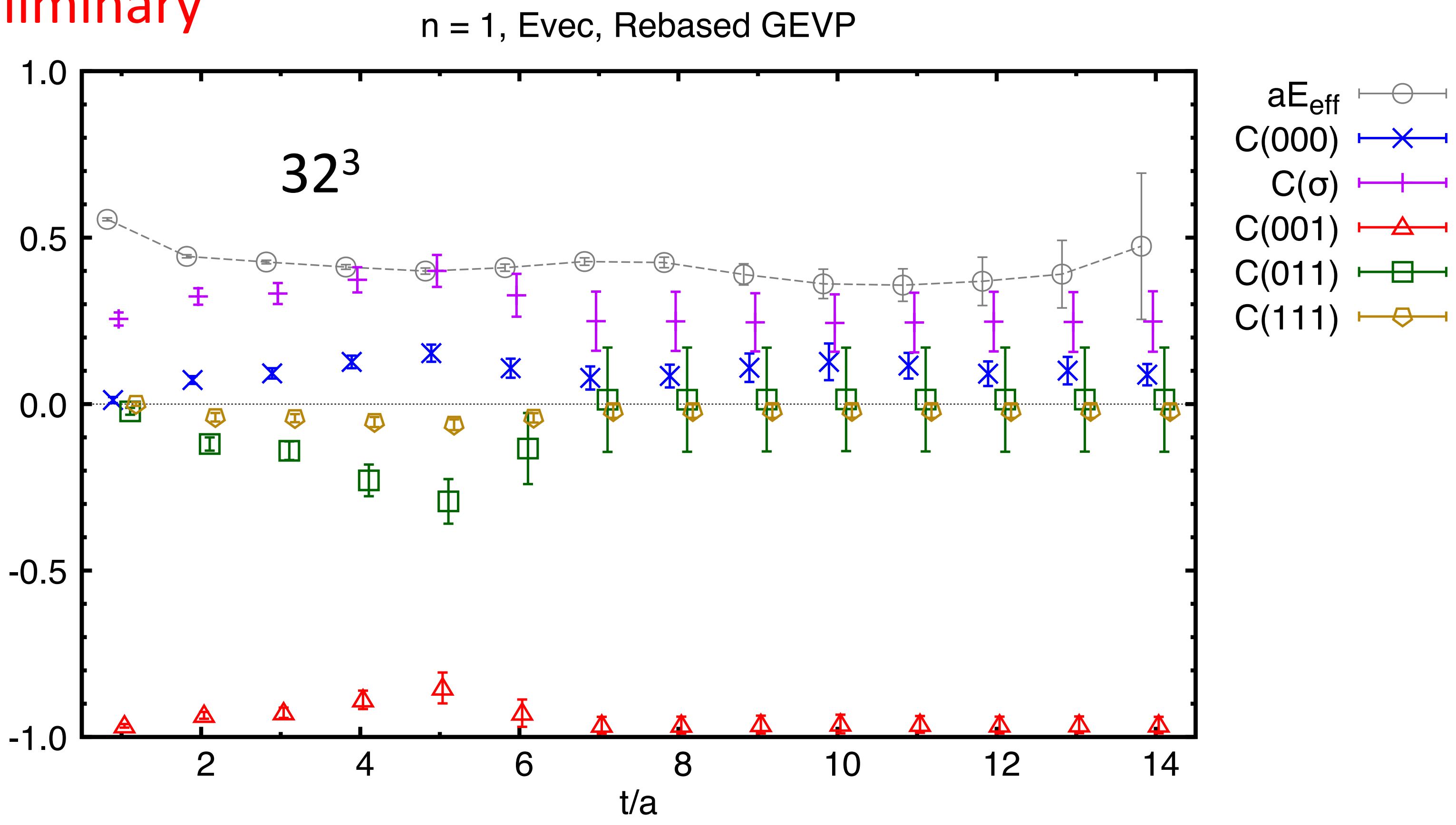
I=0 harder

$\pi\pi \to \pi\pi$ operators



- Elements of eigenvector *n* are coefficients of linear combination of operators that create state *n*

preliminary



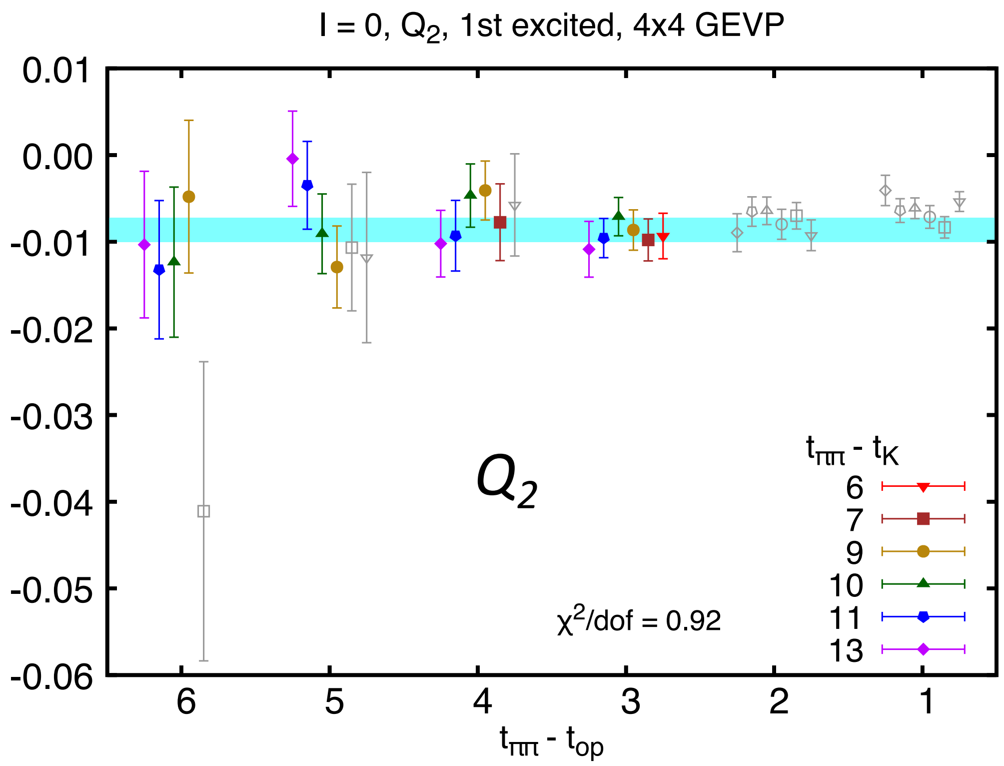
- *I*=0
- 5x5 GEVP
- Noise from higher states
- “Rebase”: use linear combination from small *t* for reduced GEVP at larger *t*

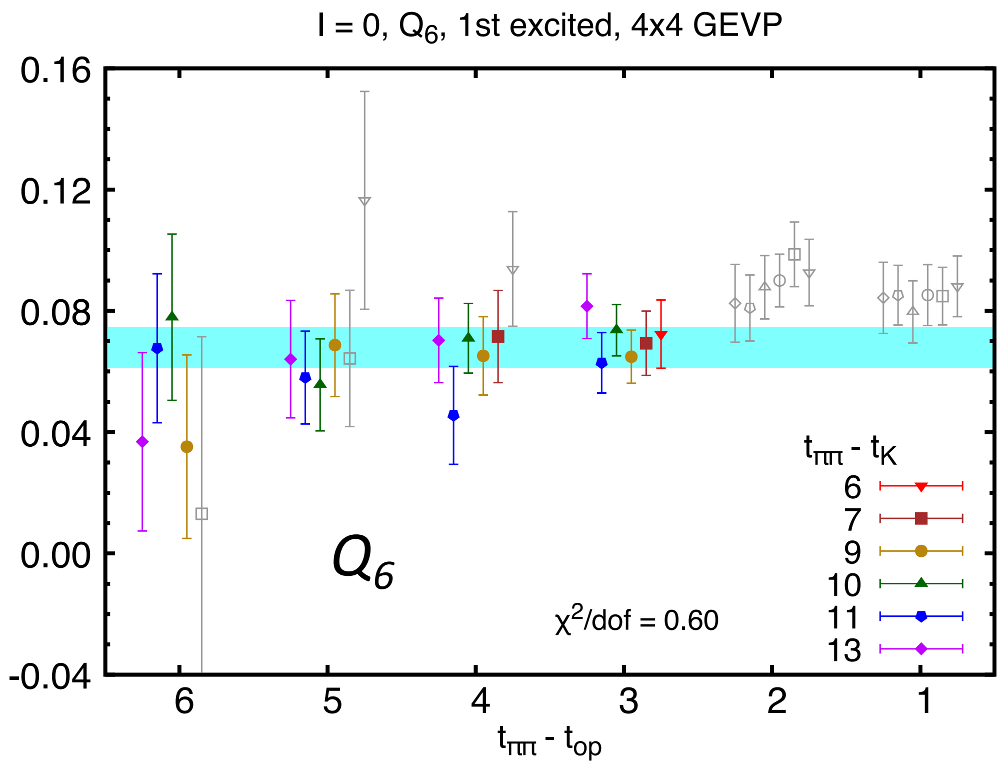
$K \to \pi\pi$ (Masaaki Tomii)

- ✓Use 2-pion states from $\pi\pi$ scattering, *i.e.*, construct linear combinations of correlation functions using coefficients from $\pi\pi$ analysis

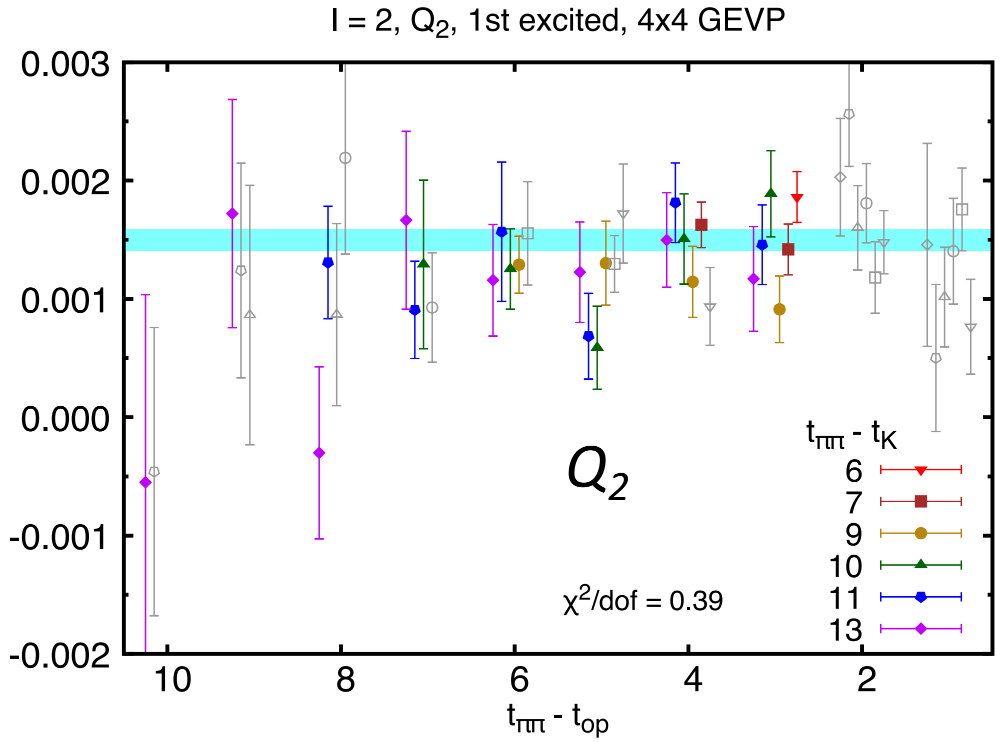
- ✓Could also do GEVP for kaon (under investigation, but not yet)

$K \to \pi\pi$ matrix elements



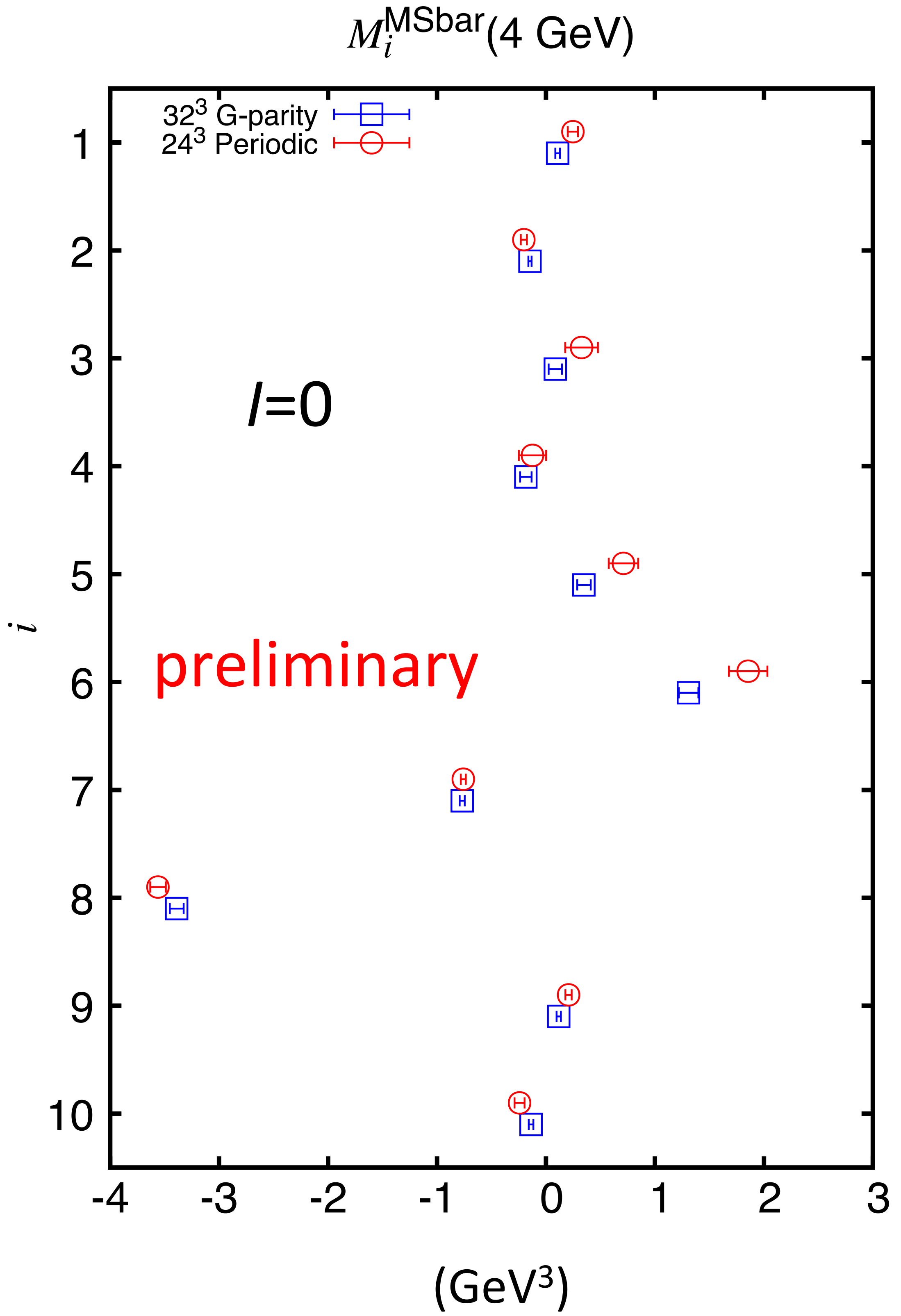


(preliminary)



- Filled symbols used in fit
- Fit to constant
- Several (fixed) $t_{\pi\pi} - t_K$ separations
- unrenormalized
- *I*=2 not optimized (sparsening for *I*=0)

Comparison with G-parity BC

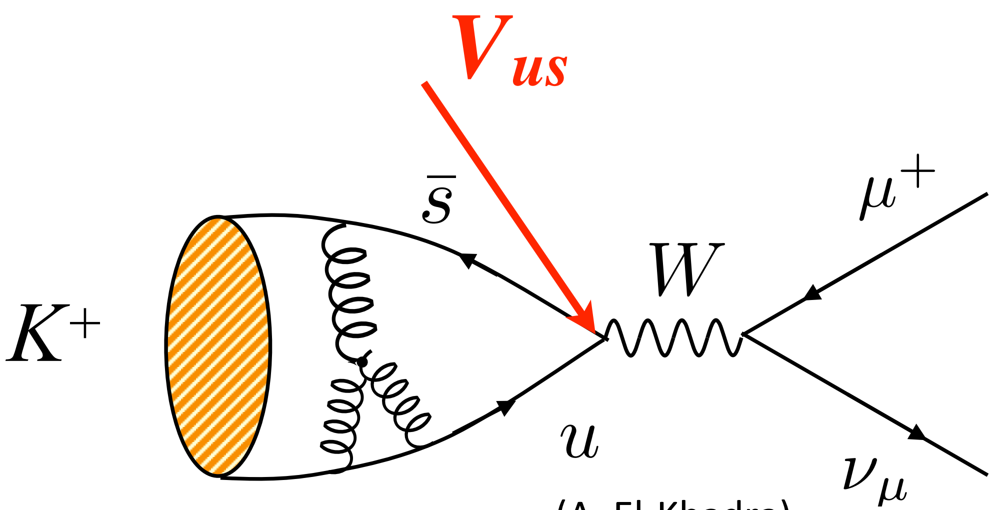


- PBC: 24^3, a^{-1}= 1 GeV
- GPBC: 32^3, a^{-1}= 1.4 GeV (3x stats)
- Renormalized matrix elements
- Consistency is encouraging
- 32^3 in progress
- Results: A_0, A_2, ϵ' at Lattice 2022 in August

Outlook

- GEVP method, with similar statistics, appears promising
- Improving statistics on both ensembles
- Moving to finer lattice spacings (including for G-Parity)
- Method for EM corrections under development– large hurdle cleared with PBC setup
- Apply to next generation of 2+1+1 MDWF ensembles
- 10% precision within next 10 years

Leptonic decay rates from lattice QCD+QED



(A. El-Khadra)

$$\Gamma\left(K^+ \to \mu^+ \nu_\mu\right) \propto (1 + \delta_{EM})|V_{us}|^2 f_{K^+}^2$$

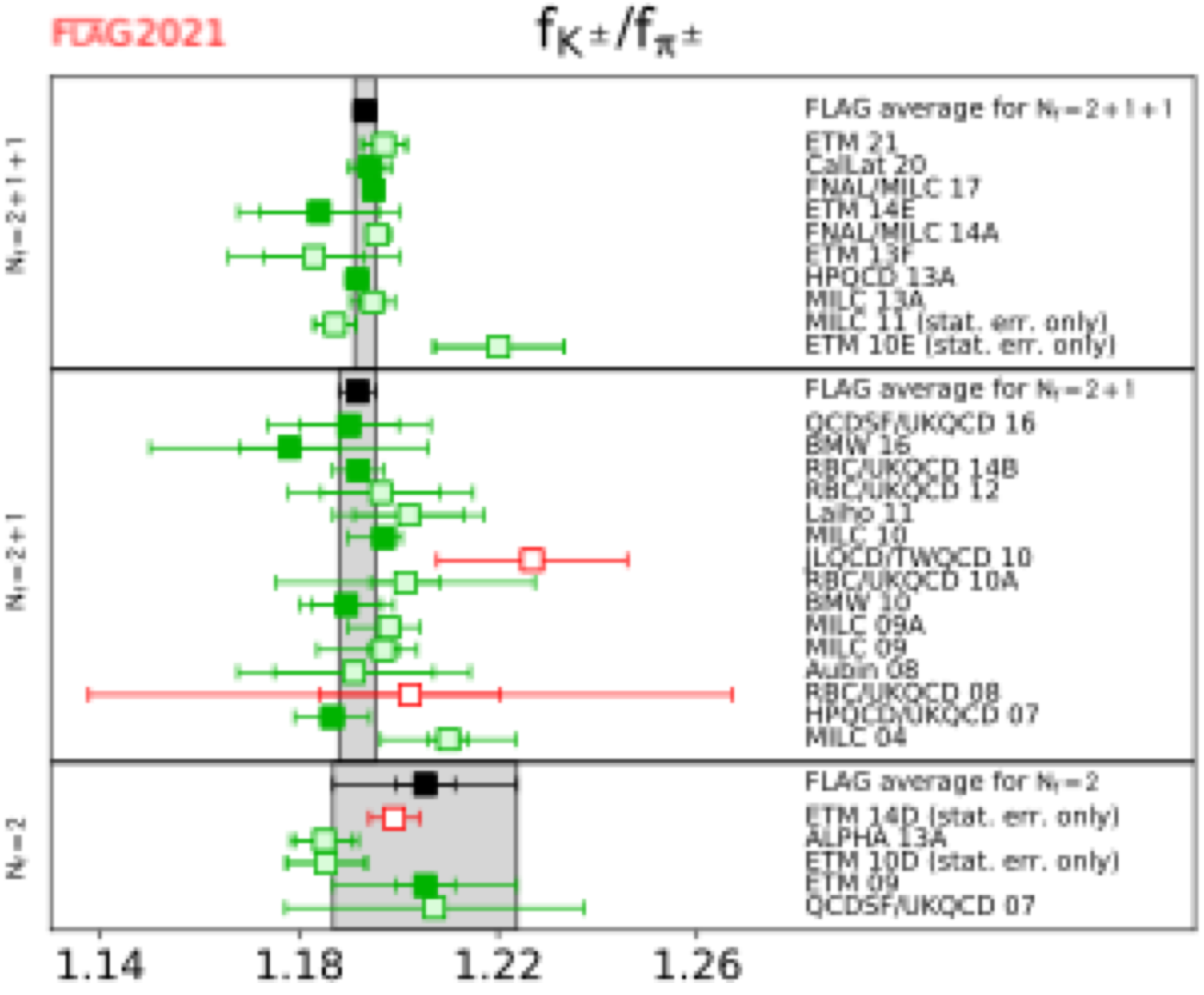
$$\frac{\Gamma(K \to \mu\nu_\mu)}{\Gamma(\pi \to \mu\nu_\mu)} = 1.3367(28)$$ (PDG)

$$\frac{|V_{us}|f_{K^+}}{|V_{ud}|f_{\pi^+}} = 0.27600(37)$$
(ChiPT, Cirigliano, et al, 2012)

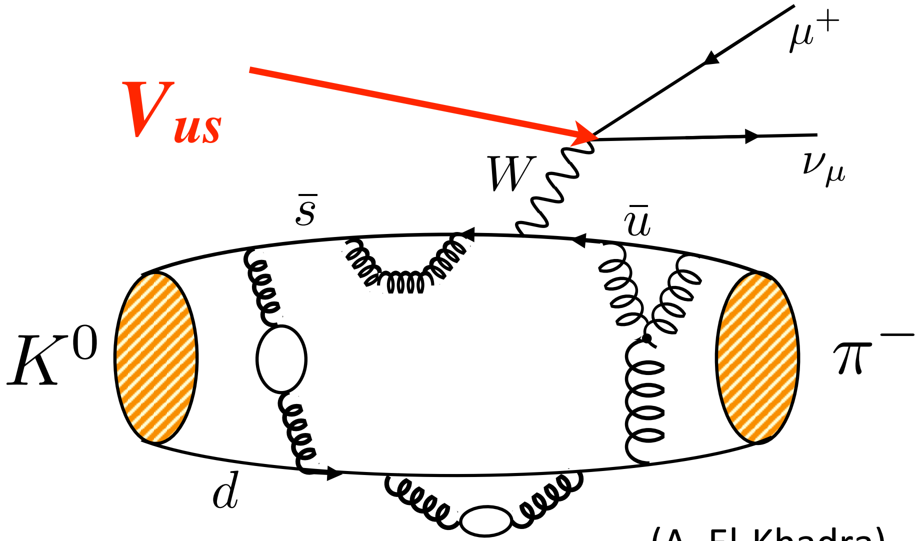
$$\frac{|V_{us}|f_{K^+}}{|V_{ud}|f_{\pi^+}} = 0.27683(29)(20)$$
(1st lattice, Di Caprio, et al, 2019)

$$\frac{|V_{us}|f_{K^+}}{|V_{ud}|f_{\pi^+}} = 0.2760(4)$$
(FLAG 2021)

$$\frac{f_{K^+}}{f_{\pi^+}} = 1.1932(21)$$
(FLAG 2021) 0.18%



Semi-leptonic decay rates from lattice QCD



(A. El-Khadra)

$$\Gamma\left(K^0 \to \pi^- \mu^+ \nu_\mu\right) \propto (1 + \delta_{EM} + \delta_{SU(2)})|V_{us}|^2 f_+^2(0)$$

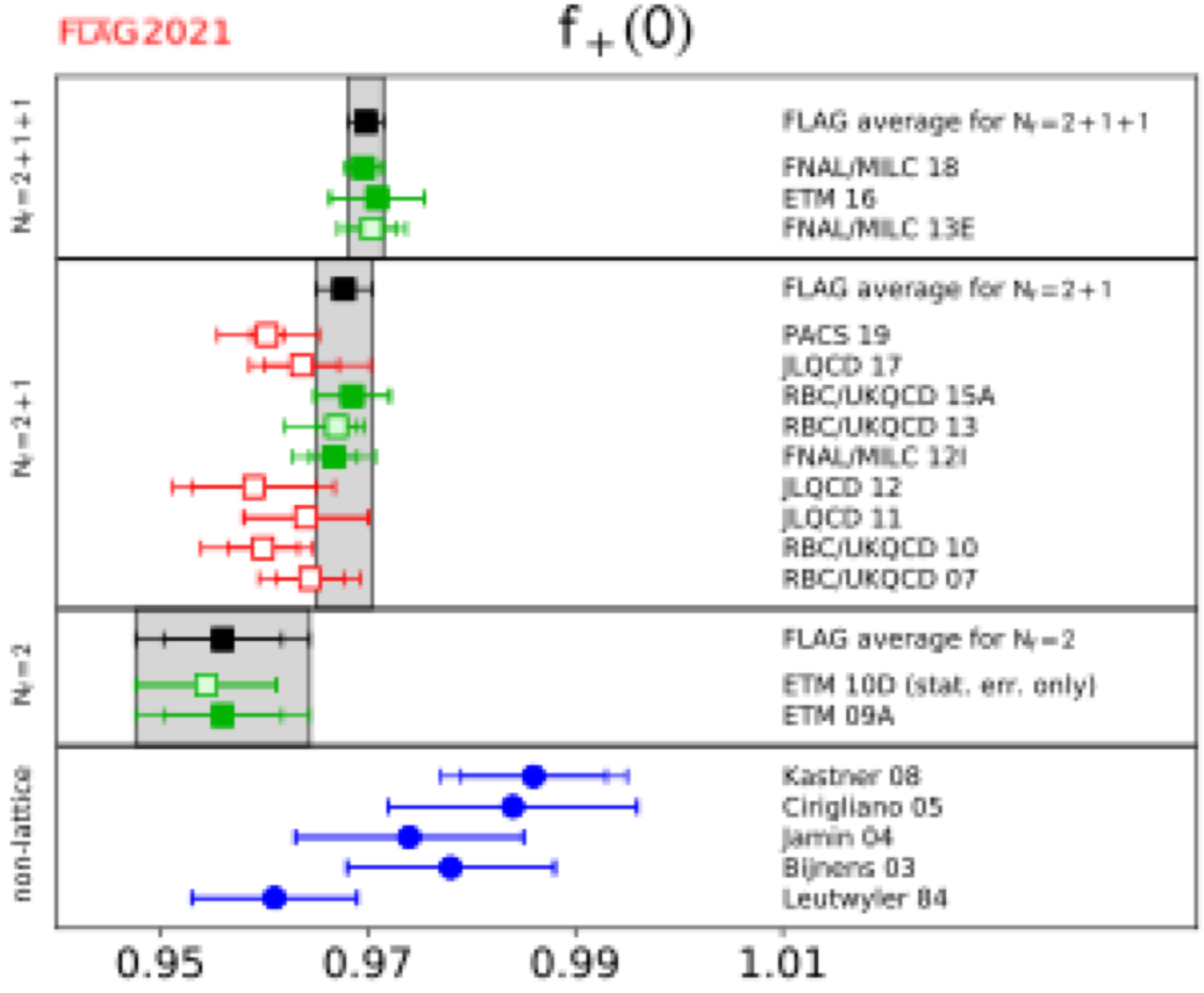
$$|V_{us}|f_+(0) \ = 0.21635(39)(3)$$

(C-Y Seng, et al, 2022)

$$|V_{us}|f_+(0) \ = 0.2165(4)$$

$$f_+(0) \ = 0.9698(17)$$

(FLAG 2021)



Tests of 1st row CKM unitarity

$$\Delta_u \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1$$

$\Delta_u = -0.0021(2)(2)(5)(2)(2)$ $f_+(0)$

(C-Y Seng, et al, 2022)

$\Delta_u = -0.0187(66)$ Lattice only

$\Delta_u = -0.00206(37)$ $f_+(0) + \beta$ decay

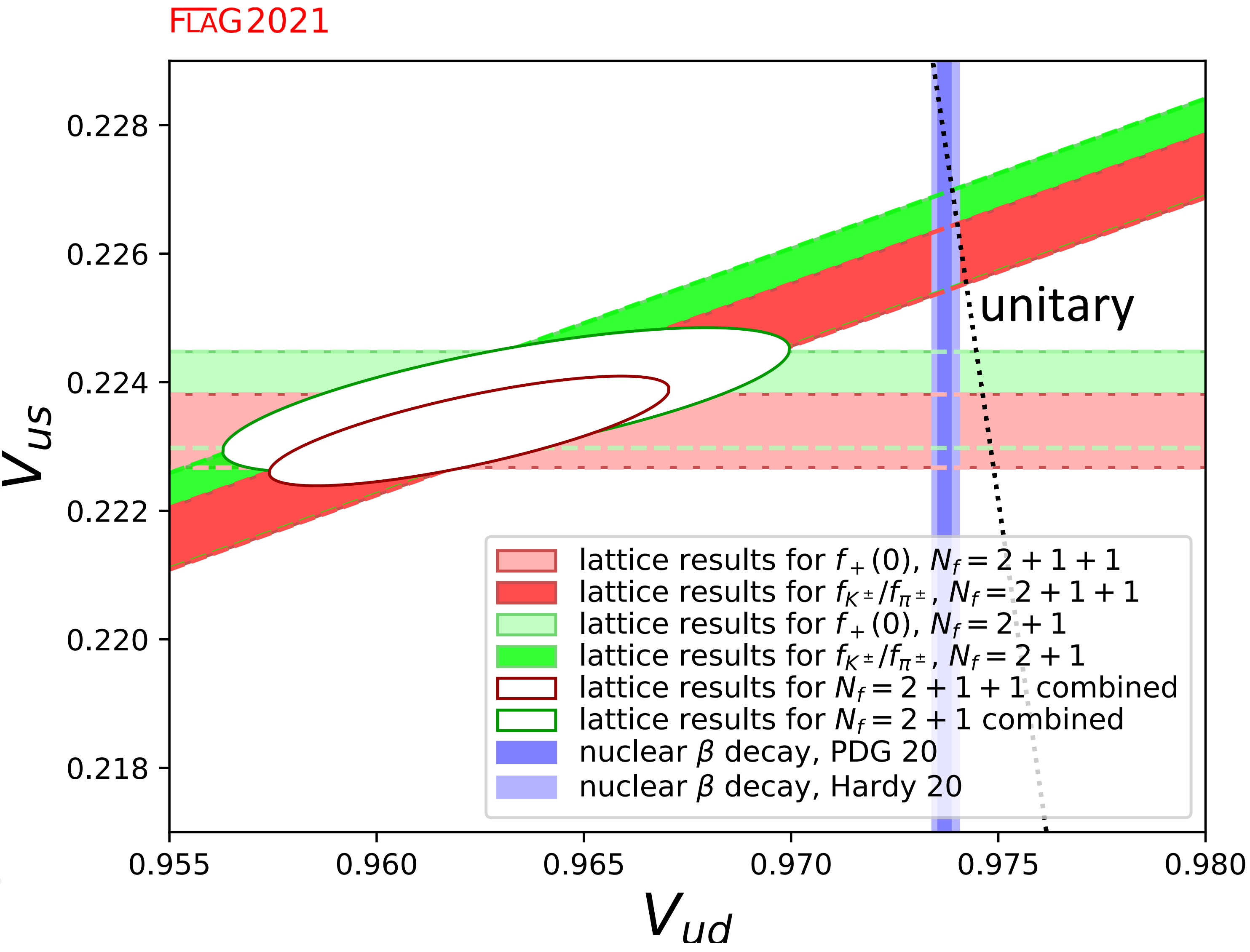
$\Delta_u = -0.00117(37)$

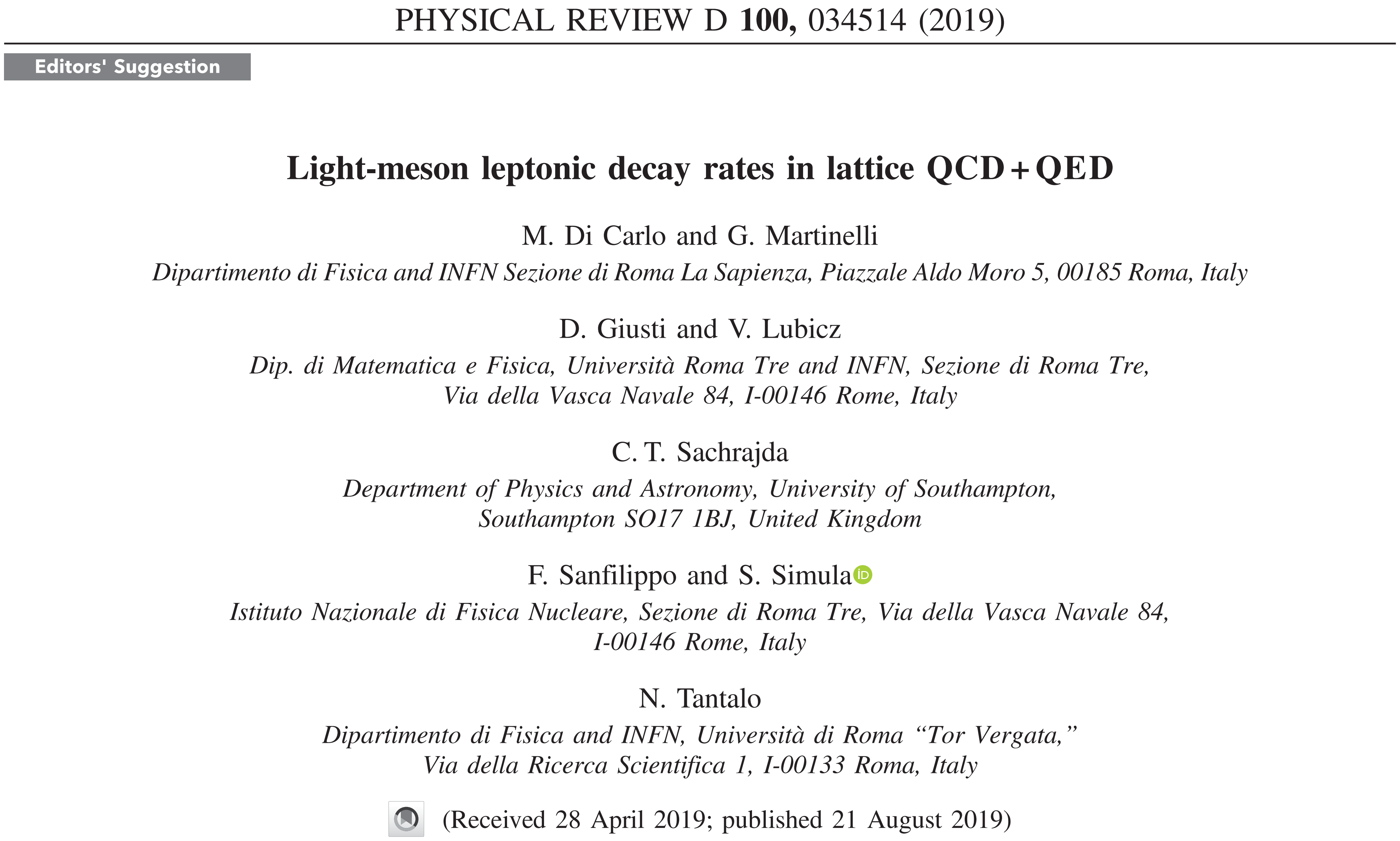
$\Delta_u = -0.00110(68)$

$\dfrac{f_{K^+}}{f_{\pi^+}} + \beta$ decay

(FLAG 2021)

Improve lattice calculations of $\dfrac{f_{K^+}}{f_{\pi^+}}$, $f_+(0)$





PHYSICAL REVIEW D **100,** 034514 (2019)

Editors' Suggestion

Light-meson leptonic decay rates in lattice QCD + QED

M. Di Carlo and G. Martinelli
Dipartimento di Fisica and INFN Sezione di Roma La Sapienza, Piazzale Aldo Moro 5, 00185 Roma, Italy

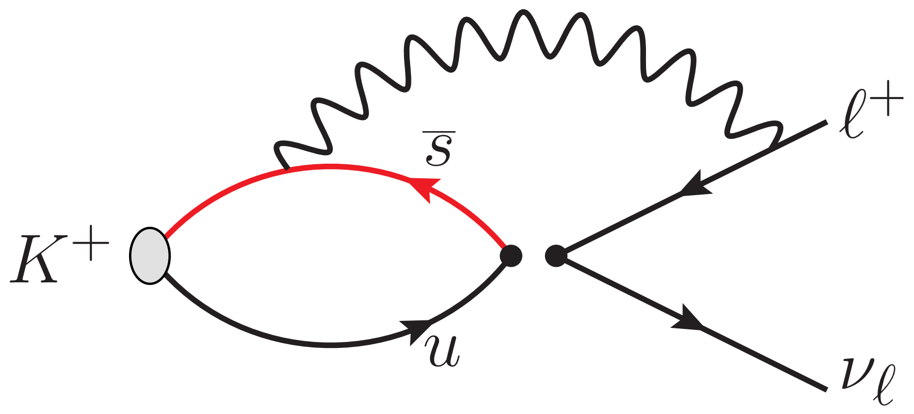
D. Giusti and V. Lubicz
Dip. di Matematica e Fisica, Università Roma Tre and INFN, Sezione di Roma Tre,
Via della Vasca Navale 84, I-00146 Rome, Italy

C. T. Sachrajda
Department of Physics and Astronomy, University of Southampton,
Southampton SO17 1BJ, United Kingdom

F. Sanfilippo and S. Simula
Istituto Nazionale di Fisica Nucleare, Sezione di Roma Tre, Via della Vasca Navale 84,
I-00146 Rome, Italy

N. Tantalo
Dipartimento di Fisica and INFN, Università di Roma "Tor Vergata,"
Via della Ricerca Scientifica 1, I-00133 Roma, Italy

(Received 28 April 2019; published 21 August 2019)

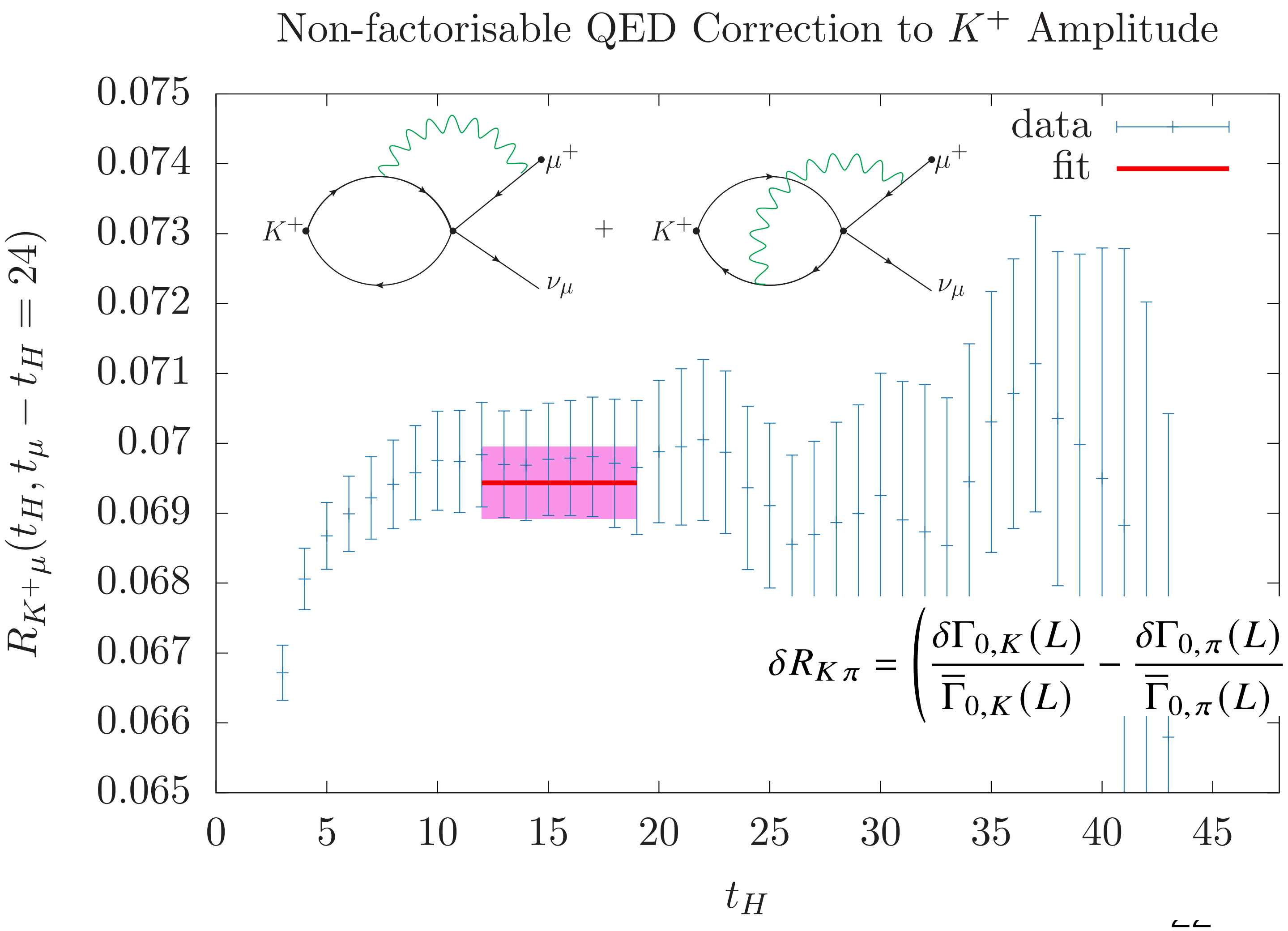


- First lattice QCD+QED calculation
- 1.53(19) % and 0.24(10) % effects on π, K leptonic decay rates
- V_{us}=0.22538(46) (2x red. over PDG)
- QED$_L$ (power law FV effects)

PROCEEDINGS OF SCIENCE

Near-Physical Point Lattice Calculation of Isospin-Breaking Corrections to $K_{\ell 2}/\pi_{\ell 2}$

Andrew Zhen Ning Yong,[a,*] Peter Boyle,[a,b] Matteo Di Carlo,[a] Felix Erben,[a] Vera Gülpers,[a] Maxwell T. Hansen,[a] Tim Harris,[a] Nils Hermansson-Truedsson,[c] Raoul Hodgson,[a] Andreas Jüttner,[d,e] Antonin Portelli[a] and James Richings[a]



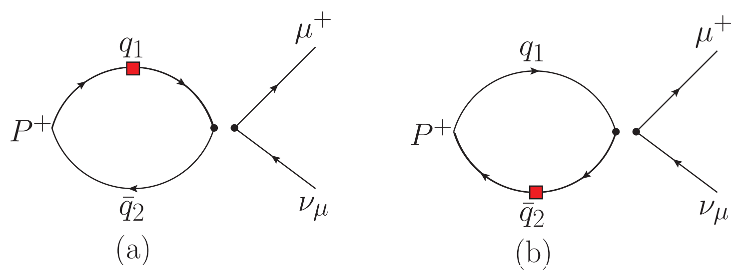


Figure 1: Feynman diagrams of scalar insertions on quark legs (marked with red boxes).

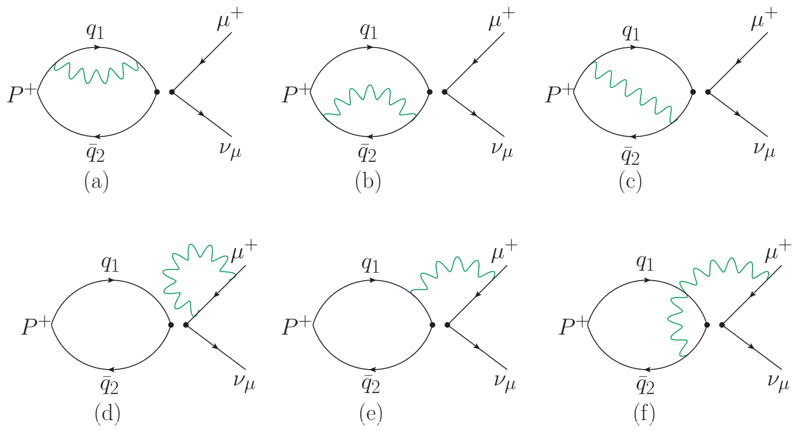


Figure 2: Feynman diagrams of all possible insertions of the electromagnetic current (marked with green squiggle lines) at $O(\alpha)$. QED interactions with sea quarks are neglected.

- Similar to di Carlo, *et al.*
- Domain wall fermions
- New result coming soon

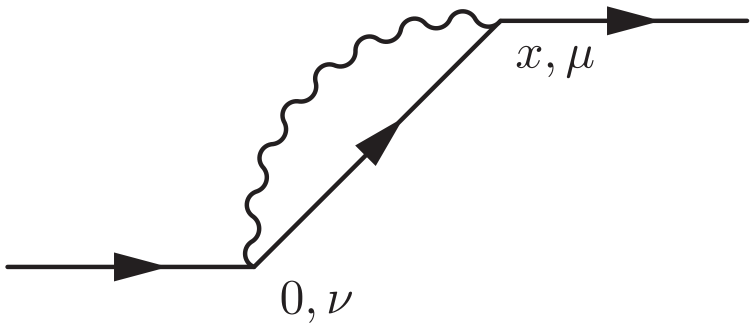
$$\delta R_{K\pi} = \left(\frac{\delta\Gamma_{0,K}(L)}{\overline{\Gamma}_{0,K}(L)} - \frac{\delta\Gamma_{0,\pi}(L)}{\overline{\Gamma}_{0,\pi}(L)}\right) - 2\frac{\alpha}{4\pi}\left(Y_K^{(2)}(L) - Y_\pi^{(2)}(L)\right) + \frac{\alpha}{4\pi}\left(\delta\Gamma_{1,K}(\Delta E) - \delta\Gamma_{1,\pi}(\Delta E)\right)$$

Infinite volume reconstruction for EM corrections

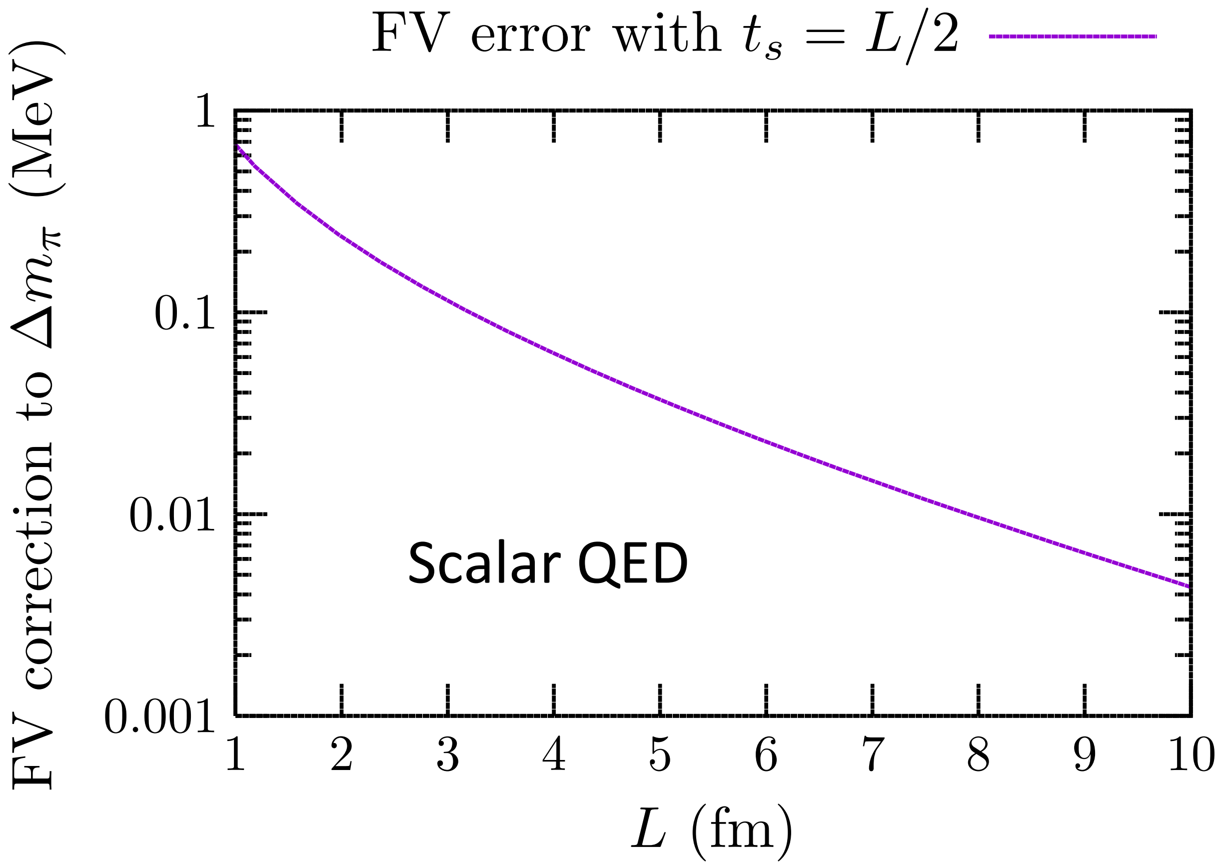
QED self-energies from lattice QCD without power-law finite-volume errors

Xu Feng[1,2,3,4,*] and Luchang Jin[5,6,†]
[1]School of Physics, Peking University, Beijing 100871, China
[2]Collaborative Innovation Center of Quantum Matter, Beijing 100871, China
[3]Center for High Energy Physics, Peking University, Beijing 100871, China
[4]State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China
[5]Physics Department, University of Connecticut, Storrs, Connecticut 06269-3046, USA
[6]RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA

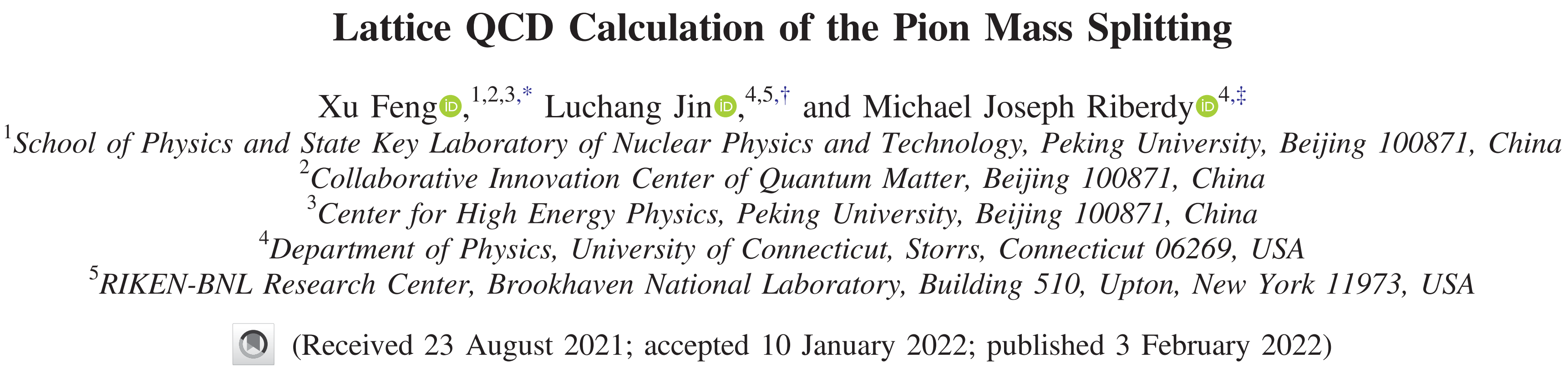
(Received 22 January 2019; published 26 November 2019)



- Naïve treatment: long-distance part has power-law FVE
- Break up time integral for loop into short and long distance pieces (t_s)
- If excited states suppressed: QCD in finite volume, QED treated infinite volume
- FVE are exponentially suppressed, *not power-law*



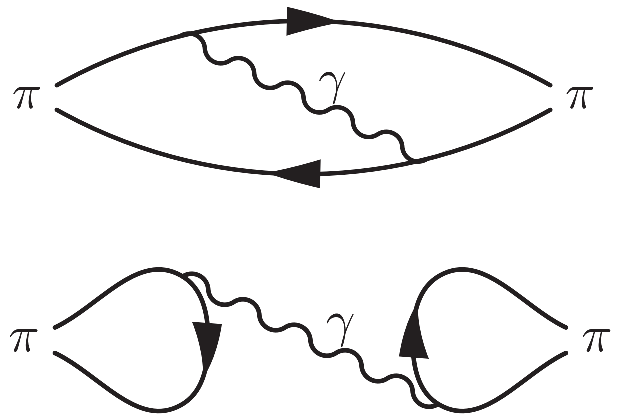
Infinite volume reconstruction: pion mass splitting

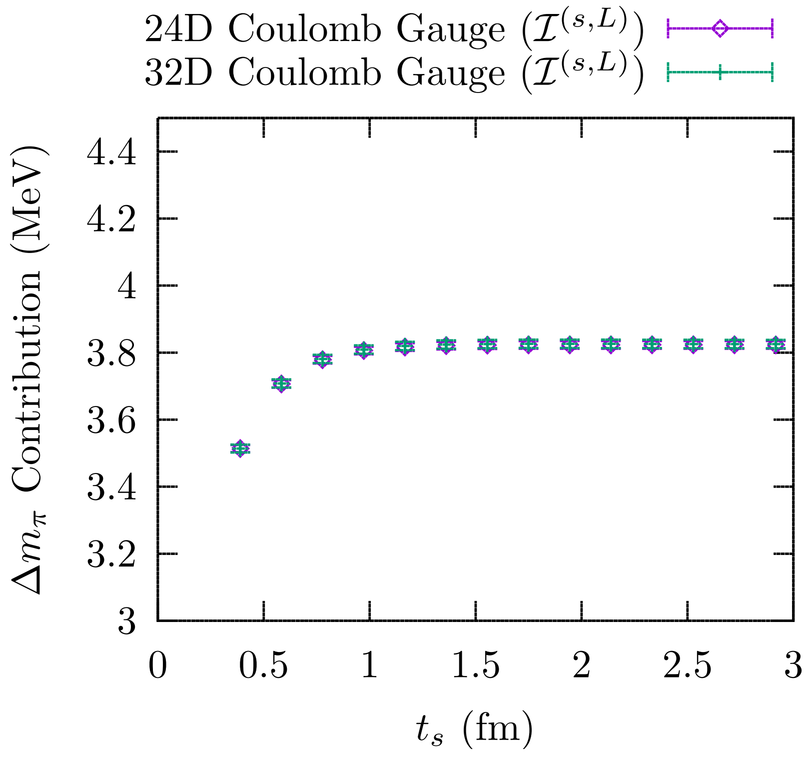


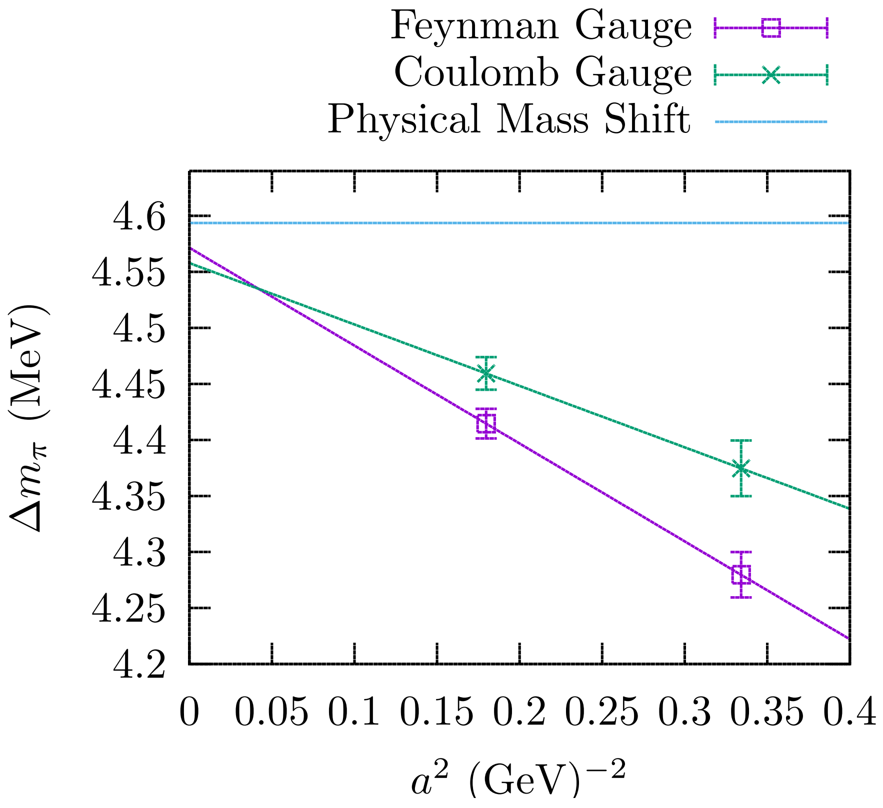
Lattice QCD Calculation of the Pion Mass Splitting

Xu Feng,[1,2,3,*] Luchang Jin,[4,5,†] and Michael Joseph Riberdy[4,‡]
[1] *School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*
[2] *Collaborative Innovation Center of Quantum Matter, Beijing 100871, China*
[3] *Center for High Energy Physics, Peking University, Beijing 100871, China*
[4] *Department of Physics, University of Connecticut, Storrs, Connecticut 06269, USA*
[5] *RIKEN-BNL Research Center, Brookhaven National Laboratory, Building 510, Upton, New York 11973, USA*

(Received 23 August 2021; accepted 10 January 2022; published 3 February 2022)







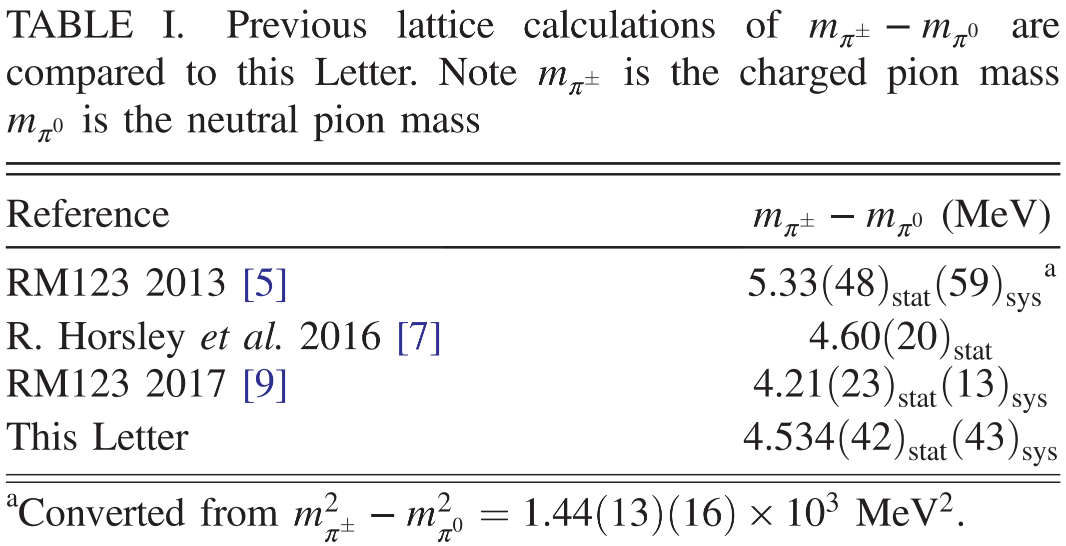


TABLE I. Previous lattice calculations of $m_{\pi^\pm} - m_{\pi^0}$ are compared to this Letter. Note $m_{\pi^\pm}$ is the charged pion mass m_{π^0} is the neutral pion mass

Reference	$m_{\pi^\pm} - m_{\pi^0}$ (MeV)
RM123 2013 [5]	$5.33(48)_{\text{stat}}(59)_{\text{sys}}$ [a]
R. Horsley *et al.* 2016 [7]	$4.60(20)_{\text{stat}}$
RM123 2017 [9]	$4.21(23)_{\text{stat}}(13)_{\text{sys}}$
This Letter	$4.534(42)_{\text{stat}}(43)_{\text{sys}}$

[a] Converted from $m^2_{\pi^\pm} - m^2_{\pi^0} = 1.44(13)(16) \times 10^3$ MeV^2.

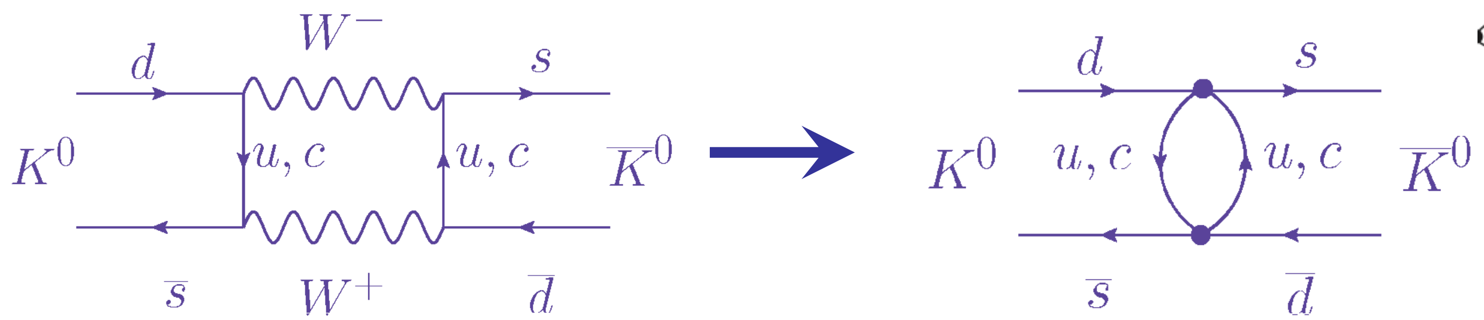
- 5x error reduction
- Pion, kaon leptonic decays soon

Outlook

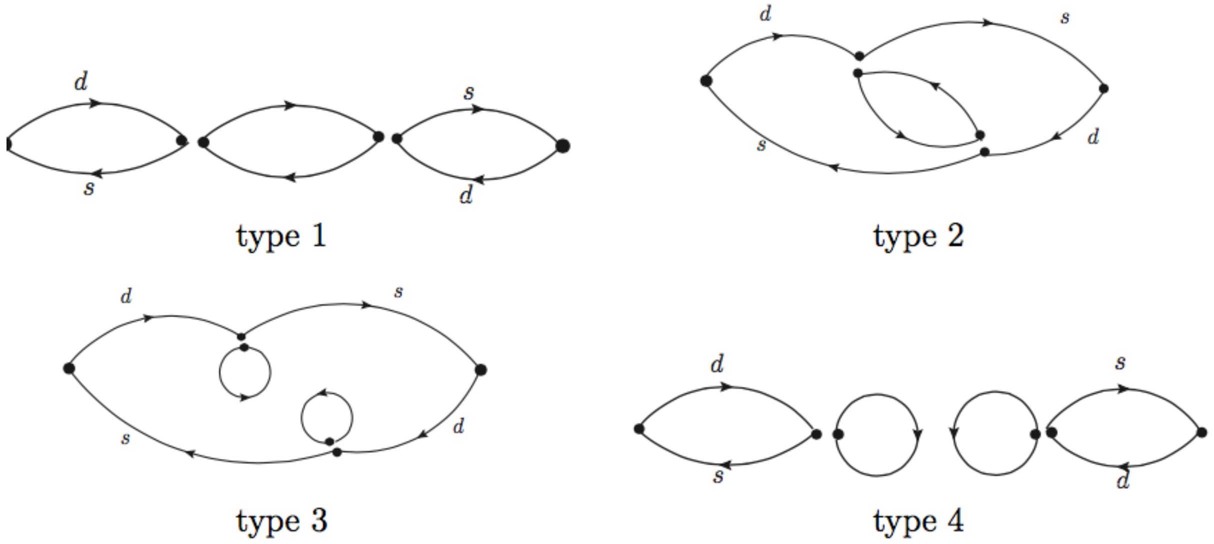
- ✓ 1st row CKM unitarity interesting, needs further attention from lattice community
- ✓ Improvement of lattice calculations of $\frac{f_K}{f_\pi}$, $f_+(0)$ from 0.2% precision → $\mathrm{sub}\ 0.1\ \%$ seems doable
- ✓ Treatment of EM effects important at this level of precision, encouraging progress in several directions

K_L-K_S mass difference Δm_K

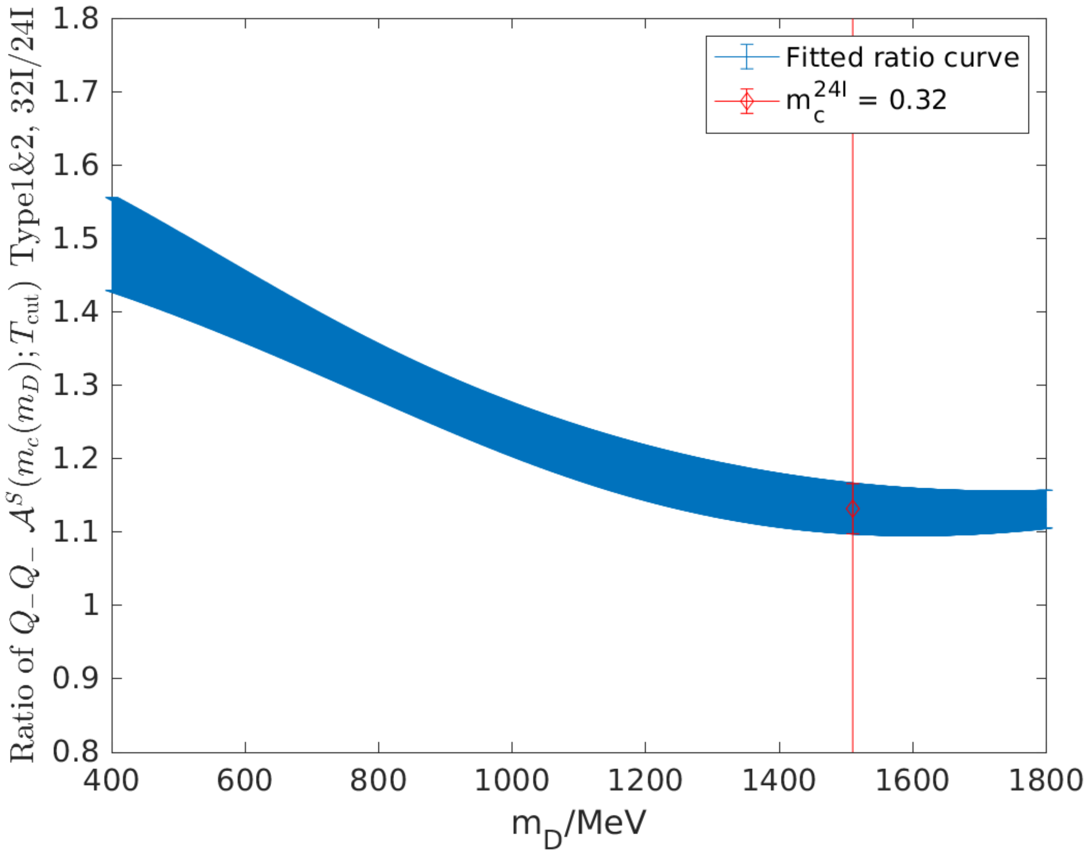
(Bigeng Wang, RBC-UKQCD)



(N. Christ)



- Exp: 3.482(6) × 10^{-12} MeV
- 1000 TeV reach
- Lattice: $5.8(0.6)_{\text{stat}}(2.3)_{\text{sys}}10^{-12}$ MeV
- Single lattice, a^{-1}=2.36 GeV
- Lattice: 10% stat errors, 1% in 10 yrs
- 40% non-zero lattice spacing error biggest hurdle (unexpected)
- c-quark important (GIM)
- Dynamical charm, small *a:* exascale resources needed (coming soon)
- 5% precision goal by 2026

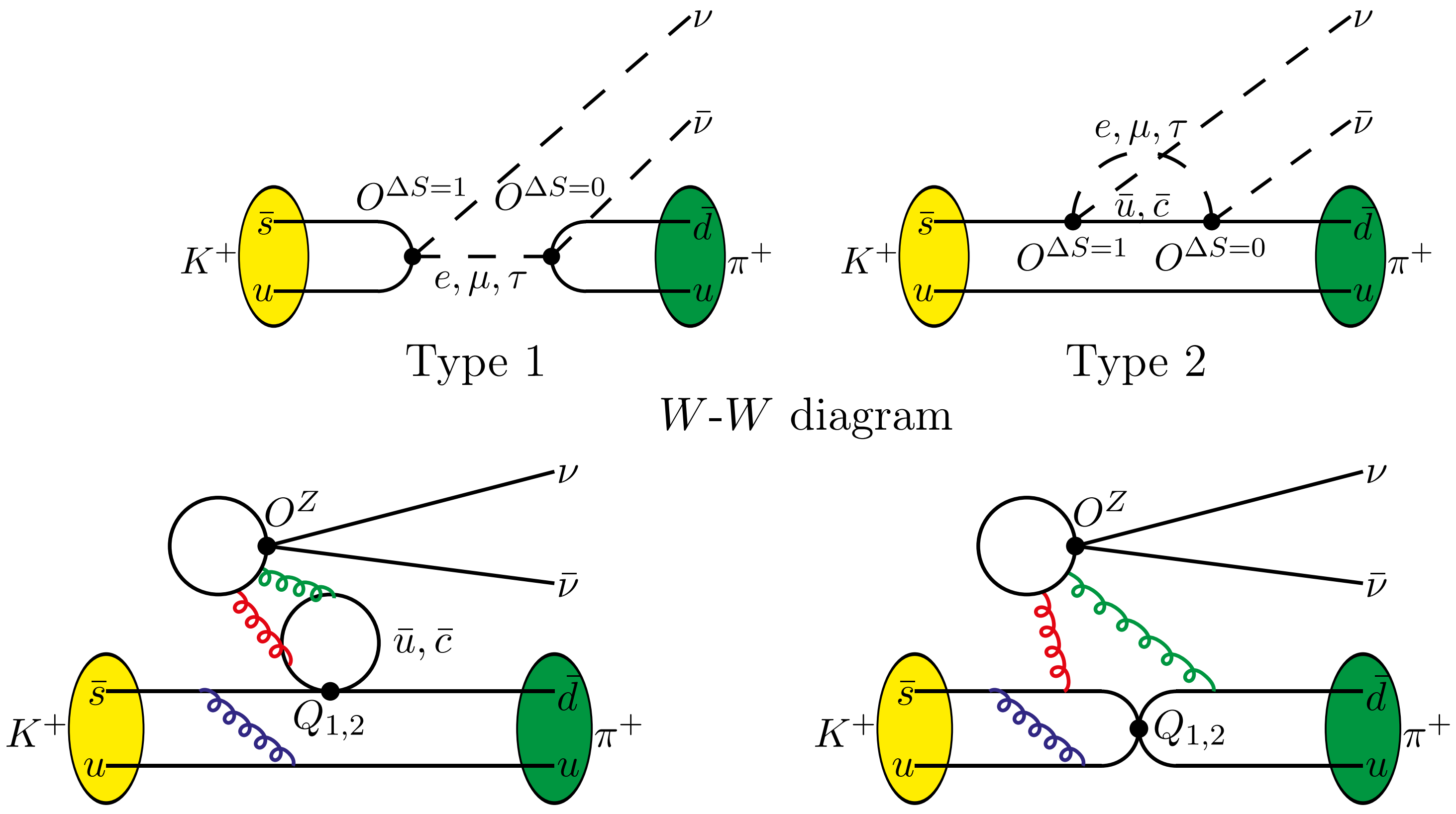


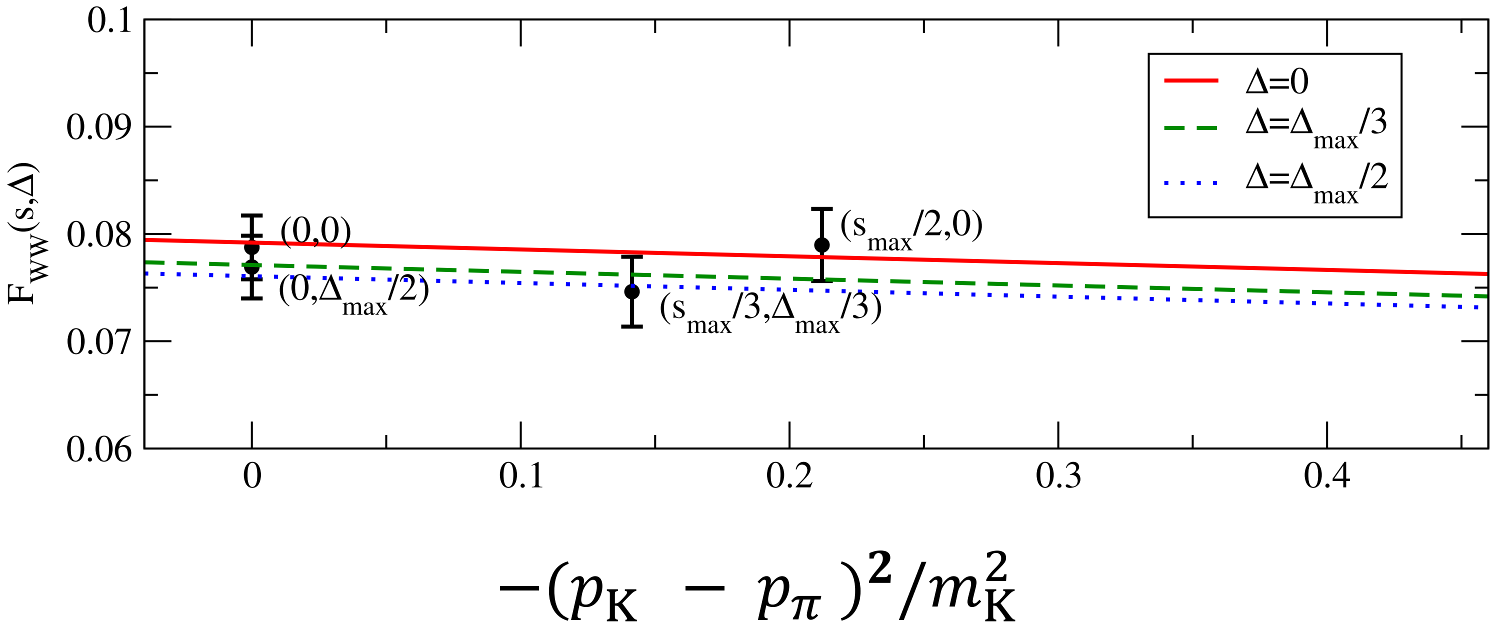
Indirect *CP* violation and ε

- sub-% precision: need ~6 % long-distance contributions
- Closely related to mixing matrix for Δm_K
- Added complication: log divergence, needs non-perturb. Renormalization
- Subtraction of divergence induces new 4-quark operator with low-energy constant computable in QCD PT
- Similar systematics / goals as for Δm_K

$K^+ \to \pi^+\nu\bar{\nu}$ decays [RBC/UKQCD 2016-2019]

- Highly suppressed, good BSM probe
- E949, 2x SM, 60-70% uncertainty
- NA68 10% precision goal
- Theory clean, but long-distance (LD) effects 5-10%
- Developed method for LD effects, completed pilot lattice calculations
- Physical kinematics calculation underway on a^{-1}=2.7 GeV lattice with 30% target precision





Acknowledgements

- Thanks to RBC/UKQCD colleagues, especially Masaaki Tomii and Norman Christ for material for this talk
- TB partially supported by US DOE, Office of Science (HEP)
- Computations done on
  - ALCF at Argonne (MIRA, ALCC), NERSC (Perlmutter Early Science Time), OLCF at Oakridge (INCITE) , USQCD KNL cluster at BNL and JLAB (US DOE)