Theoretical Overview of CP Violation and Mixing in Charm

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Charm CP Violation:

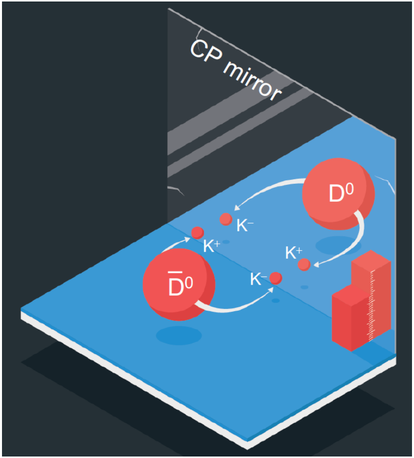
New unique gate to flavor structure of up-type quarks.

[LHCb 1903.08726, HFLAV 2021]

$$a_{CP}^{\text{dir}}(D^0 \to K^+K^-) - a_{CP}^{\text{dir}}(D^0 \to \pi^+\pi^-)$$
$$= (-0.161 \pm 0.028)\% \,.$$

Please note:

This is my personal list, so the overview is biased towards my own work.



Signs of a new era? Anomalies in Flavor Physics

- There are several anomalies. We are not sure what is behind them.
- Semileptonic and rare B decay data: Lepton-flavor non-universality?
- CP is not a fundamental symmetry of nature.
- Therefore, generically, BSM physics will also violate CP.
- If anomalies confirmed: Expect deviations from SM also in CPV.

Outline

- Direct Charm CP Violation
- Charm Mixing and Indirect CP Violation
- Higher Order Sum Rules

Direct Charm CP Violation

Direct CP Violation is an Interference Effect

$$a_{CP}^{\mathrm{dir}}(f) \equiv \frac{|\mathcal{A}(D^0 \to f)|^2 - |\mathcal{A}(\overline{D}^0 \to f)|^2}{|\mathcal{A}(D^0 \to f)|^2 + |\mathcal{A}(\overline{D}^0 \to f)|^2} \approx 2(r_{\mathrm{CKM}} \sin \varphi_{\mathrm{CKM}})\,(r_{\mathrm{QCD}} \sin \delta_{\mathrm{QCD}})\,.$$

f = CP-eigenstate.

The decay amplitude:

$$\mathcal{A} = 1 + r_{\mathrm{CKM}}\, r_{\mathrm{QCD}}\, e^{i(\varphi_{\mathrm{CKM}} + \delta_{\mathrm{QCD}})}$$

- r_{CKM} : real ratio of CKM matrix elements.
- φ_{CKM} : weak phase.
- r_{QCD} : real ratio of hadronic matrix elements.
- δ_{QCD} : strong phase.

Where does the interference come from?

$$D^0 \xrightarrow{V_{cd}^* V_{ud}} \pi^+\pi^-$$

$$D^0 \xrightarrow{V_{cs}^* V_{us}} K^+K^- \xrightarrow{\text{QCD}} \pi^+\pi^-$$

$$D^0 \xrightarrow{V_{cd}^* V_{ud}} \pi^+\pi^- \xrightarrow{\text{QCD}} K^+K^-$$

$$D^0 \xrightarrow{V_{cs}^* V_{us}} K^+K^-$$

$KK \leftrightarrow \pi\pi$ rescattering into same final state.

Weak and strong factors

$$\frac{\mathcal{A}(D \to \pi\pi \to KK)}{\mathcal{A}(D \to KK)} = \left(r_{\text{CKM}} e^{i\varphi_{\text{CKM}}}\right)\left(r_{\text{QCD}} e^{i\delta_{\text{QCD}}}\right)$$

- r_{QCD}: ratio of rescattering amplitudes.
- $\delta_{\text{QCD}} = \mathcal{O}(1)$: strong phase.
- $r_{\text{CKM}} = 1$: ratio of CKM factors, $\left|V_{cd}^* V_{ud} / (V_{cs}^* V_{us})\right|$
- $\varphi_{\text{CKM}} \approx 6 \cdot 10^{-4}$: deviation from 2×2 unitarity.

Prediction

$$\Delta a_{CP}^{dir} \sim 10^{-3} \times r_{\text{QCD}}$$

- U-spin decomposition: $r_{\text{QCD}} = r_{\text{QCD}}^{\Delta U=0} \equiv \mathcal{A}^{\Delta U=0} / \mathcal{A}^{\Delta U=1}$.

“$\Delta U = 0$ rule”: $r_{\text{QCD}} \sim 1$ [Grossman StS 1903.10952]

- We claim $\Delta U = 0$ follows similar pattern as generalized $\Delta I = 1/2$ rule.
- Both due to low energy QCD, rescattering.

“$\Delta I = 1/2$ rules” for isospin in $P^+ \to \pi^+\pi^0$, $P^0 \to \pi^+\pi^-$, $P^0 \to \pi^0\pi^0$

- Relevant ratio of strong isospin matrix elements:

$r_{QCD}^{\Delta I=1/2} \equiv A^{\Delta I=1/2}/A^{\Delta I=3/2}$	Kaon	Charm	Beauty
Data	22	2.5	1.5
“No QCD” limit	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$
Enhancement	$\mathcal{O}(10)$	$\mathcal{O}(1)$	$\mathcal{O}(\alpha_s)$

[D: Franco Mishima Silvestrini 2012, B: Grinstein Pirtskhalava Stone Uttayarat 2014]

- Rescattering most important in K decays, less important but still significant in D decays, and small in B decays.

Comparison of approaches: What is r_{QCD}?

Data

Assuming the SM, and $\delta_{\text{QCD}} = \mathcal{O}(1)$, the data implies $r_{\text{QCD}}^{\Delta U=0} \sim 1$.

Ref.	Theory Method/Assumptions	$r_{QCD}^{\Delta U=0}$	SM/NP
[Grossman StS 1903.10952]	Analogy to $\Delta I = 1/2$ rules Low energy QCD, rescattering is $\mathcal{O}(1)$	$\mathcal{O}(1)$	SM
[Brod Kagan Zupan 1111.5000]	Phenomenological analysis	$\mathcal{O}(1)$	SM
[Soni 1905.00907, StS Soni 2110.07619]	Resonance model	$\mathcal{O}(1)$	SM
[Petrov Khodjamirian 1706.07780] [Chala Lenz Rusov Scholtz 1903.10490]	Light Cone Sum Rules Resonances in principle incorporable.	$\mathcal{O}(\alpha_s/\pi)$	NP

What next? Apply methods to $\Delta I = 1/2$ rule in charm!

Reproduction of $\Delta I = 1/2$ crucial for NP case in $\Delta U = 0$.

The jury is still out: Is it SM or not?

- No matter what it is, we learn sth new.
- We have a good argument why it is QCD.
- Assumption of large rescattering at low energy agrees with the data.

$$\text{Loop/Tree} = \mathcal{O}(1)$$

Key insight: Charm is not heavy.

A_{CP} Sum Rules: Overconstrain the SM

Challenge for predicting CP asymmetries

- New hadronic quantities appear.
- These cannot be extracted from $\mathcal{B}$ measurements.

Solution

Make up $\mathrm{SU}(3)_F$ sum rules in which these cancel.

$\mathrm{SU}(3)_F$ limit sum rules

$$a_{CP}^{\mathrm{dir}}(D^0 \to \pi^+\pi^-) + a_{CP}^{\mathrm{dir}}(D^0 \to K^+K^-) = 0\,,$$
$$a_{CP}^{\mathrm{dir}}(D_s^+ \to K_S\pi^+) + a_{CP}^{\mathrm{dir}}(D^+ \to K_SK^+) = 0\,.$$

Key Measurements for $D \to PP'$.

A_{CP} sum rules including breaking effects [Müller Nierste StS 1506.04121]

- SM sum rule 1: $D^0 \to K^+K^-$, $D^0 \to \pi^+\pi^-$, $D^0 \to \pi^0\pi^0$.
- SM sum rule 2: $D^+ \to K_SK^+$, $D_s^+ \to K_S\pi^+$, $D_s^+ \to K^+\pi^0$.

Isospin Analysis [Grossman Kagan Zupan 1204.3557]

- Extract $\Delta I = 1/2$ and $\Delta I = 3/2$ MEs from

$$D^0 \to \pi^+\pi^-, D^+ \to \pi^+\pi^0, D^0 \to \pi^0\pi^0.$$

- $a_{CP}^{\text{dir}}(D^+ \to \pi^+\pi^0) = 0.$ Higher orders < sensitivity.

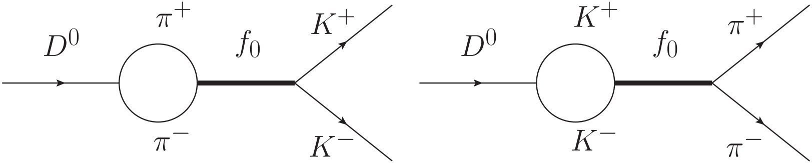
What next?

- Measurements of CP asymmetries in all SCS $D \to PP'$ decays.
- Need sum rules for multi-body decays at higher order in $SU(3)_F$.

What next? Check dynamical mechanism from data.

$$D^0 \xrightarrow{V_{cd}^* V_{ud}} \pi^+\pi^-$$

$$D^0 \xrightarrow{V_{cs}^* V_{us}} K^+K^- \xrightarrow{\text{QCD}} \pi^+\pi^-$$



Assumptions

[StS and A. Soni, 2110.07619]

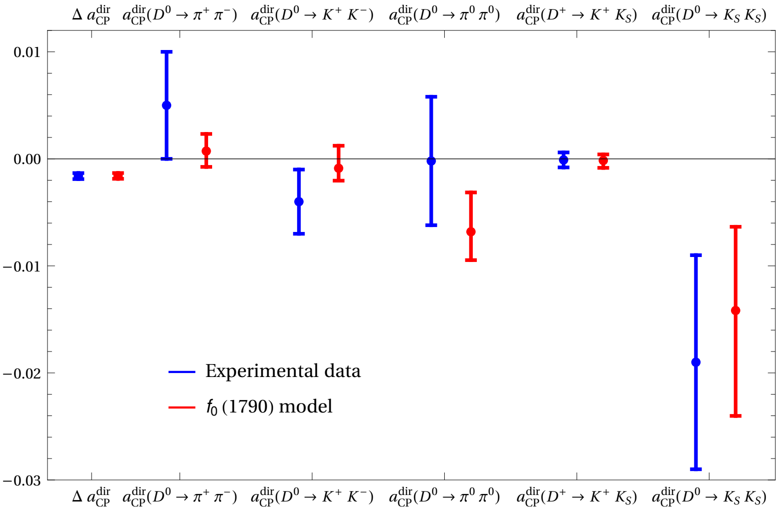
- Amplitudes to $I = 0$ states dominated by f_0 close to D^0 mass.
- Amplitudes into $I = 1$ states relatively suppressed.

Resonance structure can also be incorporated in future LCSR calculations.

[Khodjamirian Petrov 1706.07780]

Predictions in Scalar Resonance Model

[StS and A. Soni, 2110.07619]



What next? Study of $\Delta U = 0$ in three-body decays

[Dery Grossman StS Soffer 2101.02560]

$$\mathcal{A}(D^0 \to \pi^+\rho^-) = -\lambda\, T^{P_1V_2} - V_{cb}^* V_{ub}\, R^{P_1V_2}$$
$$\mathcal{A}(D^0 \to \pi^-\rho^+) = -\lambda\, T^{P_2V_1} - V_{cb}^* V_{ub}\, R^{P_2V_1}$$

- Time-integrated CP asym. of 2-body decays give only combinations

$$|\widetilde{R}^{P_1V_2}|\sin(\delta_{P_1V_2}) \qquad \text{and} \qquad |\widetilde{R}^{P_2V_1}|\sin(\delta_{P_2V_1})\,,$$

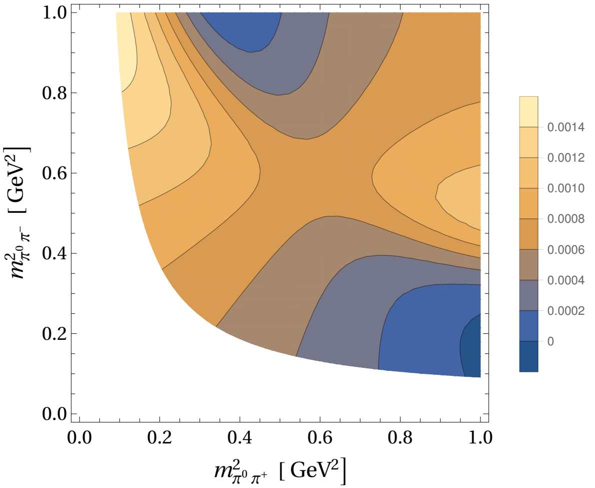
  but **not** magnitudes and phases separately.

- Three body decay changes 2 things:
  - We have additional kinematic dependences.
  - Only in a three-body decay we have interference between $D^0 \to \pi^+(\rho^- \to \pi^-\pi^0)$ and $D^0 \to \pi^-(\rho^+ \to \pi^+\pi^0)$.

↳ Extraction of all parameters from time-integrated CP meas.

Local $a_{CP}^{\text{dir}}(D^0 \to \pi^+\pi^-\pi^0)$ in overlap region of $\rho^\pm$

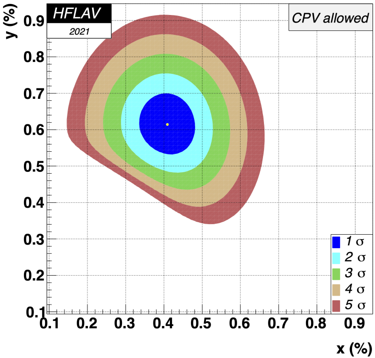
[Dery Grossman StS Soffer 2101.02560]

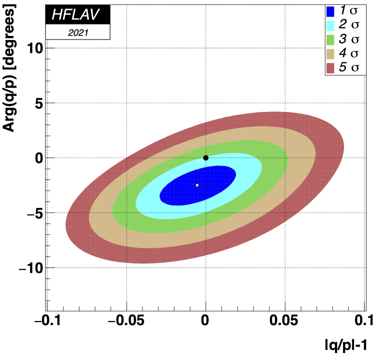


Numerical example: $\widetilde{R}^{P_1V_2} = \exp(i\pi/2)$, $\widetilde{R}^{P_2V_1} = \frac{1}{4}\exp(i\pi/3)$

Charm Mixing and Indirect CP Violation

Charm Mixing





- Mixing parameters $x \equiv \Delta m/\Gamma$ and $y \equiv \Delta\Gamma/(2\Gamma)$.
- 2021: First observation of $x \neq 0$ with $> 7\sigma$. [LHCb 2106.03744].
- Uncertainty of y reduced by a factor two in [LHCb 2110.02350].
- $|q/p| \neq 1$ would indicate CPV in mixing.
- $\mathrm{Arg}(q/p) \neq 0$ would indicate CPV from interference mixing/decay.
- SM: hard to calculate. Qualitative agreement with SM.

Exclusive Approach: Hadron-Level

$$\Gamma_{12}^{D} = \sum_{n} \rho_n \left\langle \overline{D^0} \right| \mathcal{H}_{eff}^{\Delta C=1} |n\rangle \langle n| \mathcal{H}_{eff}^{\Delta C=1} \left| D^0 \right\rangle ,$$

$$M_{12}^{D} = \sum_{n} \left\langle \overline{D^0} \right| \mathcal{H}_{eff}^{\Delta C=2} \left| D^0 \right\rangle + \mathcal{P} \sum_{n} \frac{\left\langle \overline{D^0} \right| \mathcal{H}_{eff}^{\Delta C=1} |n\rangle \langle n| \mathcal{H}_{eff}^{\Delta C=1} \left| D^0 \right\rangle}{m_D^2 - E_n^2}$$

- n: all possible hadronic states. ρ_n: density of state. $\mathcal{P}$: principal value.
- Result: $y \sim 1\%$, agreeing with measurements.

What next?

- More experimental input needed (BRs and phases).
- Theory: Need to take into account more $SU(3)_F$ breaking effects.
- Long-term: Lattice predictions?

Inclusive Approach: Quark-Level

- Heavy-Quark Expansion (HQE), motivated by $\tau(D^+)/\tau(D^0)$.
- Needed non-perturbative matrix elements from sum rules or Lattice
- Severe GIM-cancellations may take place.

Recent Developments

[Lenz Piscopo Vlahos 2007.03022]

- GIM depends on scales entering different box contributions. These contain different amounts of strangeness.
- No need that these scales are the same ⇒ GIM cancellation broken.
- HQE uncertainty gets larger, including y^{exp}.

What next?

- Higher orders in HQE expansion.
- After Γ_{12} also M_{12}, e.g. with dispersion relations.

Higher Order Sum Rules

SU(3)-flavor

- SU(3): Approximate symmetry for the light quarks u, d, s.
- Very useful, but $\mathcal{O}(30\%)$ breaking from corrections.
- Going to higher order: complicated.

$$(\mathbf{15}) \otimes (\mathbf{8}) = (\mathbf{42}) \oplus (\mathbf{24}) \oplus (\mathbf{15_1}) \oplus (\mathbf{15_2}) \oplus (\mathbf{15'}) \oplus (\bar{\mathbf{6}}) \oplus (\mathbf{3})$$

$$(\bar{\mathbf{6}}) \otimes (\mathbf{8}) = (\mathbf{24}) \oplus (\mathbf{15}) \oplus (\bar{\mathbf{6}}) \oplus (\mathbf{3})$$

Decay d	$B_1^{3_1}$	$B_1^{3_2}$	$B_8^{3_1}$	$B_8^{3_2}$	$B_8^{\bar{6}_1}$	$B_8^{\bar{6}_2}$	$B_8^{15_1}$	...
$D^0 \to K^+K^-$	$\frac{1}{4\sqrt{10}}$	$\frac{1}{8}$	$\frac{1}{10\sqrt{2}}$	$\frac{1}{4\sqrt{5}}$	$\frac{1}{10}$	$-\frac{1}{10\sqrt{2}}$	$-\frac{7}{10\sqrt{122}}$	...
$D^0 \to \pi^+\pi^-$	$\frac{1}{4\sqrt{10}}$	$\frac{1}{8}$	$\frac{1}{10\sqrt{2}}$	$\frac{1}{4\sqrt{5}}$	$-\frac{1}{10}$	$\frac{1}{10\sqrt{2}}$	$-\frac{11}{10\sqrt{122}}$	...
$D^0 \to \bar{K}^0K^0$	$-\frac{1}{4\sqrt{10}}$	$-\frac{1}{8}$	$\frac{1}{5\sqrt{2}}$	$\frac{1}{2\sqrt{5}}$	0	0	$-\frac{9}{5\sqrt{122}}$	...
$D^0 \to \pi^0\pi^0$	$-\frac{1}{8\sqrt{5}}$	$-\frac{1}{8\sqrt{2}}$	$-\frac{1}{20}$	$-\frac{1}{4\sqrt{10}}$	$\frac{1}{10\sqrt{2}}$	$-\frac{1}{20}$	$\frac{11}{20\sqrt{61}}$	...
...	...	...	...	...	...	...	...	...

Solving the Problem of Higher Order U-spin

[Gavrilova Grossman StS, 2205.soon]

We proved several theorems enabling calculations to arbitrary order.

- We are able to determine a priori up to which order sum rules exist.
- We do not need explicit Clebsches. Big complexity reduction.
- Hope: Opens the door for precision in hadronic decays.
- Close a gap between theory and experiment.

Take advantage of precision data on nonleptonic decays.

Systematics of U-spin breaking

- U-spin breaking from mass difference of strange and down quarks:

$$\varepsilon = \frac{m_s - m_d}{\Lambda_{\rm QCD}} \sim 0.3\,.$$

- Parametrized by triplet-operator H_ε:

$$\mathcal{H}_{\rm eff} = \sum_{m,b} f_{u,m} \left(H^u_m \otimes H_\varepsilon^{\otimes b}\right)\,, \qquad H_\varepsilon^{\otimes b} \equiv \underbrace{H_\varepsilon \otimes \cdots \otimes H_\varepsilon}_{b}\,.$$

- Any system can be constructed from tensor products of doublets.
- Moving irreps (“crossing sym.”) does not affect structure of sum rules.
- Without loss of generality, consider doublet-only system with

$$0 \to \left(\frac{1}{2}\right)^{\otimes n}$$ and singlet Hamiltonian.

Properties of U-spin pairs

[Gavrilova Grossman StS, 2205.soon]

- Amplitude:

$$A_j = \underbrace{(-,-,+,-,+,\ldots,+)}_{n} = \sum_\alpha C_{j\alpha} X_\alpha\,.$$

- U-spin conjugated amplitude (complete interchange $s \leftrightarrow d$):

$$\overline{A}_j = \underbrace{(+,+,-,+,-,\ldots,-)}_{n} = (-1)^p \sum_\alpha (-1)^b C_{j\alpha} X_\alpha\,.$$

- Notation: Abbreviate m-quantum number: $\pm 1/2 \mapsto \pm$.
  X_α: Reduced matrix element. $C_{j\alpha}$: Clebsches.
- Define (anti-)symmetric combinations of U-spin pairs:

$$a_j \equiv \underbrace{A_j - (-1)^p \overline{A}_j}_{\text{odd in } b}\,, \qquad s_j \equiv \underbrace{A_j + (-1)^p \overline{A}_j}_{\text{even in } b}\,.$$

Results: Sum Rules at any order of U-spin breaking

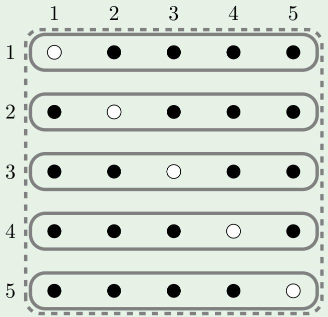
[Gavrilova Grossman StS, 2205.soon]

All sum rules at any order b can be written as:

$$\sum_j a_j = 0\,, \qquad \sum_j s_j = 0\,.$$

Example: $n = 6$ doublets. Dimension of lattice $d = n/2 - 1 = 2$.

- Each node $\Leftrightarrow$ U-spin pair.
- Each node (points): a-type sum rule valid to $b = 0$.
- Sums of nodes in lines: s-type sum rules valid to $b = 1$.
- Sum of all nodes in plane: a-type sum rule valid up to $b = 2$.



This is just the beginning of the exploration of charm CPV

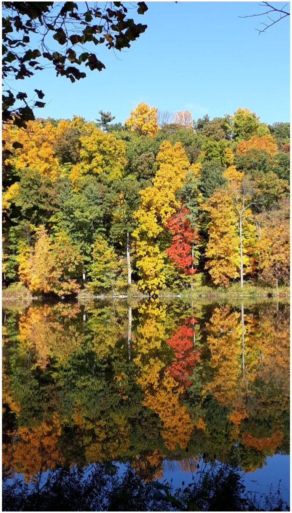
- Crucial: CP asymmetries of all SCS two-body charm decays.
- Necessary to benefit from insights of flavor symmetry sum rules.
- Most promising for next observation: $D \to K_S K_S$ and $D \to K K^*$.
- Test picture of flavor symmetry breaking: at expected level (30%)?
- Important to search for optimized observables for multi-body decays.
  How can we maximize sensitivity to CP violation?
  What is the smartest binning for multi-body decays?
- How can we formally account for the phase space effects when comparing Dalitz plots that are related by flavor symmetries?

Conclusions

- So much more data and theory ideas: New era in flavor physics.
- We need to keep:

  Theory error < Experimental error .

- No matter what, we will learn sth new: QCD or New Physics.



BACK-UP

Charm: Non-perturbative Diagrams