Muon g-2 and EDM Experiments as Dark Matter Detectors

Ryan Janish

(Fermilab)

[RJ & Ramani, 2006.10069]

Muonic Dark Matter

We lack direct measurements of DM-muon couplings

Limits exist from astrophysics and cosmology

[Dror et al, 1909.12845], [Croon et al, 2006.13942], [Grifols and Masso, 9610205]

Limits may be inferred from virtual muons

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Limits may be inferred from virtual muons

g-2 and EDM precession data can be used to detect or directly constraint DM-muon interactions

DM may provide a background field that drives distinct spin precession trajectories.

DM-muon interactions may explain the g-2 anomaly without modification to the intrinsic muon g-factor

g-2 and EDM Experiments

Measure the precession frequency $\vec{\omega}_a$ in a known background field

Magnetic dipole measurement (g – 2)

$$\vec{\omega}_a = -\frac{e}{2m_\mu}\left(g_\mu - 2\right)\vec{B}$$

Measure $|\vec{\omega}_a|$

[Albahri et al, 2104.03247]

Electric dipole measurement (EDM)

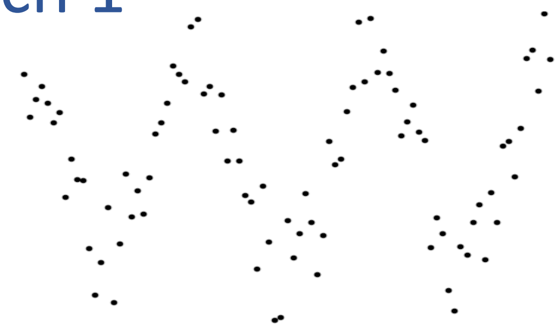
$$\vec{\omega}_a = -\frac{e}{2m_\mu}\left(g_\mu - 2\right)\vec{B} - d_\mu\,\vec{v}\times\vec{B}$$

Measure $\vec{\omega}_a \perp \vec{B}$

[Bennett et al, 0811.1207]

g-2 Stack and Fit

Bunch 1



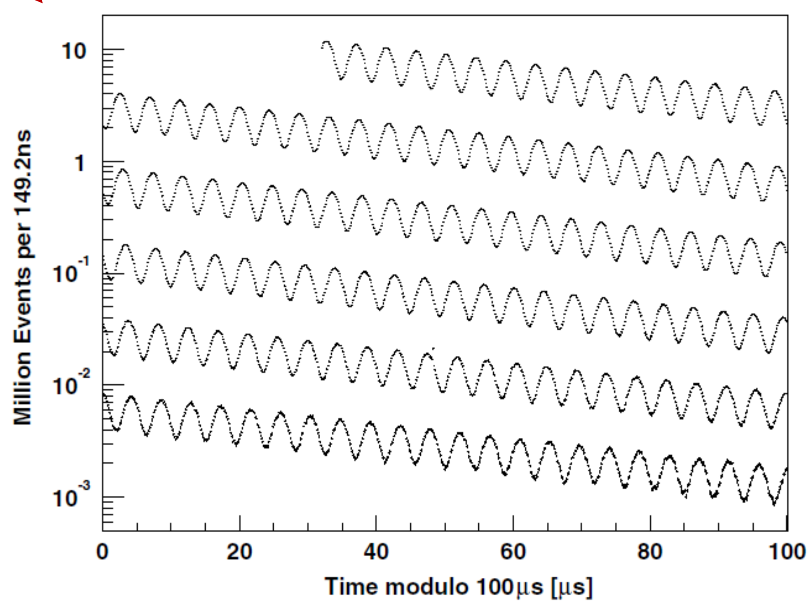
Bunch 2

Each bunch lasts $\approx 640\ \mu s$ (about 10 muon lifetimes) and observes about 1000 decay positrons.

Align and sum $\approx 10^7$ bunches, collected over years (individual bunch data is retained).

Stacked

[Miller et al, 0703049]



Fit for the frequency of the stacked signal

g-2 Precision

FNAL Run I: $\frac{\delta\omega_a}{\omega_a} \approx 0.5 \cdot 10^{-6}$

3.3σ larger than the SM prediction.

[Abi et al, 2104.03281] [Davier et al, 1908.00921]

This is similar to the previous BNL result, but will improve with future runs. [Bennet et al, 0602035]

FNAL projection: $\frac{\delta\omega_a}{\omega_a} \approx 10^{-7}$ [Grange et al, 1501.06858]

J-PARC projection: $\frac{\delta\omega_a}{\omega_a} \approx 10^{-7}$ [Abe et al, 1909.03047]

EDM Precision

BNL: $\frac{\omega_\perp}{\omega_a} \gtrsim 5 \cdot 10^{-4}$

BNL null result limits the muon EDM to a value slightly too small (4σ) to explain the total precession anomaly: $|d_e| < 1.9 \cdot 10^{-19}\, e\,\mathrm{cm}$

[Bennett et al, 0811.1207]

Fermilab and J-PARC projection: $\frac{\omega_\perp}{\omega_a} \gtrsim 5 \cdot 10^{-6}$

[Grange et al, 1501.06858]

[Abe et al, 1909.03047]

Frozen spin projection: $\frac{\omega_\perp}{\omega_\mathrm{a}} \gtrsim 10^{-9}$

[Adelmann et al, 0606034]

Ultralight Bosonic Dark Matter

Dark matter may be a classical, oscillating field

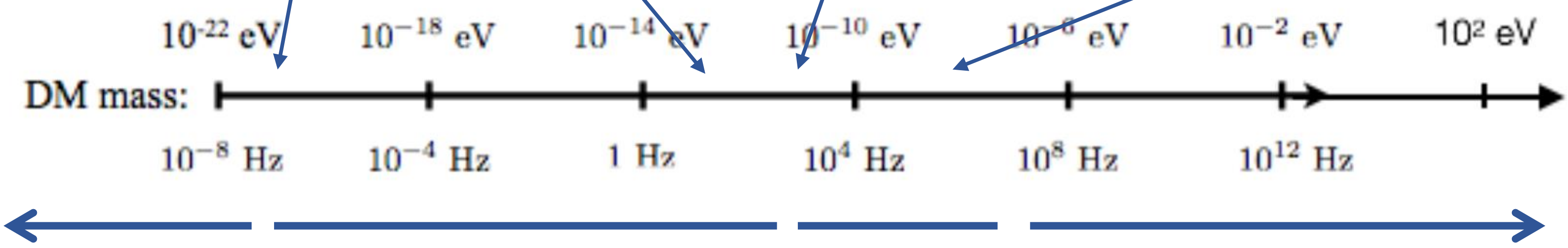
$$\phi(t) = \phi_0 \cos(m_{\mathrm{dm}} t)$$

10^{-21} eV = 1 year

10^{-13} eV = fill rate

10^{-11} eV = bunch duration

10^{-9} eV = ω_a



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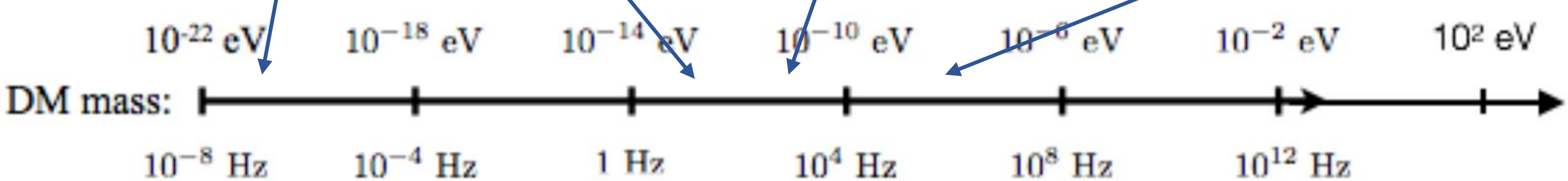


DM field is effectively static.

DM field is static over each bunch but oscillates many times over the lifetime of the experiment.

DM-perturbed Spin Precession

DM background field may apply a torque to muon spins:

$$\dot{\vec{S}} = \vec{\omega}_a(t) \times \vec{S}$$

$$\vec{\omega}_a(t) = \vec{\omega}_{\rm sm} + \vec{\omega}_{\rm dm}(t)$$

Standard Model precession

$$\vec{\omega}_{sm} = -\frac{e}{m_\mu} a_\mu \vec{B}$$

Time-dependent perturbation

Oscillates at DM mass
Amplitude set by DM-muon coupling
Exact form is model dependent

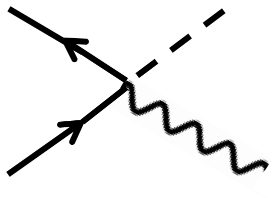
A DM-induced Muon EDM

Pseudoscalar DM of mass m_a with an interaction:

DM background

$$\mathcal{L} \supset -\frac{i}{2} g\, a\, \bar{\mu}\, \sigma_{\alpha\beta}\, \gamma_5\, \mu\, F^{\alpha\beta}$$

EDM operator



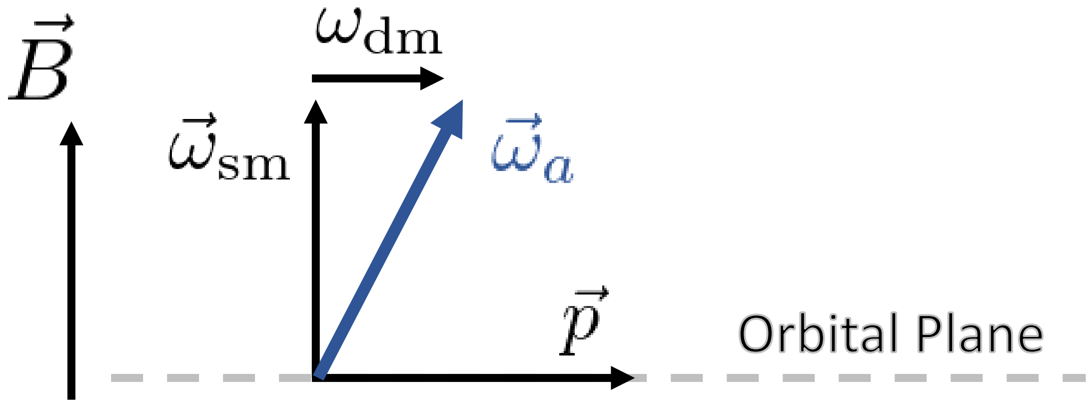
DM field generates a time-varying EDM for the muon. The precession frequency is:

$$\vec{\omega}_a = \omega_{sm} \hat{B} + \omega_{dm} \cos\left(m_a t\right) \left(\hat{p} \times \hat{B}\right)$$

$$\omega_{dm} = 2g \frac{\sqrt{2\rho_{dm}}}{m_a} v_\mu B$$

A DM-induced Muon EDM

$$\mathcal{L} \supset -\frac{i}{2} g\, a\, \bar{\mu}\, \sigma_{\alpha\beta}\, \gamma_5\, \mu\, F^{\alpha\beta}, \qquad \omega_{dm} = 2g \frac{\sqrt{2\rho_{dm}}}{m_a} v_\mu B$$



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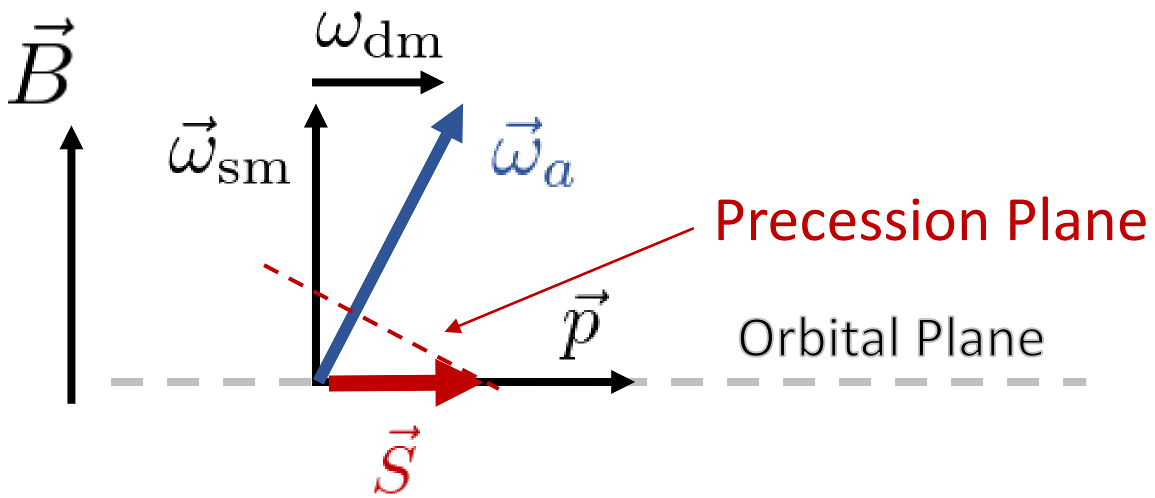

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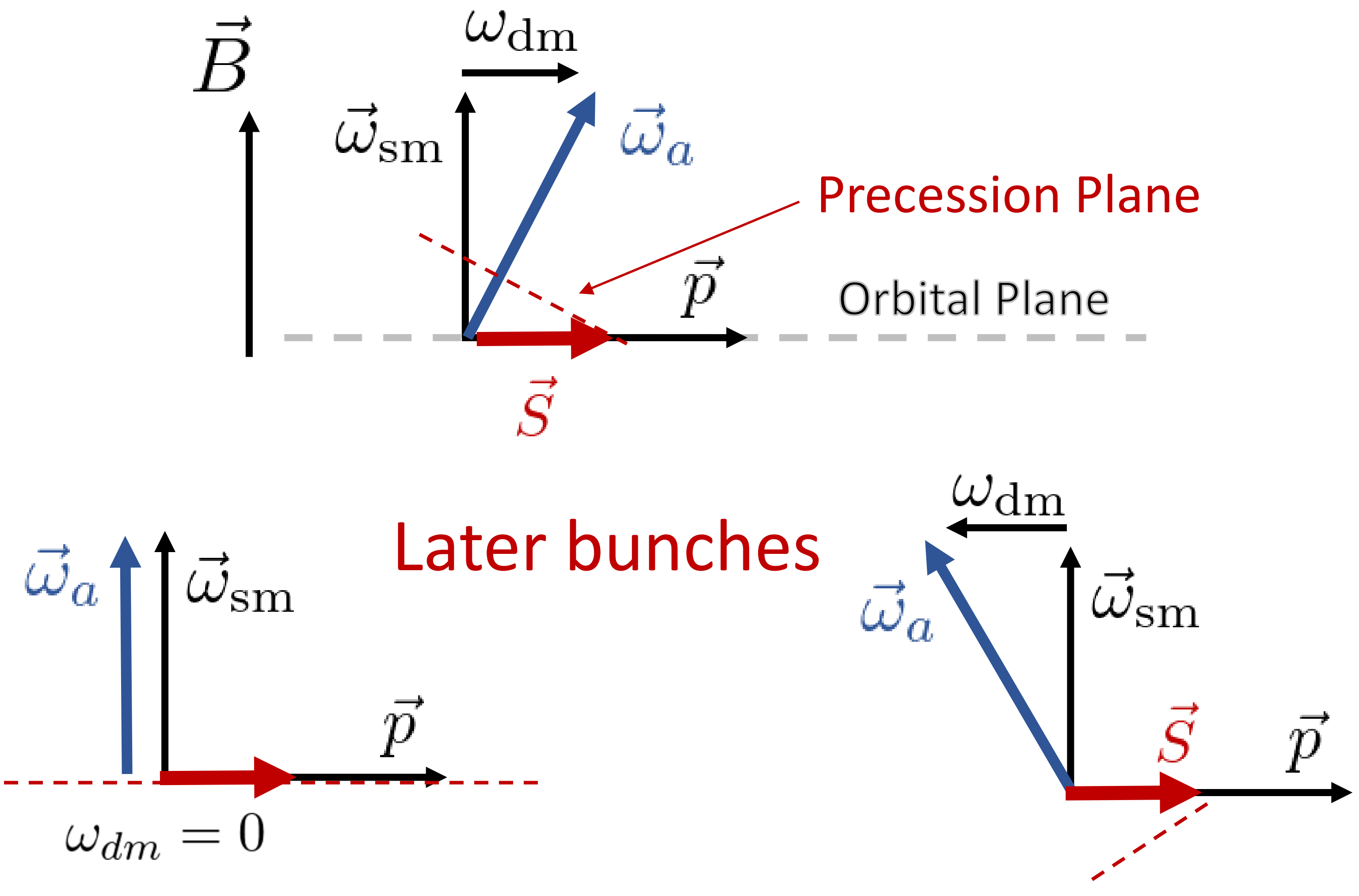


A DM-induced Muon EDM

$$\vec{\omega}_a = \omega_{sm}\hat{B} + \omega_{dm}\cos\left(m_a t\right)\left(\hat{p}\times\hat{B}\right)$$

In g-2 measurement (total positron count):

$$|\vec{\omega}_a| = \sqrt{\omega_{sm}^2 + \omega_{dm}^2\cos^2(m_a t)} \approx \omega_{sm} + \frac{\omega_{dm}^2}{2\,\omega_{sm}}\cos^2(m_a t)$$

$$= \left(\omega_{\rm sm} + \frac{1}{4}\frac{|\omega_{\rm dm}|^2}{\omega_{\rm sm}}\right) + \frac{1}{4}\frac{|\omega_{\rm dm}|^2}{\omega_{\rm sm}}\cos\left(2m_{\rm dm}t\right)$$

Positive frequency shift

Can explain g-2 anomaly

Frequency modulation

Signature of background DM-muon interactions

Stacking of the FM Precession Signal

Could FM precession be hiding in the g-2 data?

Yes - stacking averages away the modulation.

The stacked data is a sum of cosines at different frequencies:

$$S_p \sim \Big(\text{oscillation at } \langle \omega(t) \rangle\Big) \cdot \Big(\text{envelope with scale } \sigma_{\omega(t)}\Big)$$

Small net shift from the SM value

$$\langle \omega(t) \rangle \approx \omega_{\mathrm{sm}} + \frac{\omega_{\mathrm{dm}}^2}{4\omega_{\mathrm{sm}}}$$

Envelop is nearly flat over each bunch

$$\sim \left(1 - \frac{\omega_{\mathrm{dm}}^2}{4\omega_{\mathrm{sm}}} t\right)$$

Time-Resolved Frequency Tracking

Can we detect FM precession?

Yes – use archived bunch data to measure the precession frequency as a function of time.

Bunch 1

Bunch 2

Individually fit each bunch

Time series $\omega_a(t)$

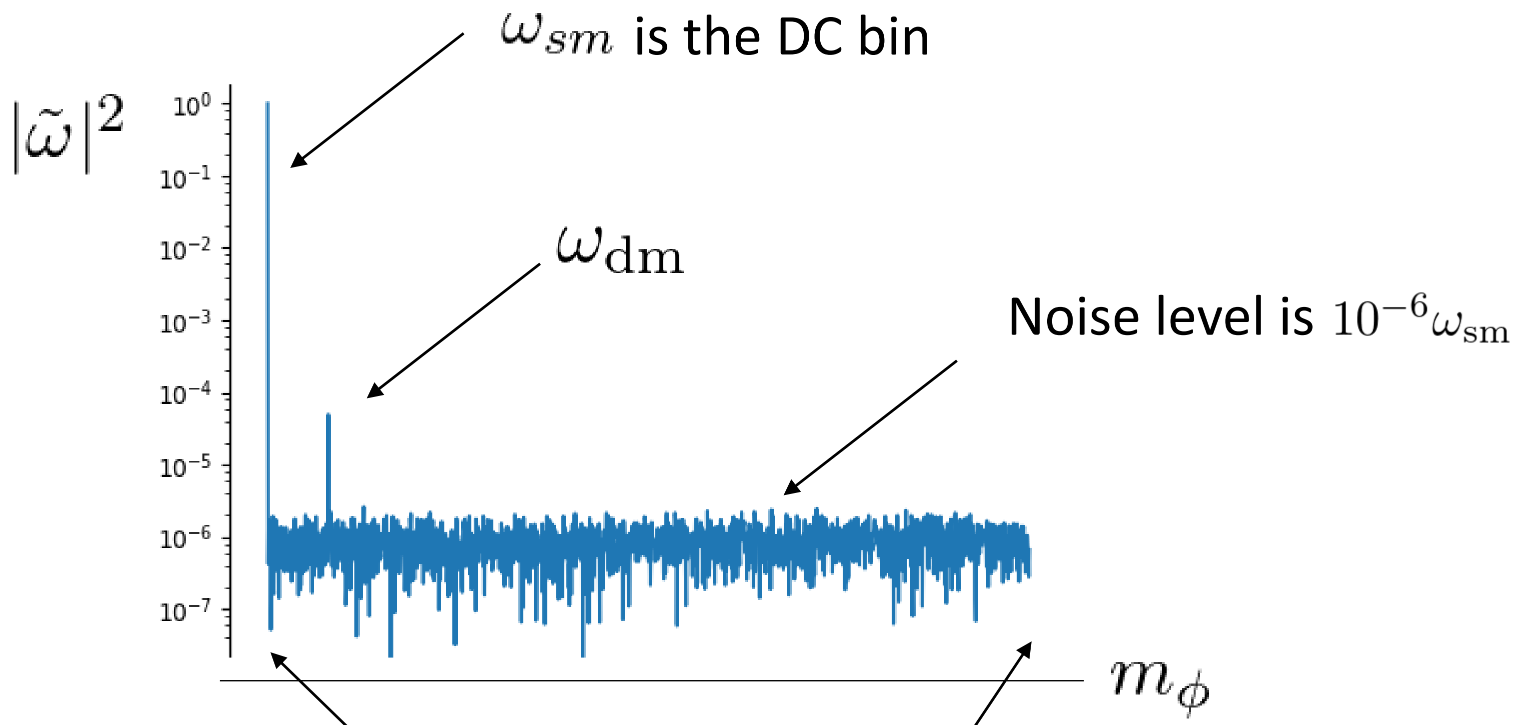
10^6 data points

Noise:

$\sigma_\omega \approx 10^{-3}\omega_{sm}$

Time-Resolved Frequency Tracking

Fourier transform of $\omega_a(t)$:



$$10^{-23}\,\mathrm{eV} \sim \frac{1}{T_{\rm run}} < m_\phi < \frac{N_{\rm bunches}}{T_{\rm run}} \sim 10^{-15}\,\mathrm{eV}$$

Assuming a uniform spacing of bunches – in reality, the mass resolution is not uniform and requires actual bunch timings

A DM-induced Muon EDM

$$\vec{\omega}_a = \omega_{sm}\hat{B} + \omega_{dm}\cos(m_a t)\left(\hat{p}\times\hat{B}\right)$$

In EDM search (net vertical count):

$$S_B = S_0 \frac{\omega_{dm}}{\omega_{sm}} \cos(m_a t) \sin\left[\left(\omega_{sm} + \frac{\omega_{dm}^2}{4\omega_{sm}}\right)t + \frac{\omega_{dm}^2}{8\omega_{sm}}\sin(2m_a t)\right]$$

Amplitude Modulation

This averages away in a stacked analysis

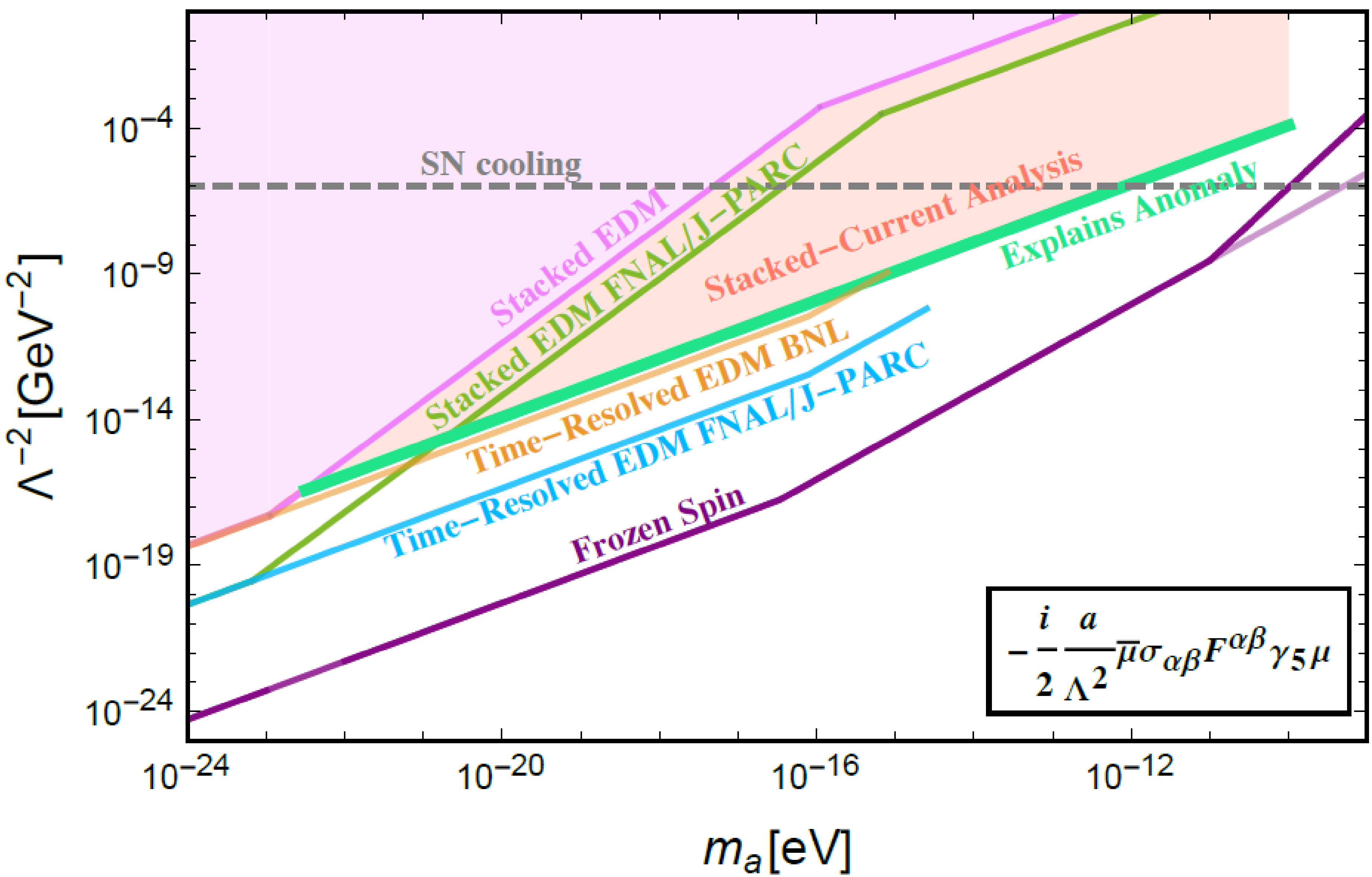
$$S_B \sim \frac{1}{\sqrt{N_{\text{bunches}}}}\frac{\omega_{dm}}{\omega_{sm}} S_p$$

Frequency shift and modulation

Identical to that of the total count oscillation

Can search for modulating EDM with bunch-by-bunch analysis, analogous to the modulating frequency search

Expected Reach Muon-ALP EDM Coupling



DM-induced Magnetic Dipole Moment

Scalar DM ϕ of mass m_ϕ with a muon Yukawa coupling:

$$\mathcal{L} \supset y\,\phi\bar{\mu}\mu$$

DM background field is:

$$\phi(t) = \sqrt{\frac{2\rho_{\rm dm}}{m_\phi}}\cos\left(m_\phi t\right)$$

DM field generates a time-varying muon mass and MDM:

$$m(t) = m_\mu + y\phi(t)$$

$$\vec{\mu} = \frac{e\,g_\mu}{2\,m(t)}\,\vec{S}$$

A Scalar DM Precession Signal

Rest frame precession frequency:

$$\vec{\omega}_a(t) = -\frac{e}{m(t)} a_\mu \vec{B}$$

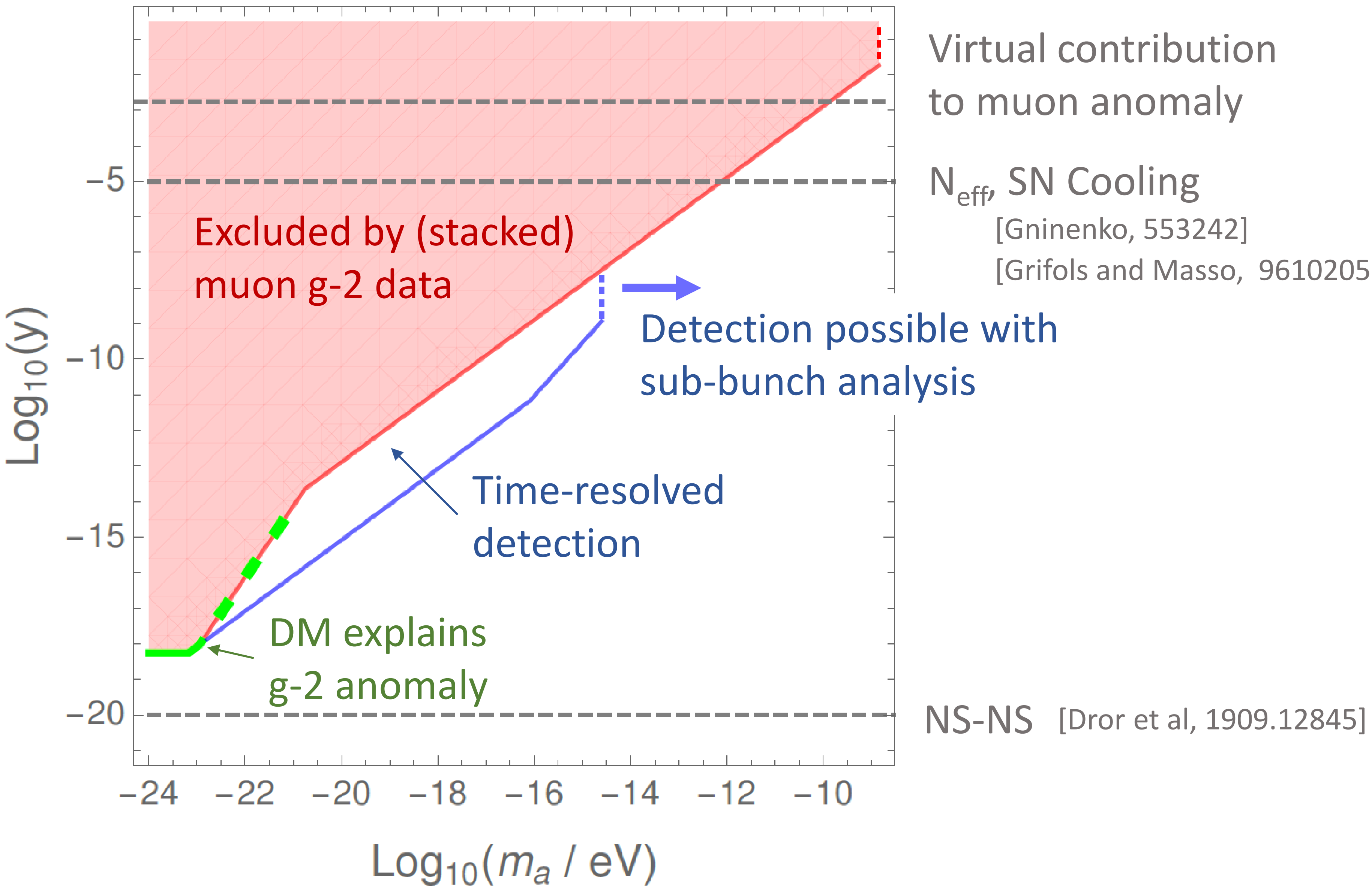
$$\approx \vec{\omega}_{\rm sm} \left[1 - \frac{y}{m_\mu}\sqrt{\frac{2\rho_{\rm dm}}{m_\phi}}\cos\left(m_\phi t\right)\right]$$

Solve the precession equation:

Spin precesses about $\hat{B}$ with an instantaneous angular frequency $|\vec{\omega}_a(t)|$.

Frequency modulation of total positron count
No net frequency shift
No EDM signals, static nor modulated

Detection Reach for DM-Muon Yukawa



Muon g-2 and EDM Experiments as DM Detectors

A new search for ultralight DM using muon spin targets.

Direct, terrestrial limits on muophillic DM.

Detection reach for DM-muon interactions, pending reanalysis of previous and upcoming g-2 data.

DM may explain the muon g-2 anomaly via coherent interaction with DM background field (not via loops).

Approach improves with ongoing development of g-2 and EDM measurement techniques (e.g., frozen spin experiments).

Muon g-2 and EDM Experiments as DM Detectors

Extra Slides

ALP-Muon Wind

$$\mathcal{L} \supset g\, \partial_\nu a\, \bar{\mu}\, \gamma^\nu \gamma_5\, \mu$$

In the rest frame of the muon: $H \supset g\, \vec{\nabla} a \cdot \vec{S}$

Muon spin precesses about the relative velocity of DM, which is essentially the muon velocity.

$$\vec{\omega_a} = \omega_{sm} \hat{B} + \omega_{dm} \cos\left(m_a t\right) \hat{v}$$

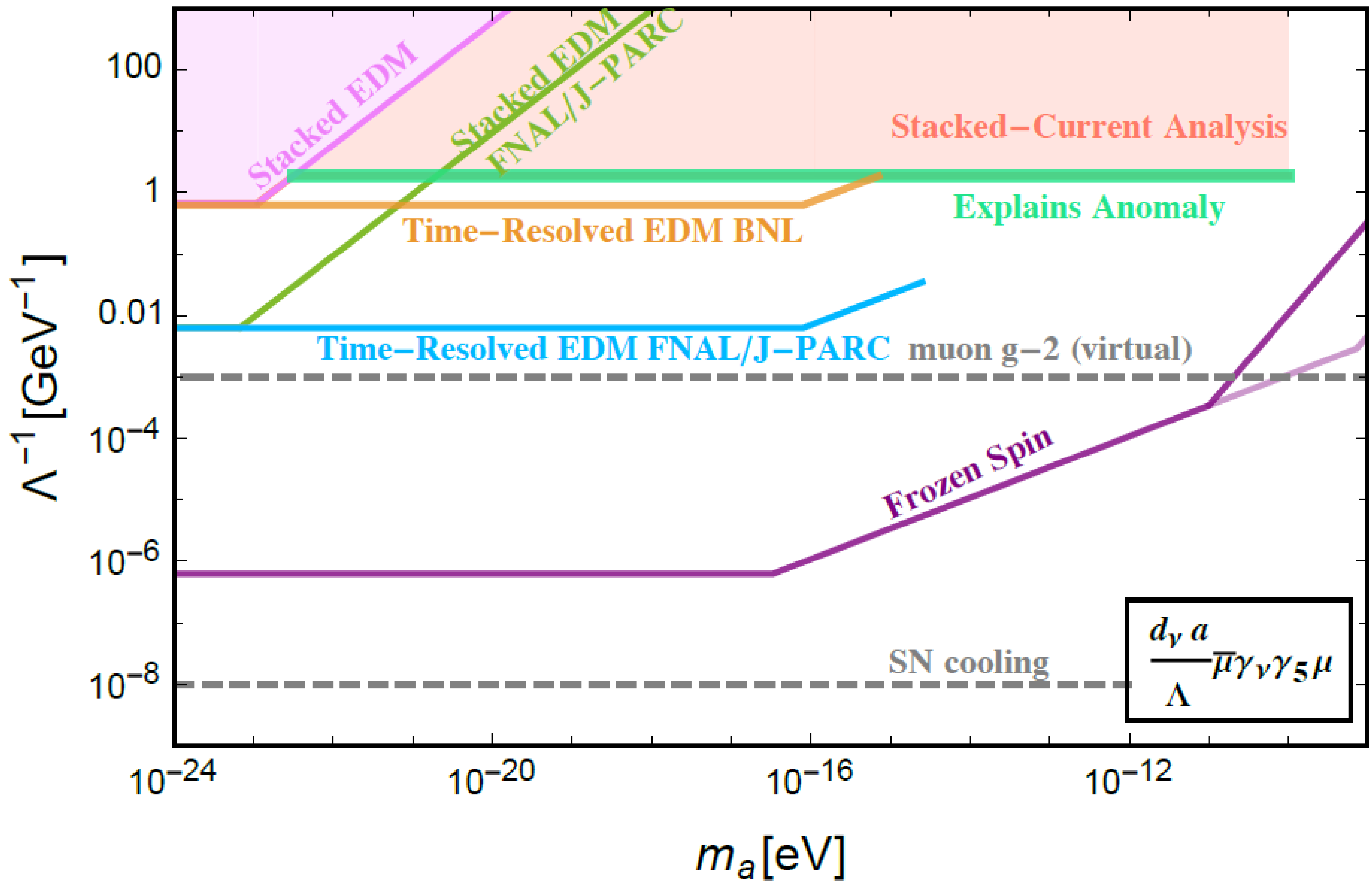
$$\omega_{dm} = g\, \sqrt{2 \rho_{dm}}$$

This is an oscillating orthogonal perturbation – the precession dynamics and detection limits are qualitatively the same as the axion EDM coupling.

The two counts are now in-phase, which may introduce additional systematic errors.

[Bennett et al, 0811.1207]

Detection Reach for ALP-Muon Wind



Muonic Vector DM

Vector DM generates a dark electric and magnetic field,

$$E_{\rm dm} = \sqrt{2\rho_{\rm dm}}\cos\left(m_{\rm dm}t+\alpha\right)$$

$$B_{\rm dm} = v_{\rm dm}\sqrt{2\rho_{\rm dm}}\sin\left(m_{\rm dm}t+\alpha\right)$$

B_{dm} is too small to be observed in existing experiments

$$\frac{\omega_{\rm dm}}{\omega_{\rm sm}} = \frac{g_{\rm dm}}{e}\frac{B_{\rm dm}}{B_0} \approx 10^{-6} g_{\rm dm}\ \left(\frac{3\,{\rm T}}{B_0}\right)$$

E_{dm} may be observed in experiments that do not use the "magic momentum" to cancel electric field precession (e.g., J-PARC, frozen spin)

* disfavored by NS-NS inspiral and solar neutrino oscillations ($L_\mu - L_\tau$)

Muonic Vector DM

Ultralight vector DM manifests as a local dark electric and magnetic field:

$$E_{\rm dm} = \sqrt{2\rho_{\rm dm}} \cos\left(m_{\rm dm} t\right)$$

$$B_{\rm dm} = v_{\rm dm} E_{\rm dm}$$

Four distinct contributions to precession – the dominant one is the component of $\vec{E}_{\rm dm}$ transverse to the orbital plane:

$$\vec{\omega}_{\rm dm} = \frac{g_{\rm dm}}{m_\mu \gamma^2} \, \vec{v} \times \vec{E}_{\rm dm}$$

Not observable at BNL or Fermilab, as vertical trapping EM fields will screen $\vec{E}_{\rm dm}$!

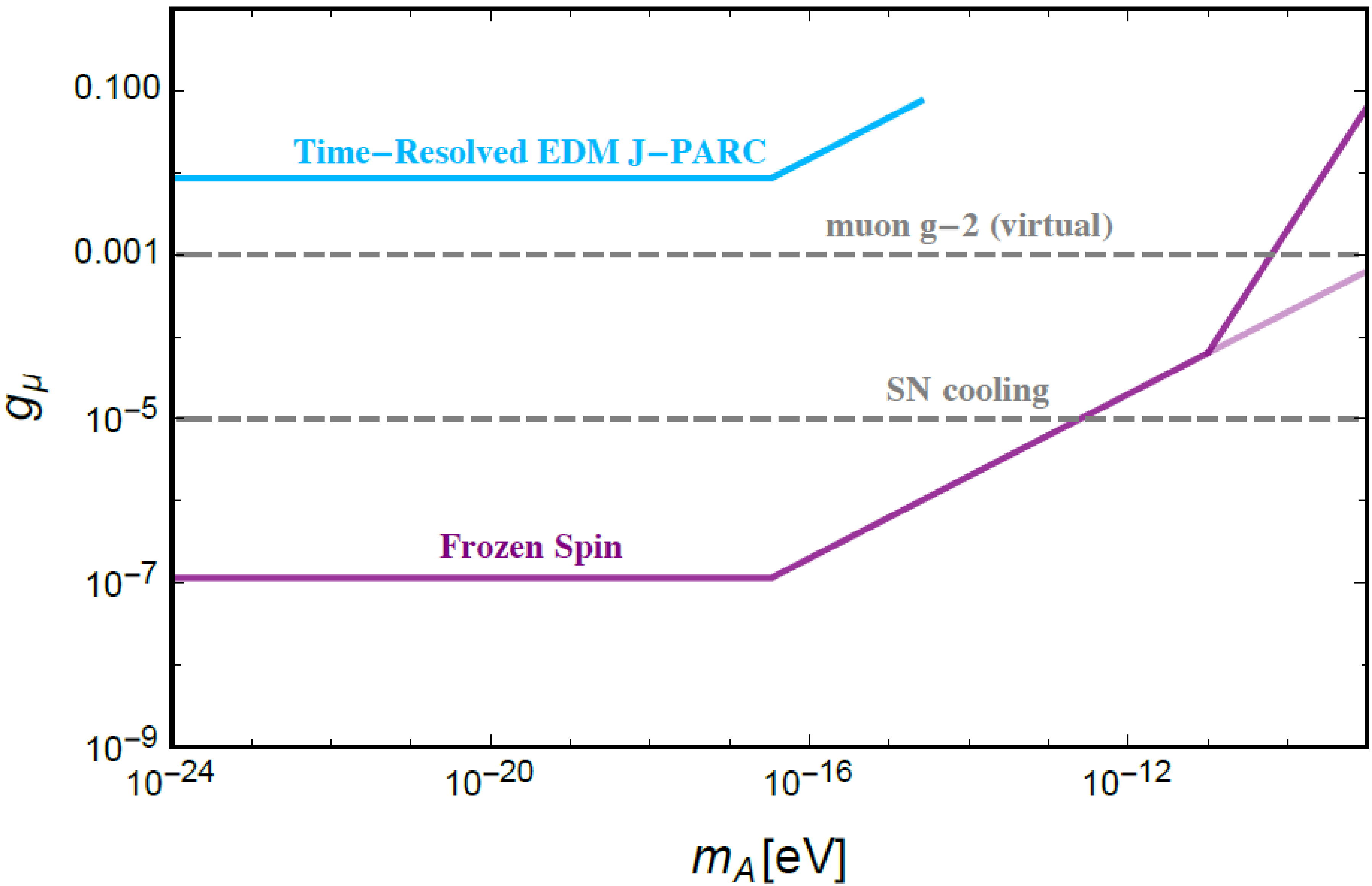
[Bennett et al, 0602035]

[Grange et al, 1501.06858]

Observed at J-PARC or future frozen spin searches.

[Abe et al, 1909.03047] [Adelmann et al, 0606034]

Muonic Vector DM



Muonic Dark Matter

Existing bounds on DM – muon interactions are from astrophysics, cosmology, or virtual effects.

Astrophysics and Cosmology

Long range forces on neutron stars	[Dror et al, 1909.12845], [Poddar et al, 1908.09732]
Supernovae cooling	[Bollig et al, 2005.07141] , [Croon et al, 2006.13942]
BBN (N_{eff})	[Grifols and Masso, 9610205]
BH Superradiance	[Arvanitaki et al, 821575], …

Loop effects

Muon g - 2	[Chen et al, 1701.07437]
Induced interactions	[Arvanitaki et al, 1405.2925], [Beznogov et al, 1806.07991], …

A direct terrestrial search is epistemically distinct and provides an opportunity for a surprising discovery.

Laboratory ceiling is higher (sensitivity will improve).

Ultralight Bosonic Dark Matter

Dark matter may be a classical, oscillating field

$$\phi(t) = \phi_0 \cos(m_{\rm dm} t)$$

Amplitude is set by the local DM energy density and DM mass for scalars

Frequency is roughly the DM mass

Frequency is properly the total energy:

$$\omega_{dm} = m_{dm} + \frac{1}{2} m_{dm} v_{dm}^2$$

$$v_{dm} \sim 10^{-3}$$

DM field coherence time $\sim \dfrac{10^6}{m_{dm}}$

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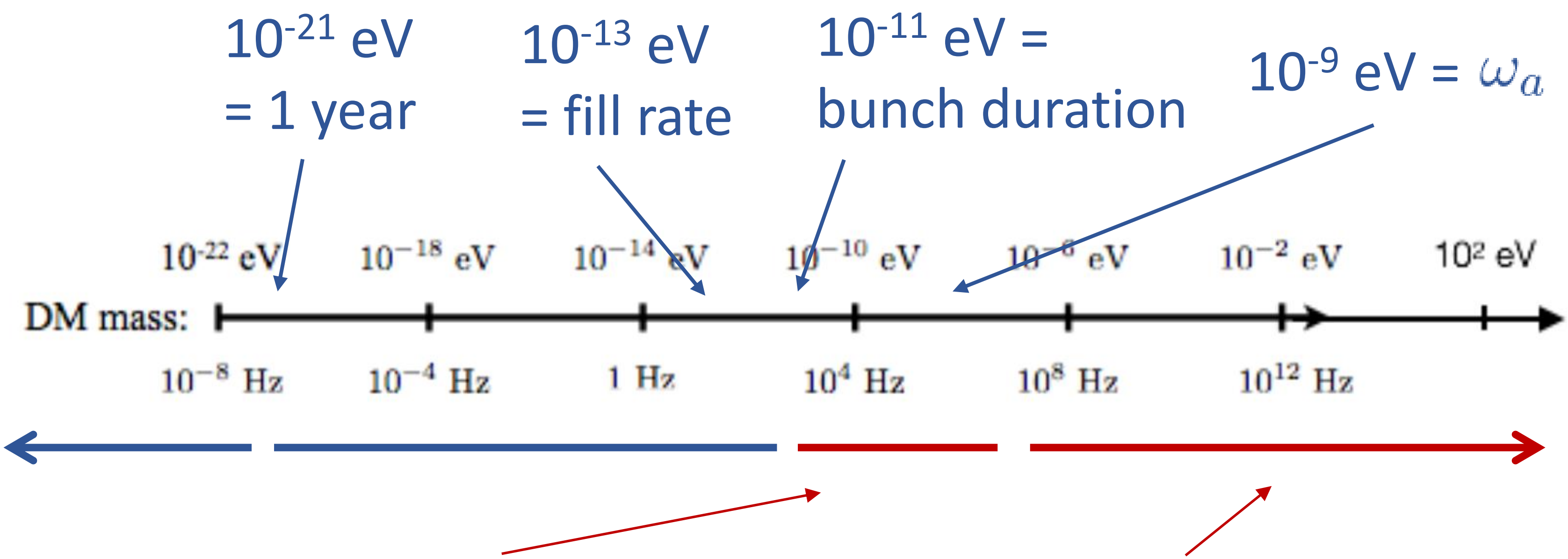

DM field oscillates within each bunch.

DM field oscillates faster than the SM precession. Signal is suppressed and mimics known systematics.

Ultralight Bosonic Dark Matter

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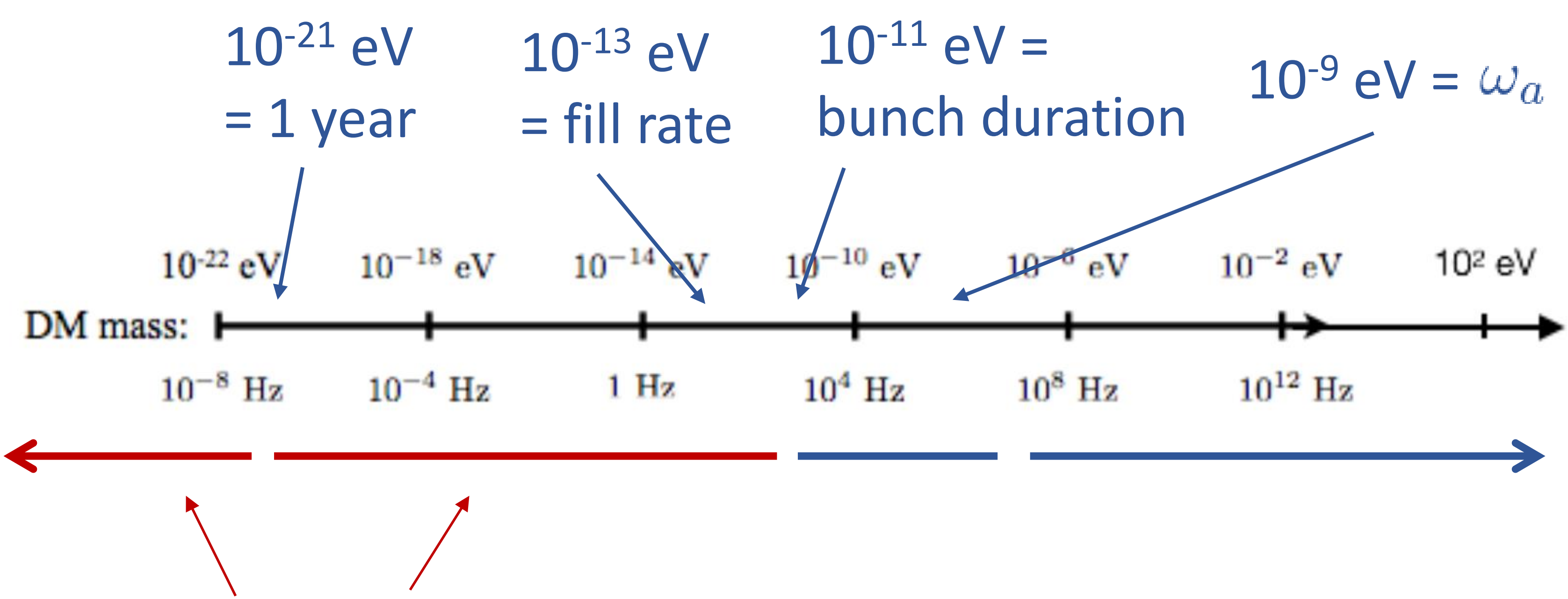
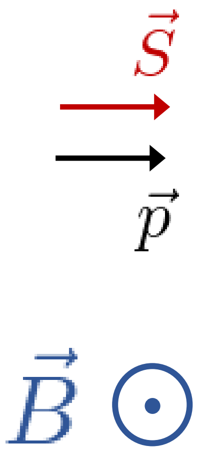


Focus of this work.

Spin Precession in a Storage Ring

Lab Frame



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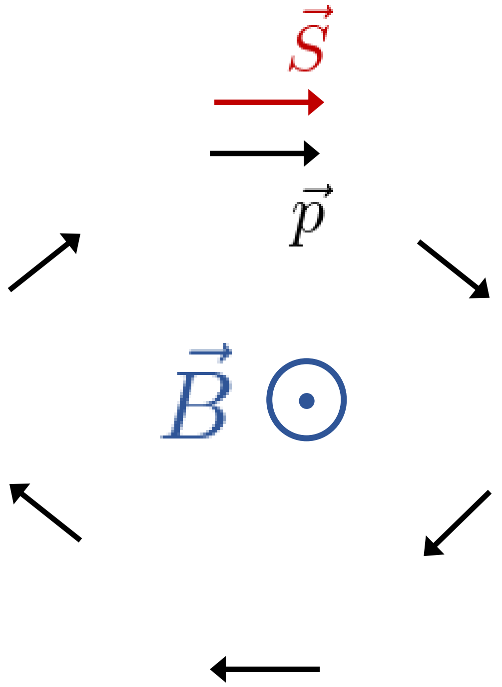

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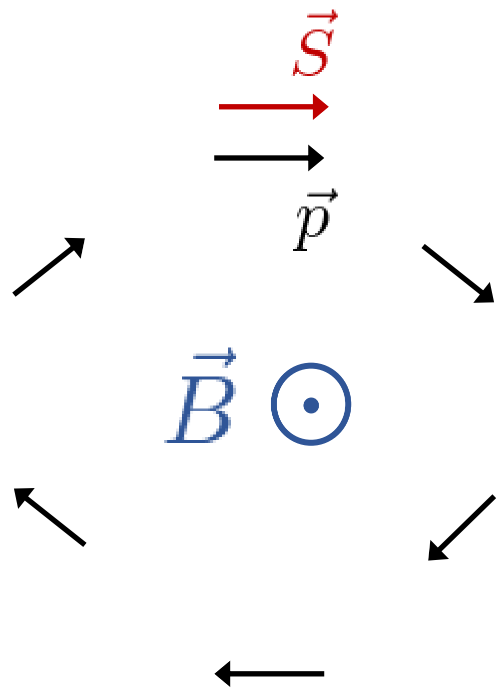
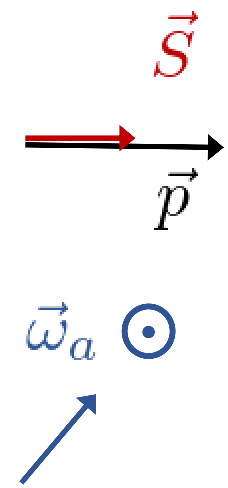


Rotated Rest Frame



Rest Frame
Precession Frequency

$$\frac{2\pi}{\omega_a} \approx 5\ \mu s$$

Spin Precession in a Storage Ring

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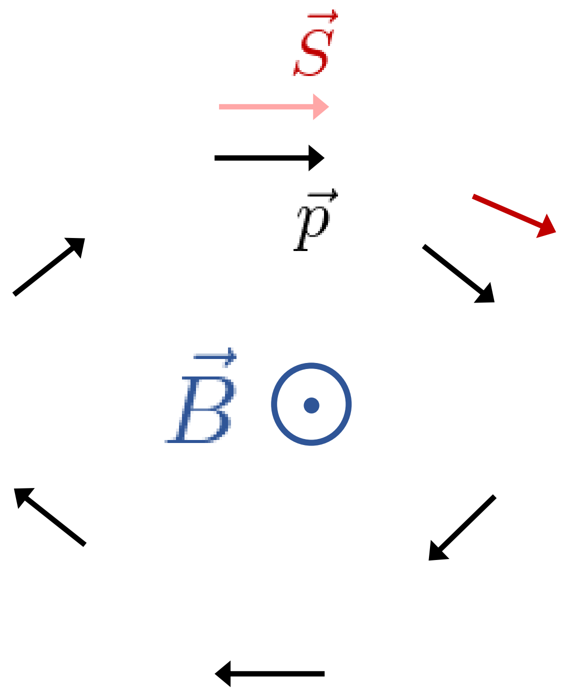

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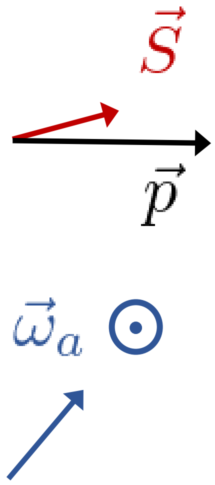

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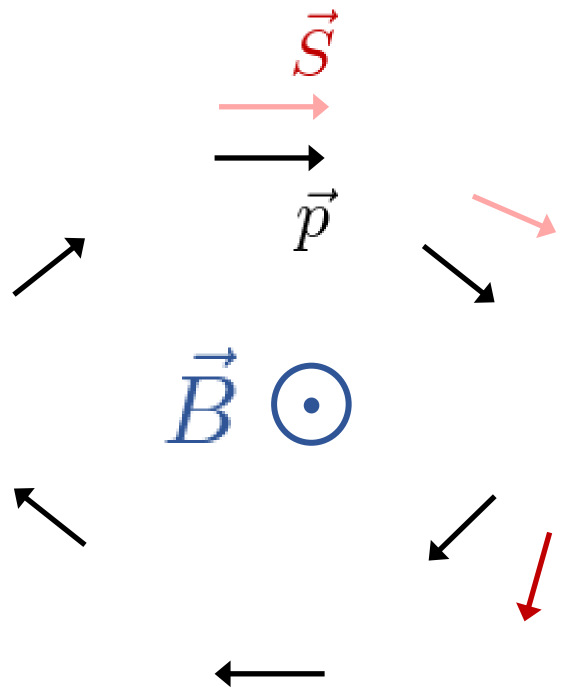

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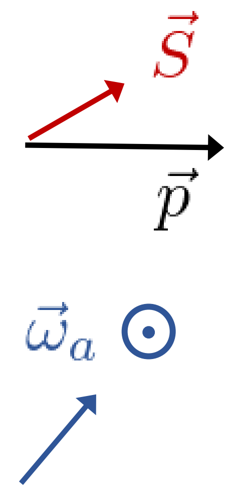

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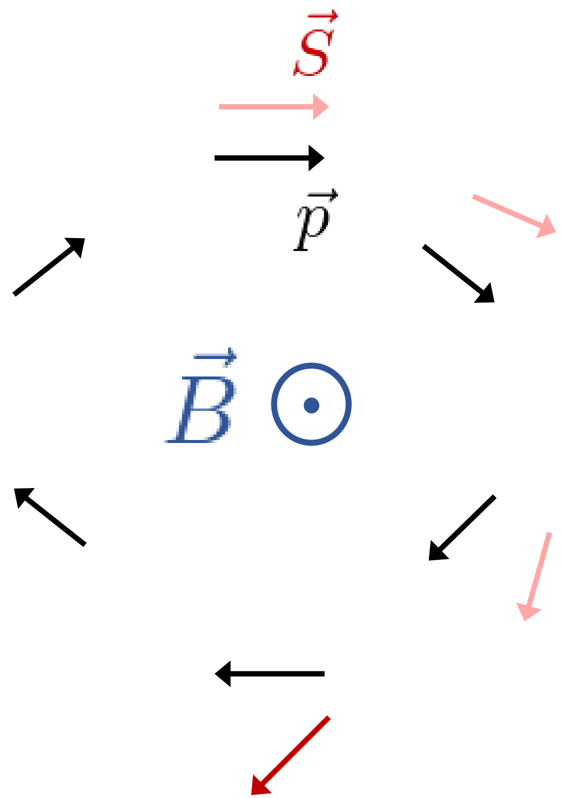

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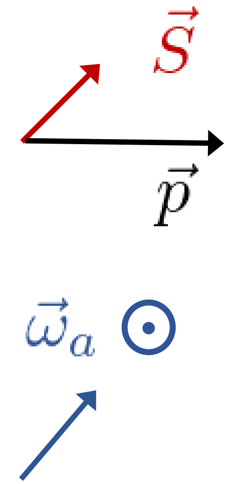

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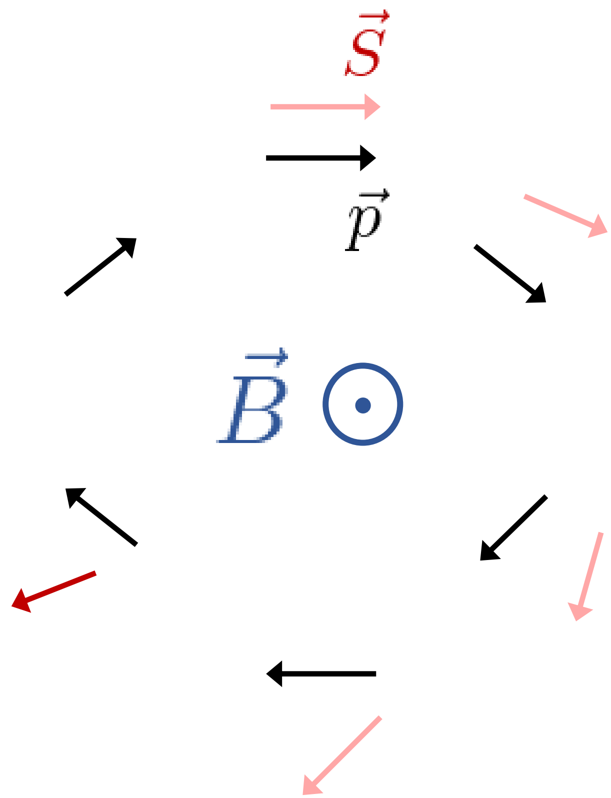

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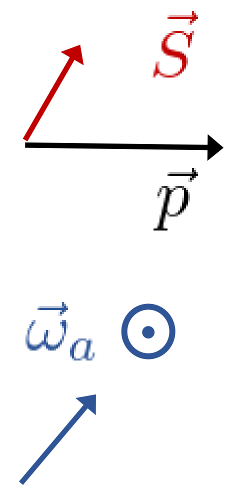

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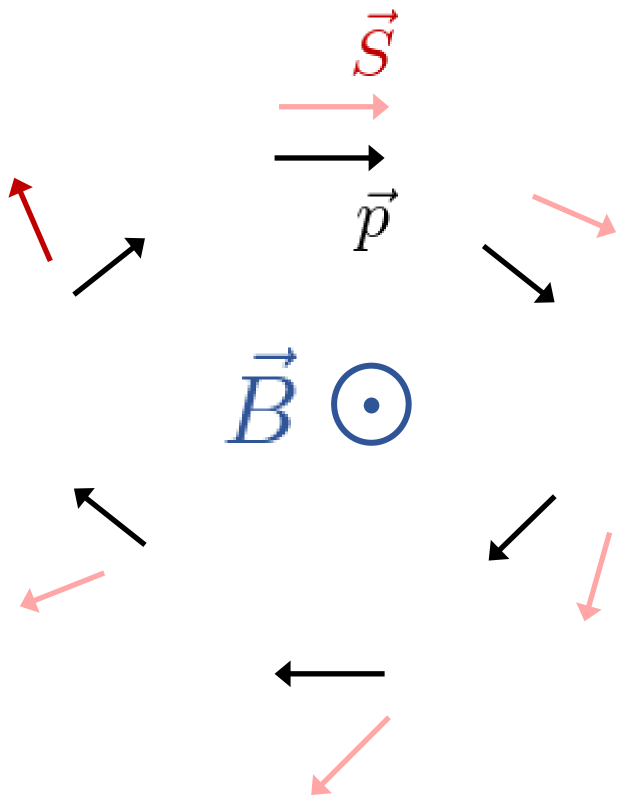

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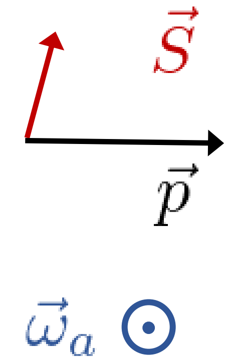

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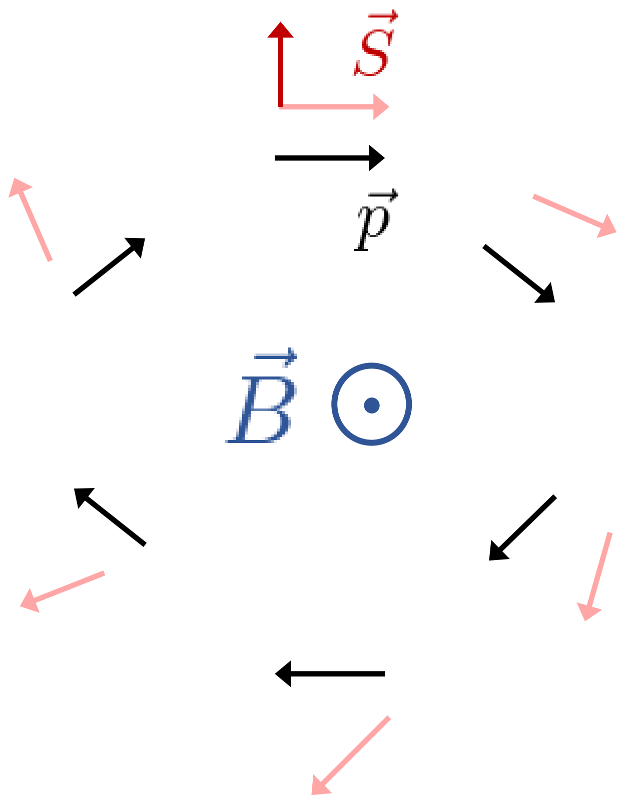

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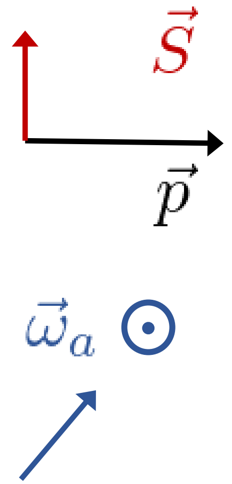

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Precession Frequency

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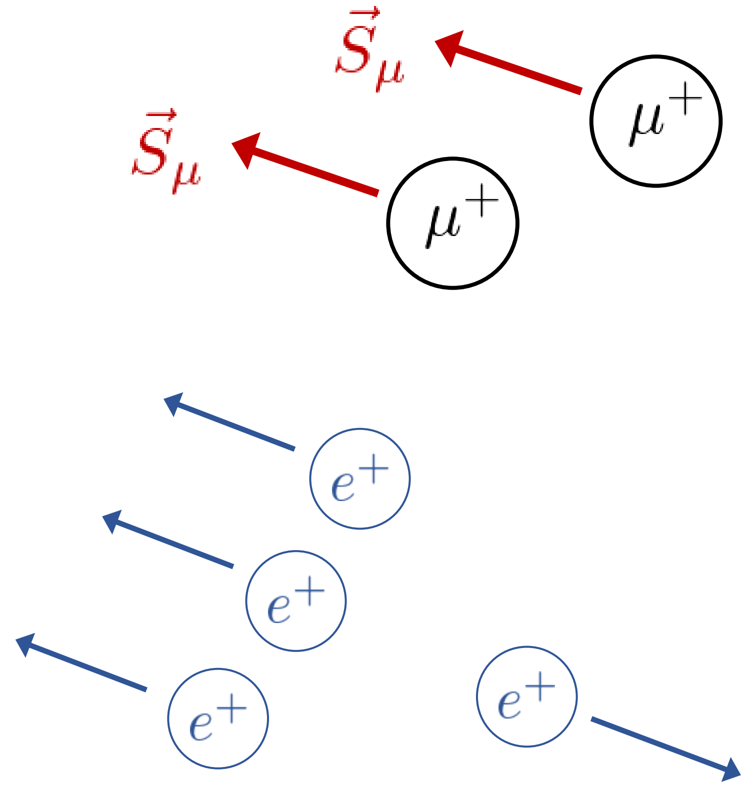
Tracking Muon Spins

Asymmetric Muon Decay

Positrons are preferentially emitted along the direction of the anti-muon spin.

Polarized muon bunch

Positron flux

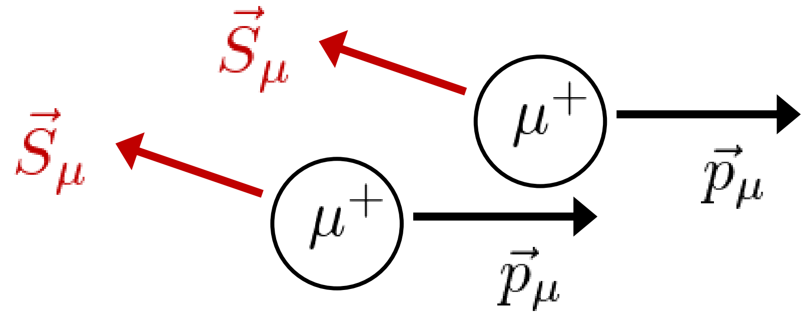


Tracking Muon Spins

Energy is a proxy for direction

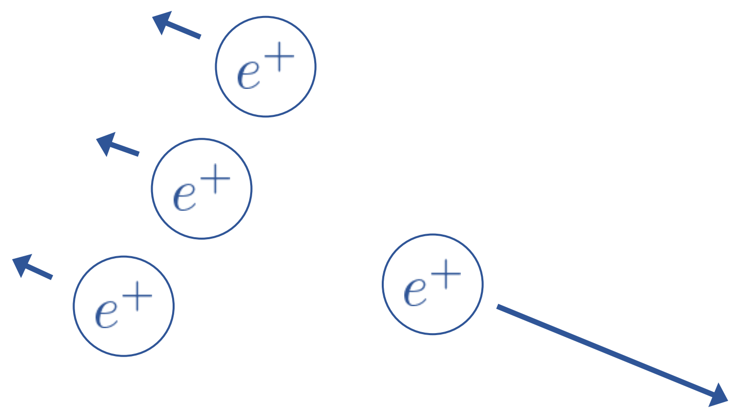
The most energetic positrons are those emitted along the muon's momentum:

Polarized muon bunch



Positron flux

Few high energy positrons

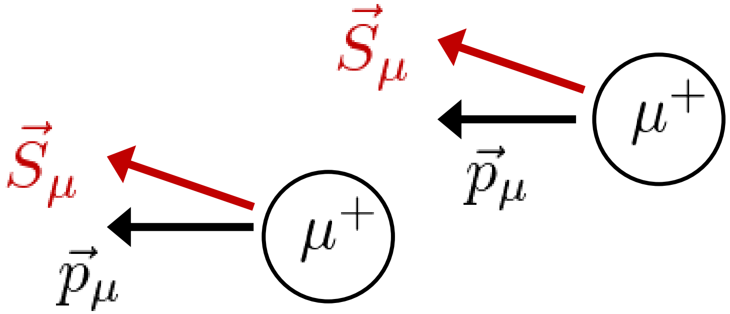


Tracking Muon Spins

Energy is a proxy for direction

The most energetic positrons are those emitted along the muon's momentum:

Polarized muon bunch



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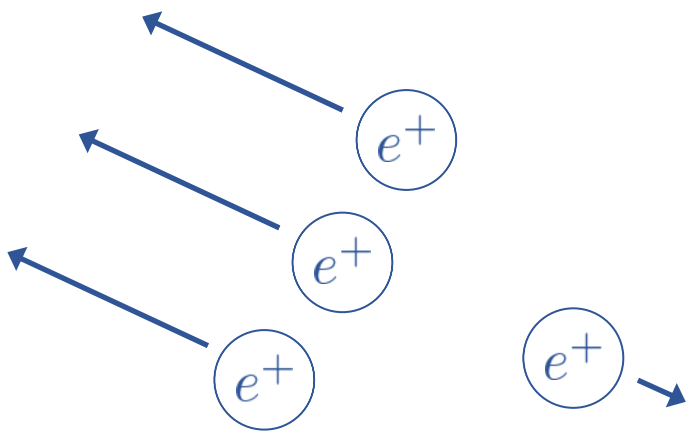
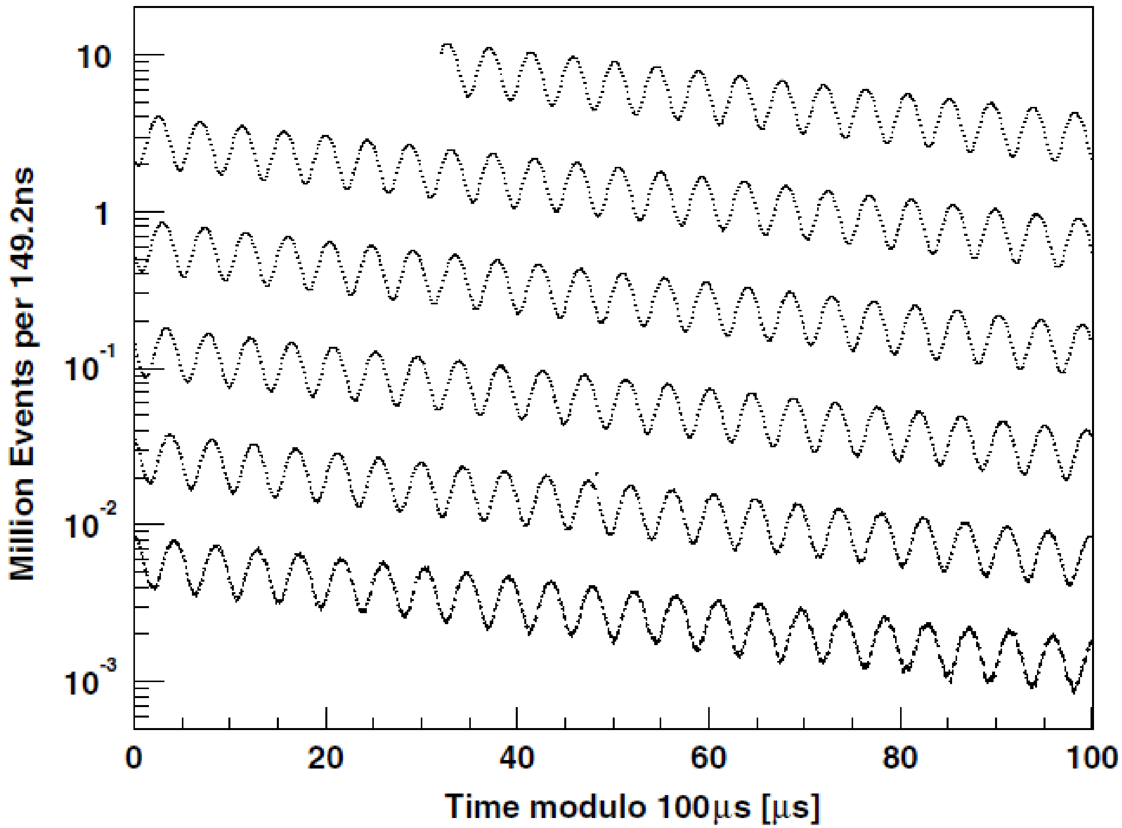


g-2 Measurement: Positron Count

Asymmetric muon decay

Construct positron counting observables that oscillate at ω_a



g-2 count:

Number of positrons in the highest-energy bin:

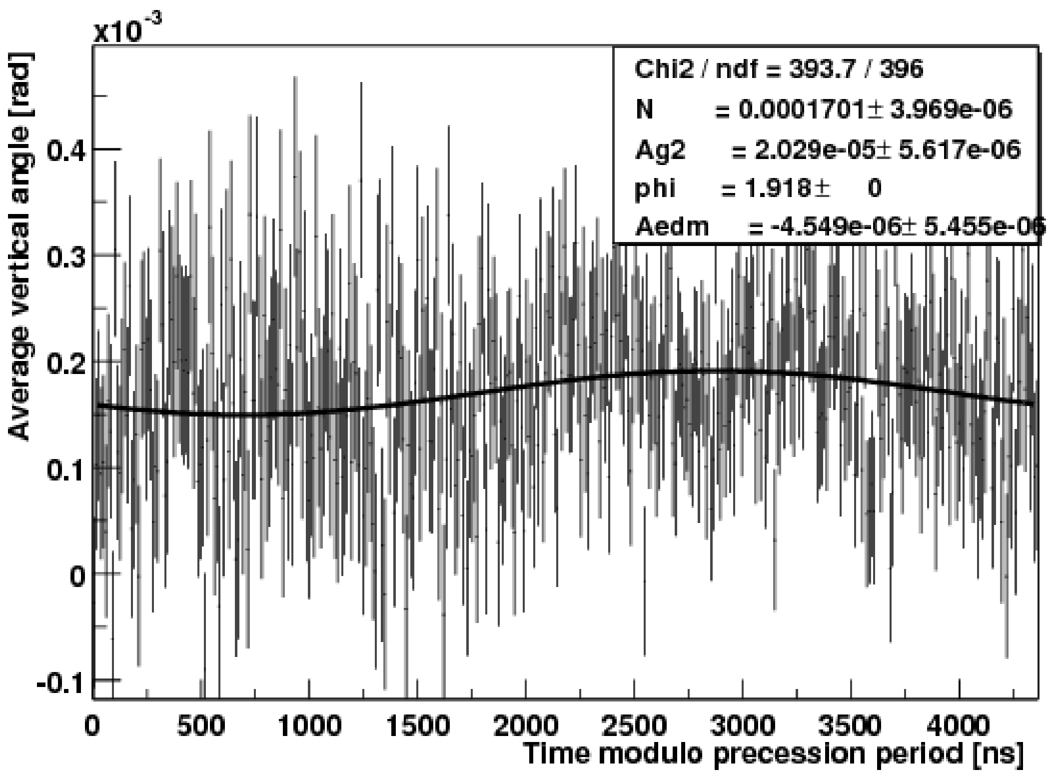
$$N_T \propto [1 + A \cos(\omega_a t)]$$

[Miller et al, 0703049]

EDM Measurement: Positron Count

Asymmetric muon decay

Construct positron counting observables that oscillate at ω_a:



Vertical Count

Excess of upward-moving positrons:

$$\Delta N_B \propto \frac{\omega_\perp}{\omega_a} \sin \omega_a$$

[Bennett et al, 0811.1207]

EDM Stack and Fit

Vertical Count, Bunch 1

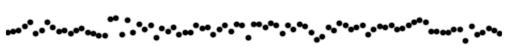
Vertical Count, Bunch 2

Momentum Count, Stacked

Sum all bunches

Fit stacked momentum count

Vertical Count, Stacked



$\omega_a, \ \phi_a$

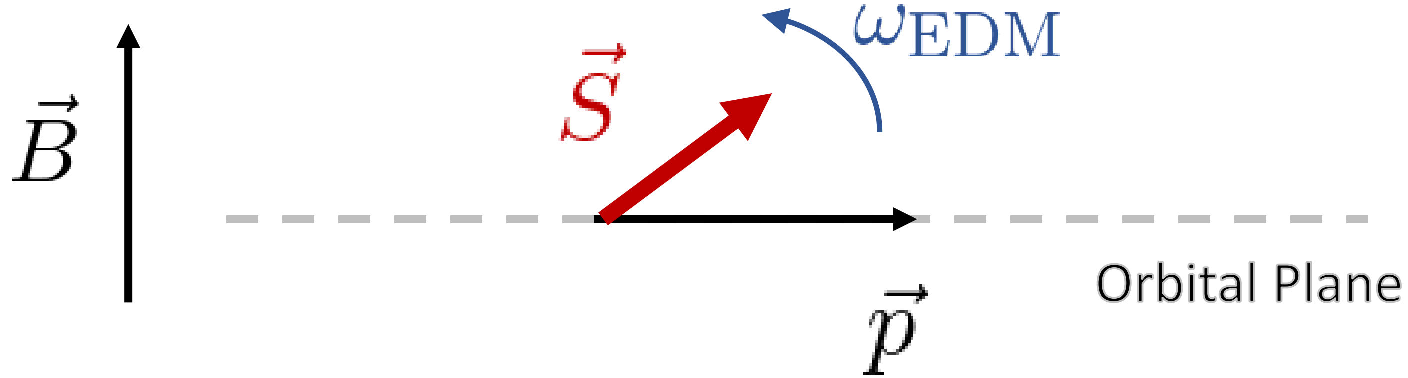
Fit stacked vertical count to: $A \sin\left(\omega_a t + \phi_a\right)$

Frozen Spin

A dedicated EDM search would do best by minimizing ω_{sm} .

Choose laboratory EM fields to set $\omega_{\mathrm{sm}} = 0$.

[Adelmann et al, 0606034]

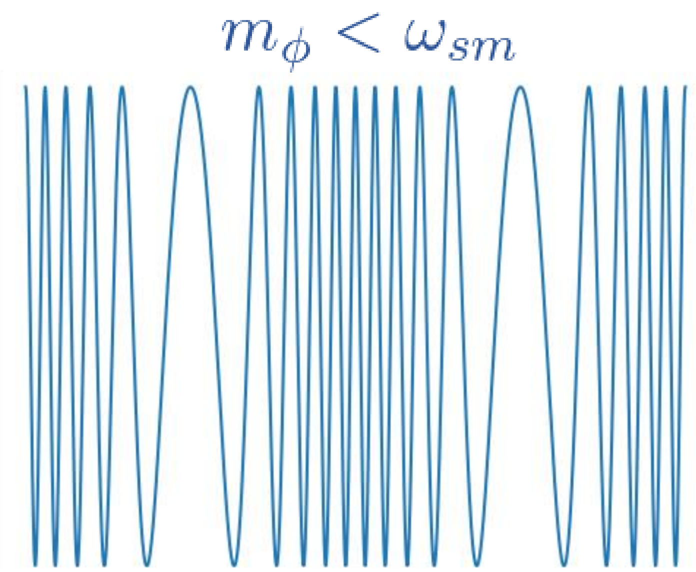


$$\Delta N_B \propto S_0 \sin\left(\omega_\perp t\right)$$

Frequency Modulation

Momentum count has frequency modulation:

$$\vec{S}\cdot\vec{p} = S_0 \cos\left[\omega_{\rm sm} t + \frac{\omega_{\rm dm}}{m_\phi}\sin\left(m_\phi t\right)\right]$$



Gradual drift in the local precession frequency

A Scalar DM Precession Signal

DM Yukawa coupling

No DM interaction

$\vec{\omega}_a \odot$

$\vec{p}$

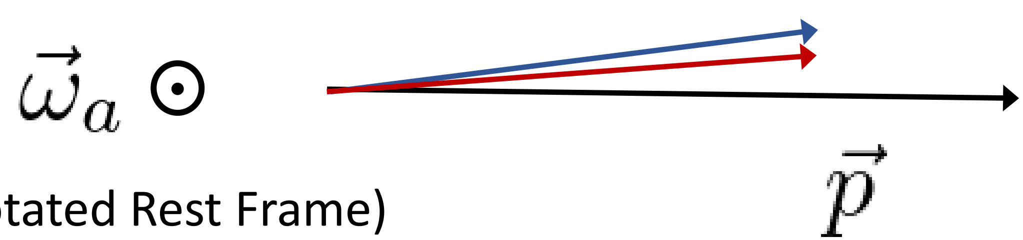
$t = 0$

(Rotated Rest Frame)

A Scalar DM Precession Signal

DM Yukawa coupling

No DM interaction



(Rotated Rest Frame)

$$0 < t < \frac{\pi}{m_\phi}$$

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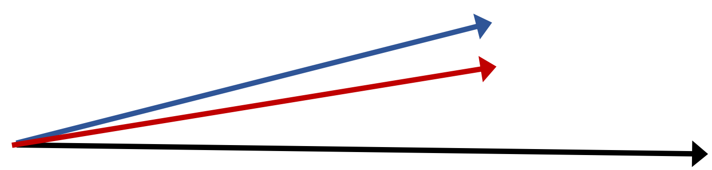
$\vec{\omega}_a \odot$

(Rotated Rest Frame)



$\vec{p}$

$0 < t < \frac{\pi}{m_\phi}$

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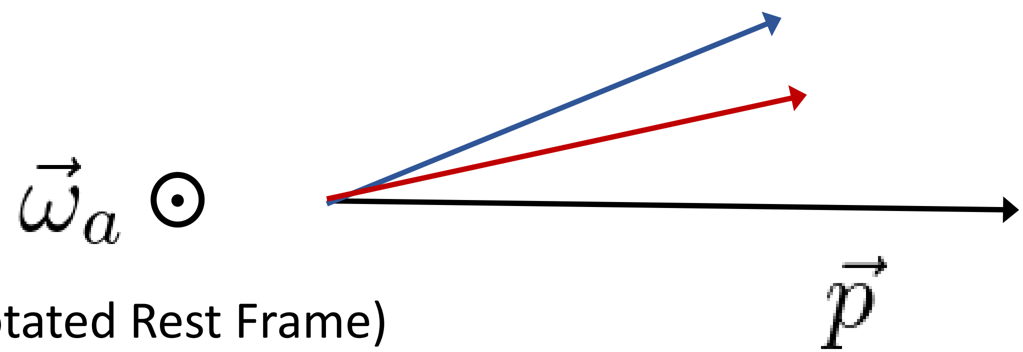

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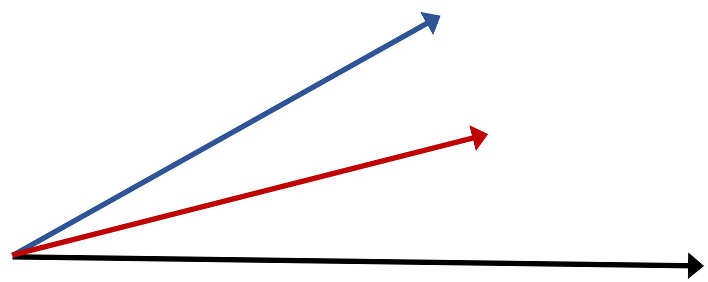

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(Rotated Rest Frame)

$\vec{p}$

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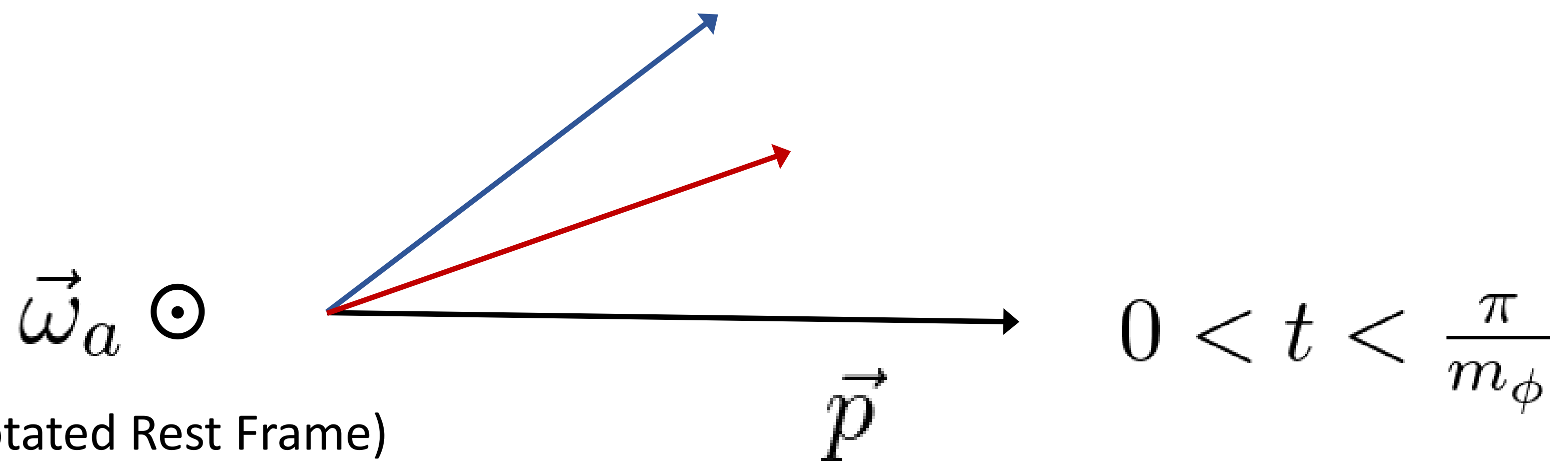

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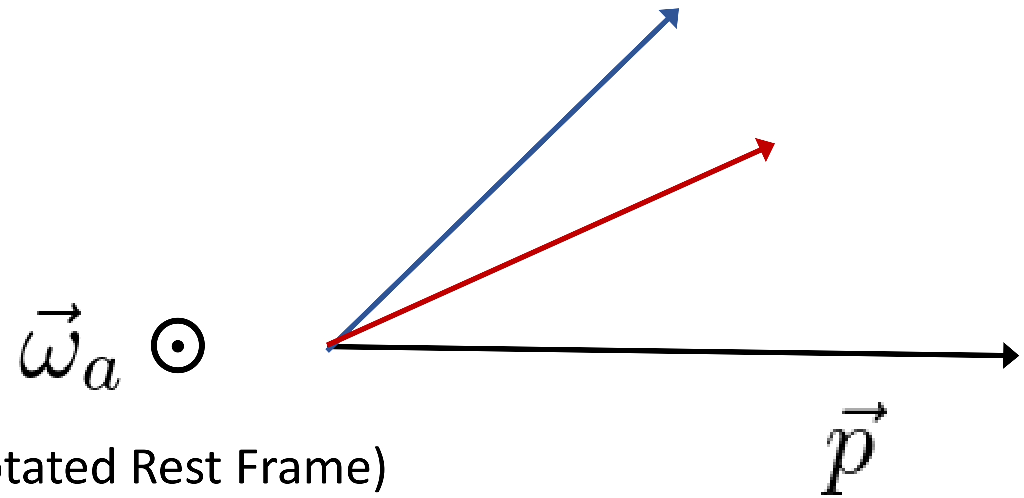

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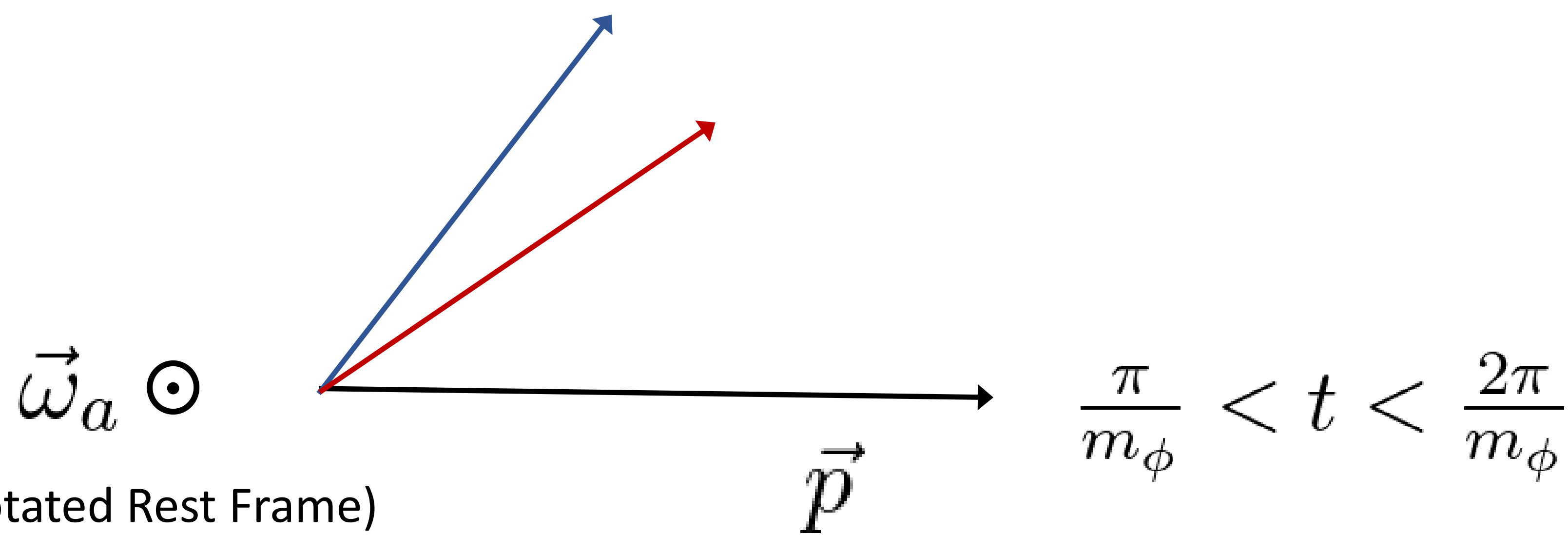

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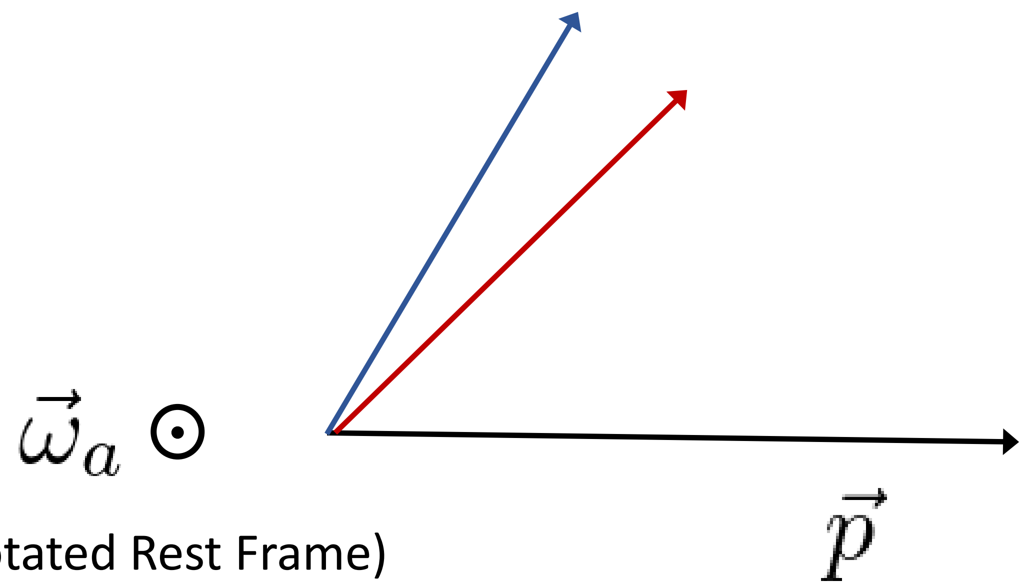


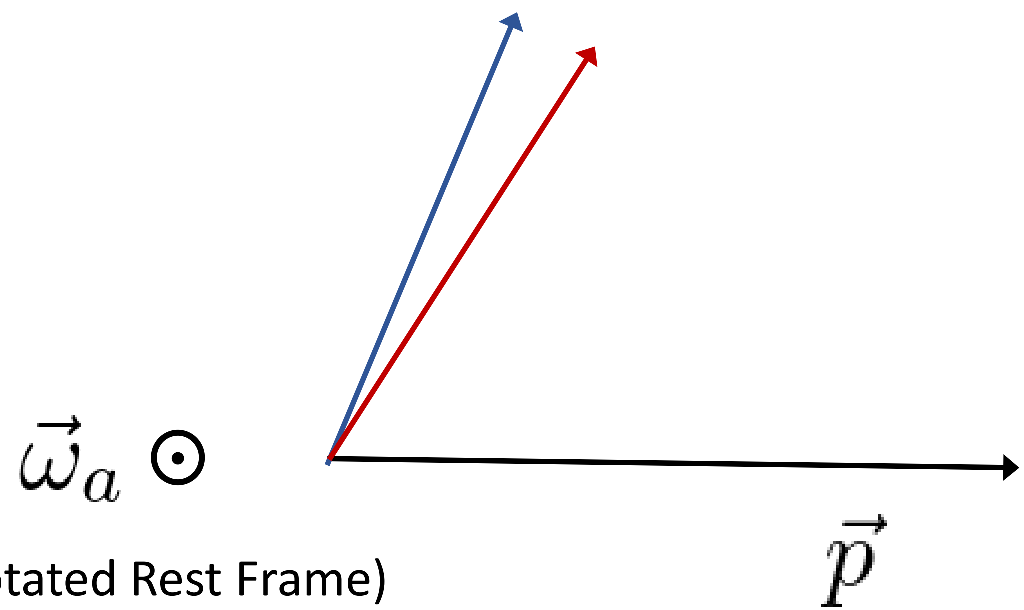
$$\frac{\pi}{m_\phi} < t < \frac{2\pi}{m_\phi}$$

(Rotated Rest Frame)

A Scalar DM Precession Signal

DM Yukawa coupling

No DM interaction



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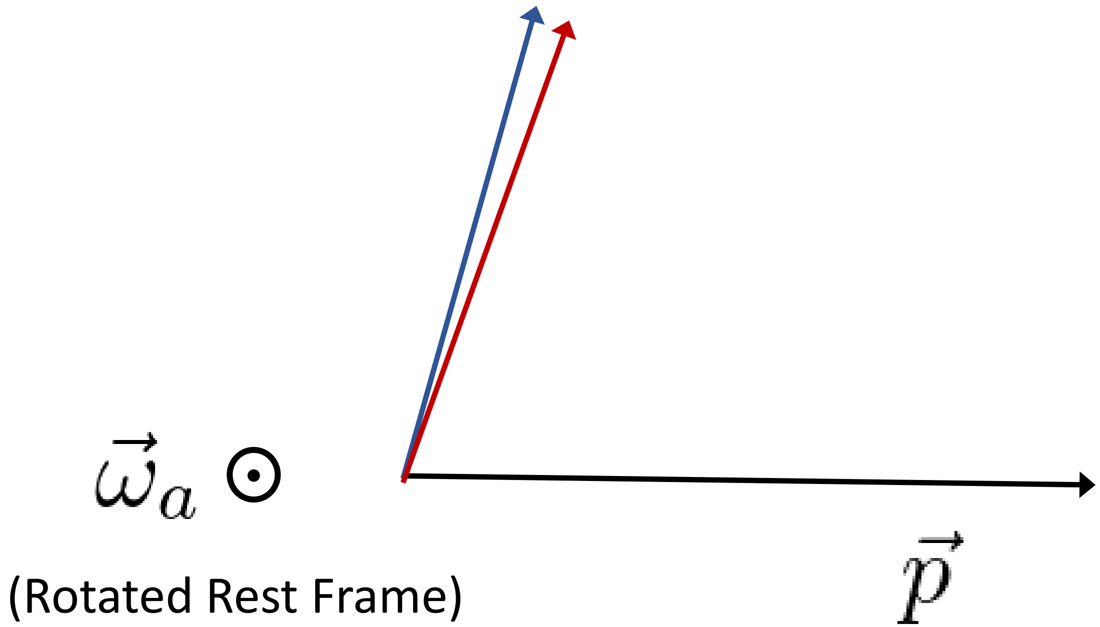

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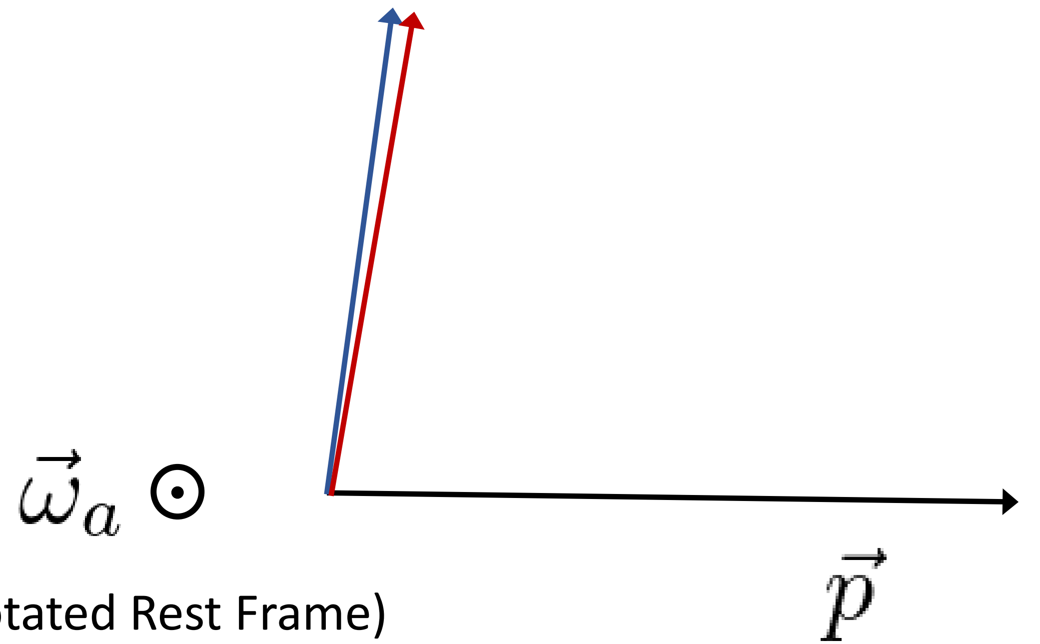

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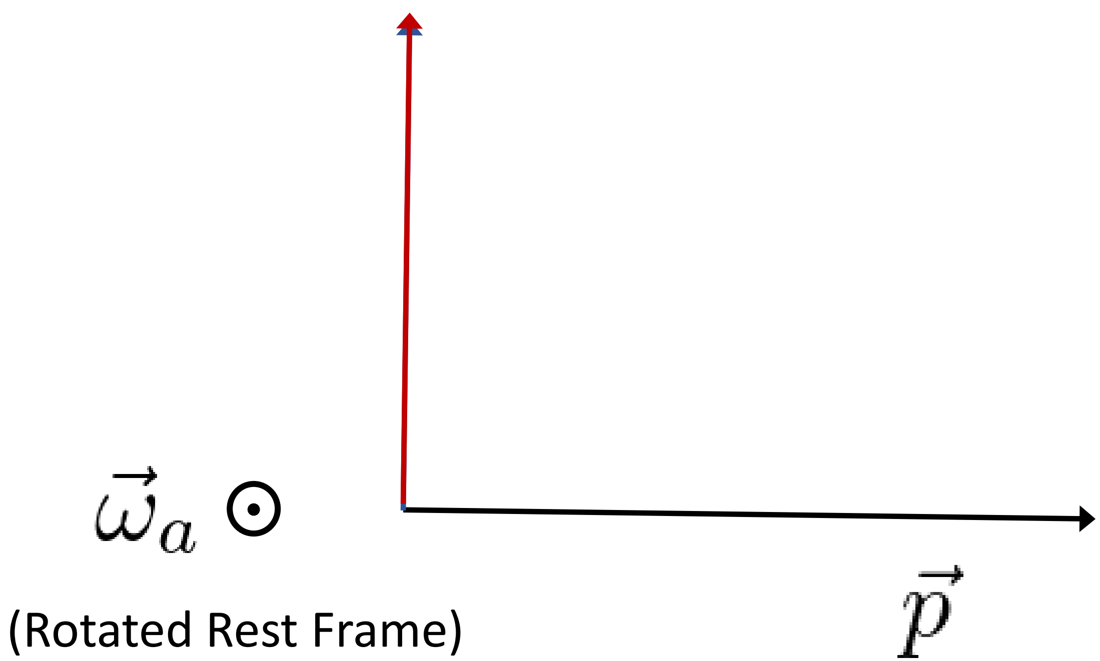


$$t = \frac{2\pi}{m_\phi}$$

Stacking of the FM DM Signal

Could FM precession be hiding in the g-2 data?

Yes - stacking averages away the modulation.

The stacked data is a sum of cosines at different frequencies:

$$S_p \sim \Big(\text{oscillation at}\ \langle \omega(t) \rangle \Big) \cdot \Big(\text{envelope with scale}\ \sigma_{\omega(t)} \Big)$$

Small deviation from the SM value

$$\langle \omega(t) \rangle \sim \omega_{\rm sm} + \omega_{\rm dm} \frac{1}{\sqrt{N_{\rm bunches}}}$$

Envelop is nearly flat over each bunch

$$\sim \left(1 - \omega_{\rm dm}^2 t^2\right)$$

Constraints from Stacking the FM DM Signal

The envelope is detectable as a failure to fit the momentum count as a pure oscillation.

A decaying envelope is already present due to muon losses, modeled and empirically fit to be an $\approx 10\%$ decay

Allowed: $(\omega_{dm} T_{\text{bunch}})^2 < 10\%$ [Bennet et al, 0602035]

If the envelop is ignorable, it follows that the stacked frequency is the discrete mean of the individual bunch frequencies

$$\omega_{\text{stack}} = \omega_{sm} + \omega_{dm} \left(\frac{1}{N_{\text{bunches}}} \Sigma_{t_i} \cos\left(m_\phi t_i\right) \right)$$

$$\Rightarrow \quad \left|\omega_{\text{stack}} - \omega_{sm}\right| \sim \frac{\omega_{dm}}{m T_{\text{run}}} \qquad \left[\text{if } m \lesssim \frac{N_{\text{bunches}}}{T_{\text{run}}}\right]$$

Deviation must be less than (or equal!) the observed frequency.

Time-Resolved Amplitude Tracking

Can we reveal AM in the vertical counts, analogous to the FM precession in the momentum counts?

Total Count, Bunch 1

Fit each momentum count

$$\omega_a(t),\ \phi_a(t)$$

EDM Count, Bunch 1

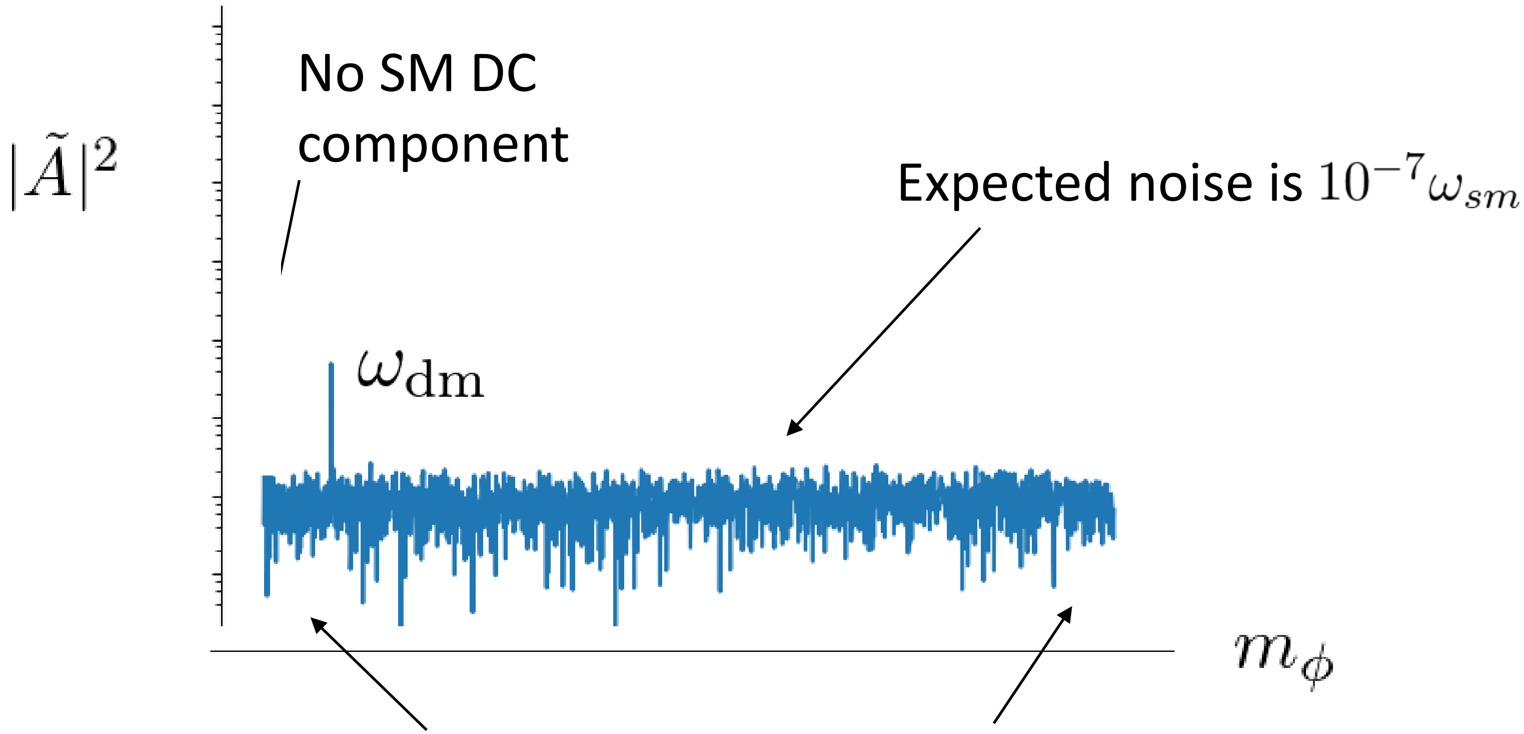
Fit corresponding vertical count to: $$A \sin\left(\omega_a t + \phi_a\right)$$

Time series

$$A(t) \propto \omega_{\perp}(t)$$

Time-Resolved Amplitude Tracking

Fourier transform of vertical amplitude:



$$10^{-23}\,\mathrm{eV} \sim \frac{1}{T_{\rm run}} < m_\phi < \frac{N_{\rm bunches}}{T_{\rm run}} \sim 10^{-15}\,\mathrm{eV}$$

(Assuming a uniform spacing of bunches)

Yukawa Coupling: Static Limit

For sufficiently small m_ϕ, the DM background provides a static contribution to m_μ which will be included in the computation of ω_{sm} – no anomaly is observed.

We then constrain the linear drift of m_μ between (g-2) experiments and the previous determination of m_μ.

In practice, use the magnetic moment ratio μ_μ/μ_p determined from the hyperfine splitting of muonium instead of m_μ [Liu et al, 1999]

$$|\omega_{stack} - \omega_{sm}| \sim \Delta T_{m_\mu} \partial_t \omega_{dm} \qquad \left[\text{if } m_\phi \lesssim \frac{1}{T_{\rm total}}\right]$$

Time between (g-2) and muonium experiments

Total span of (g-2) experiment

Yukawa: Loop Effects

$$\mathcal{L} \supset y\, \phi \bar{\mu} \mu$$

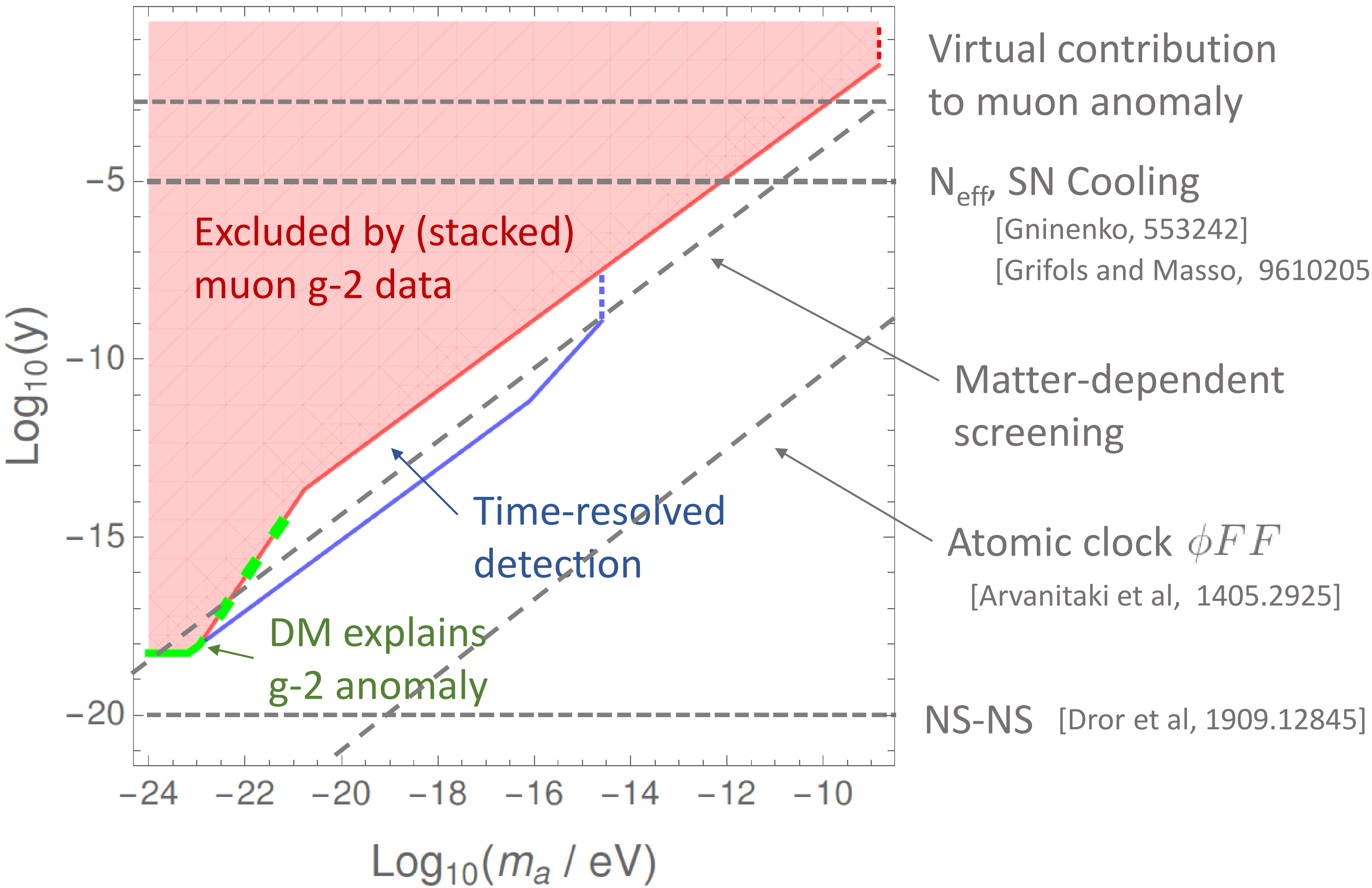
A few of the more egregious examples:

Generates couplings to electrons, nucleons and photons which are highly constrained by atomic clocks and EP tests.

[Arvanitaki et al, 1405.2925]

Induced $\phi^2 \bar{n} n$ produces a matter-dependent potential for the DM, may screen it from terrestrial experiments.

Detection Reach for DM-Muon Yukawa

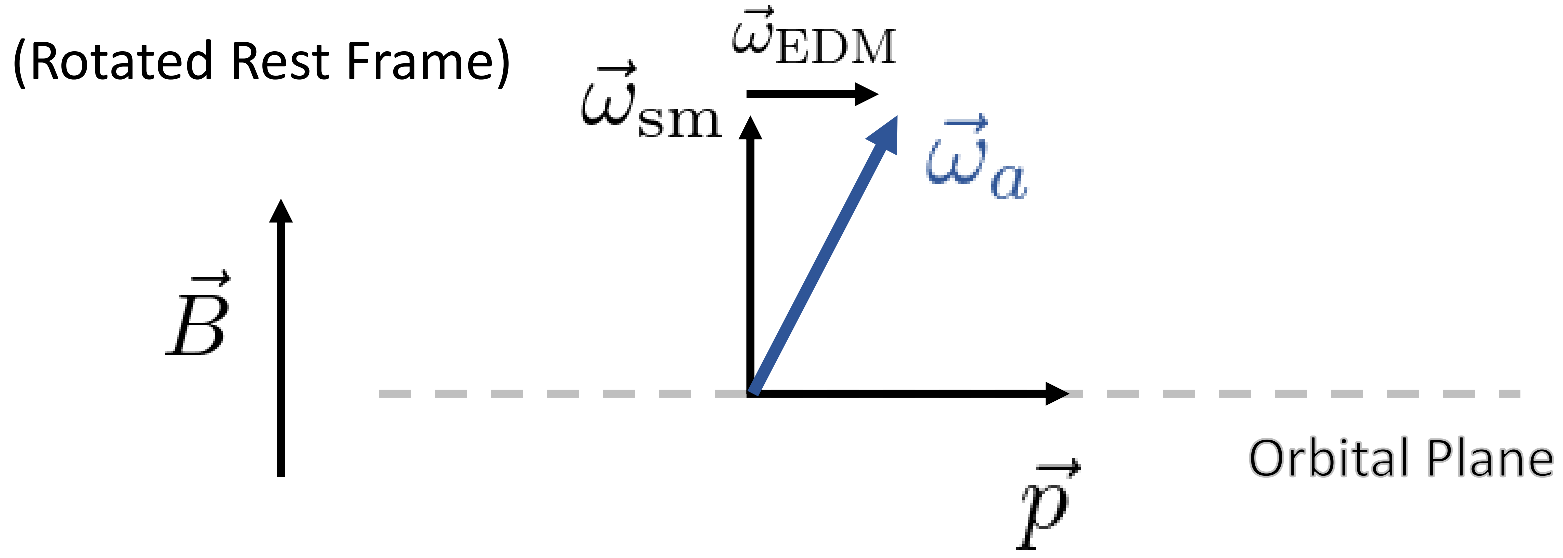


A Muon Electric Dipole Moment

A moving electric dipole will precess in a magnetic field. A muon EDM contributes to $\vec{\omega}_a$ as:

$$\vec{\omega}_{\mathrm{EDM}} = -2\, d_e \left(\vec{v} \times \vec{B} \right)$$

This is orthogonal to the magnetic field!



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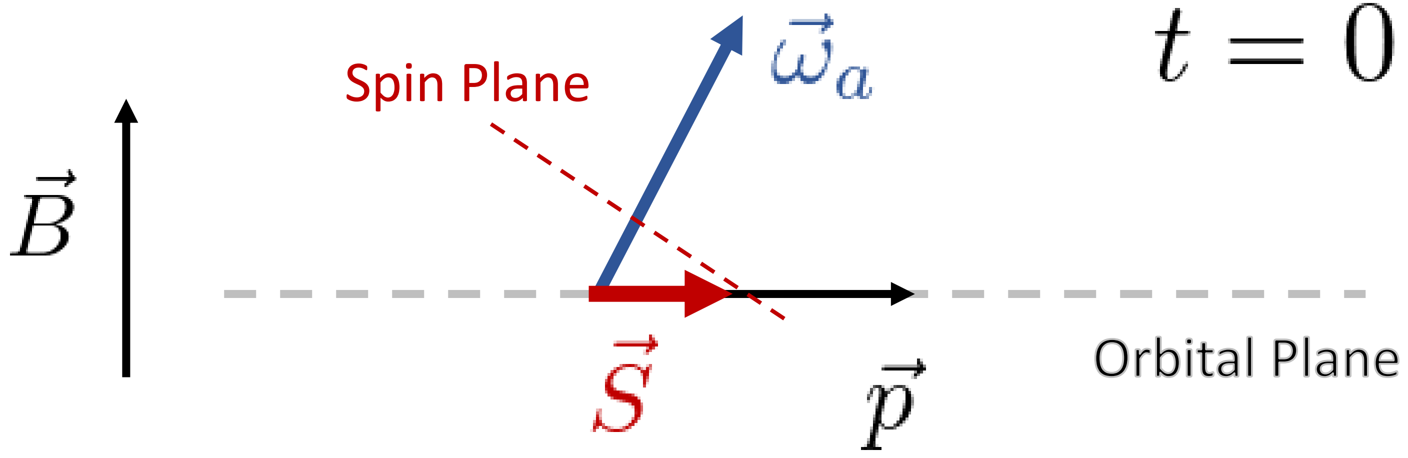

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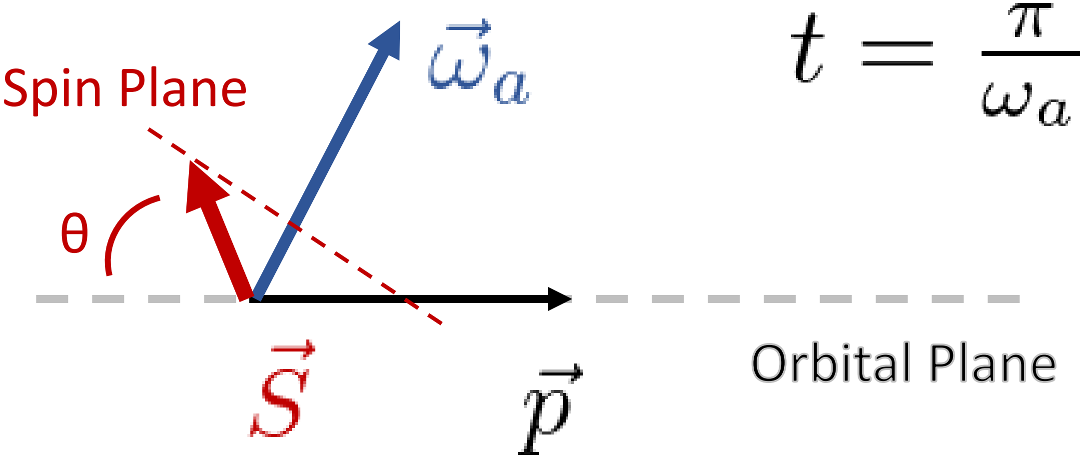
(Rotated Rest Frame)

$$t = \frac{\pi}{\omega_a}$$

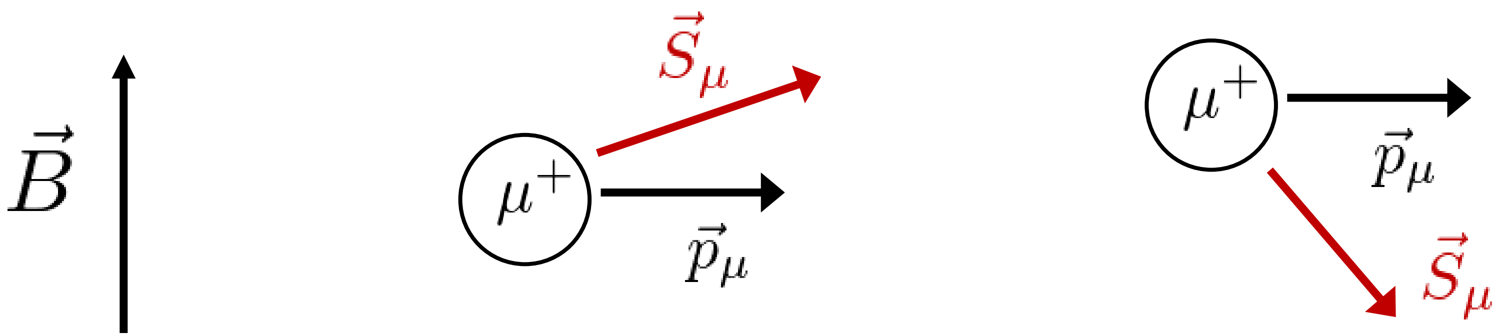
$$\theta \approx \frac{\omega_{\mathrm{edm}}}{\omega_{\mathrm{sm}}}$$



Vertical Count

Detecting an EDM requires measuring upward vs downward moving positrons.

The momentum count is blind to this.



Positrons mostly upward   Positrons mostly downward

Measure the difference ΔN_B in the number of upward versus downward moving positrons:

$$\Delta N_B \propto \vec{S} \cdot \vec{B}$$

Vertical Count

$$\vec{\omega}_{\text{EDM}} = -2\, d_e \left(\vec{v} \times \vec{B}\right)$$

The momentum and vertical components of spin are:

$$\vec{S} \cdot \hat{p} = S_0 \cos\left(\omega_a t\right)$$

$$\vec{S} \cdot \hat{B} = S_0 \frac{\omega_{\text{edm}}}{\omega_{\text{sm}}} \sin\left(\omega_a t\right)$$

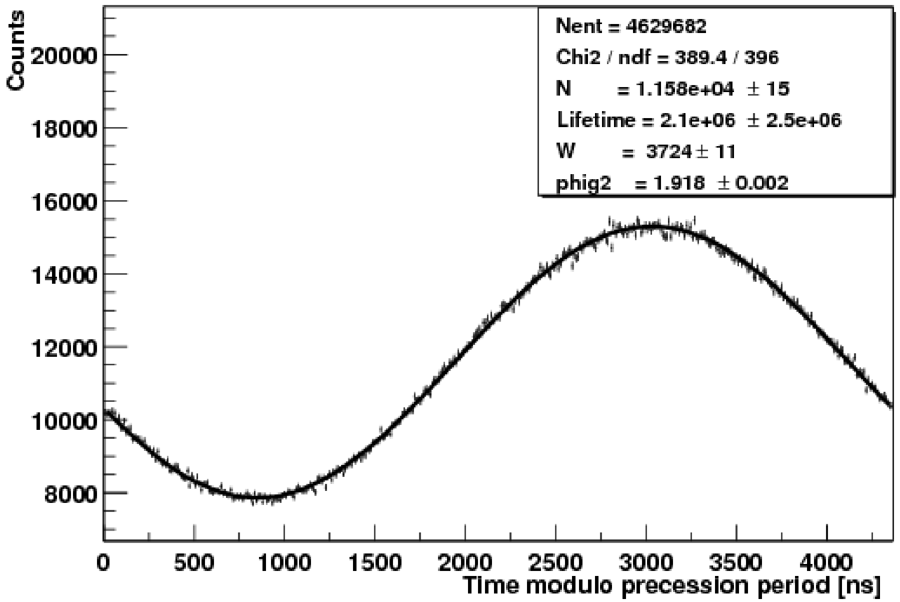
Phase shift between momentum and vertical counts.

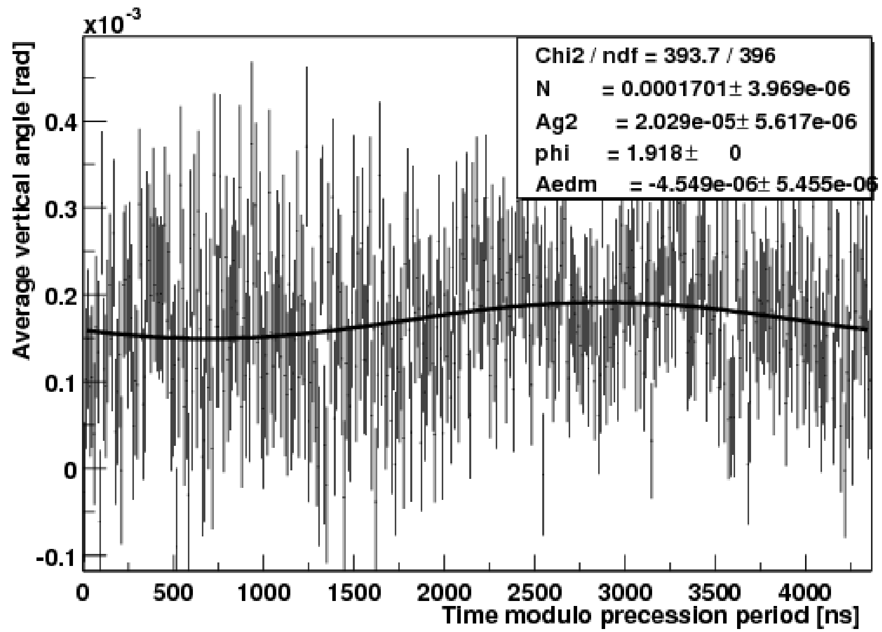
$$\omega_a = \sqrt{\omega_{\text{sm}}^2 + \omega_{\text{edm}}^2}$$

Net increase in momentum count frequency – mimics anomaly.

Stack and Fit

BNL stacked momentum and vertical counts:





[Bennett et al, 0811.1207]

A DM EDM Signal

Precession trajectory (quasi-static limit):

$$S_p = S_0 \underline{\cos} \left[\left(\omega_{sm} + \frac{\omega_{dm}^2}{4\omega_{sm}} \right) t + \frac{\omega_{dm}^2}{8\omega_{sm}} \sin(2m_a t) \right]$$

$$S_B = S_0 \frac{\omega_{dm}}{\omega_{sm}} \cos(m_a t) \underline{\sin} \left[\left(\omega_{sm} + \frac{\omega_{dm}^2}{4\omega_{sm}} \right) t + \frac{\omega_{dm}^2}{8\omega_{sm}} \sin(2m_a t) \right]$$

AC Amplitude Modulation

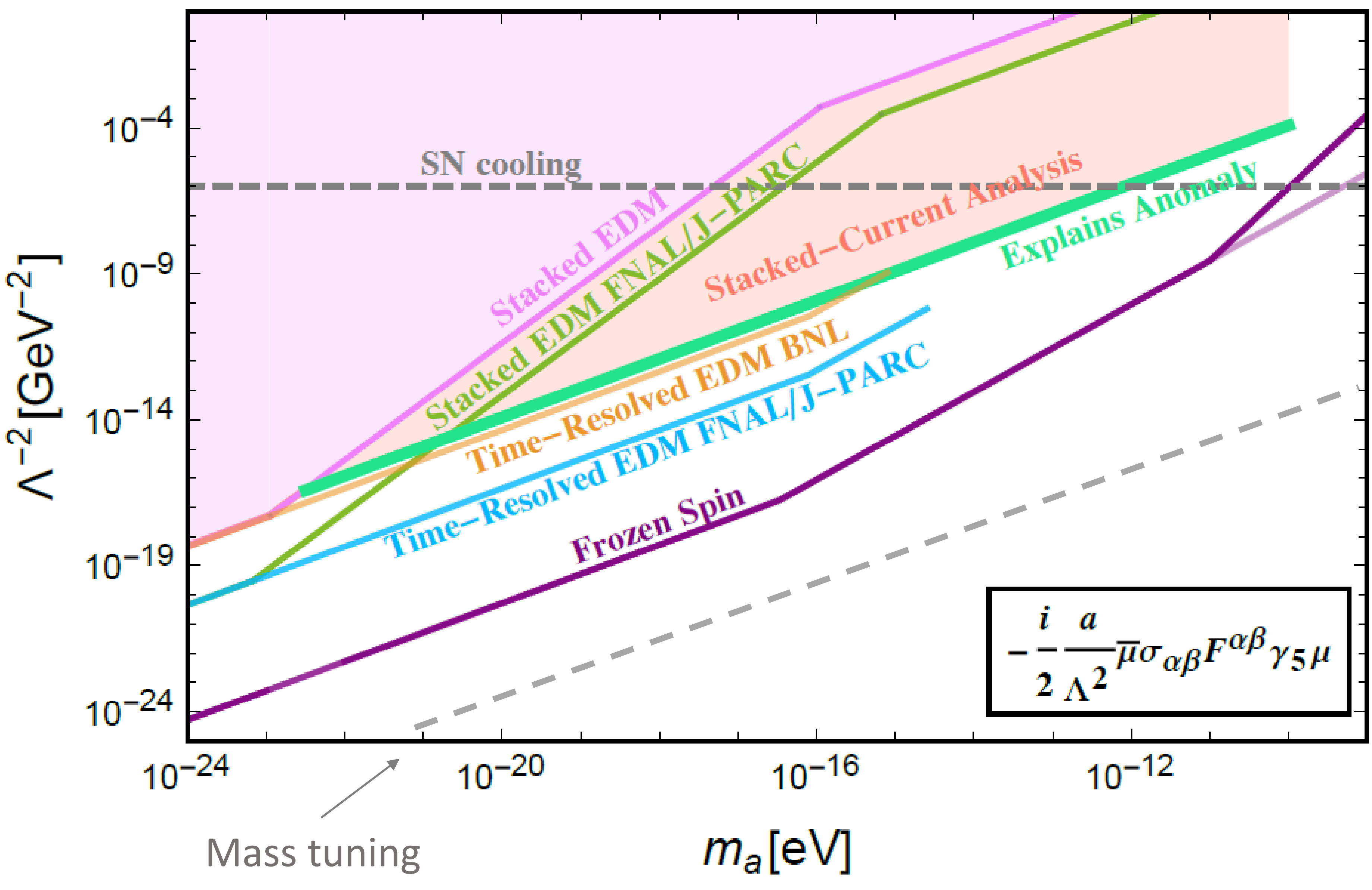
Fixed phase shift between counts

Immediate constraints (or explanation):

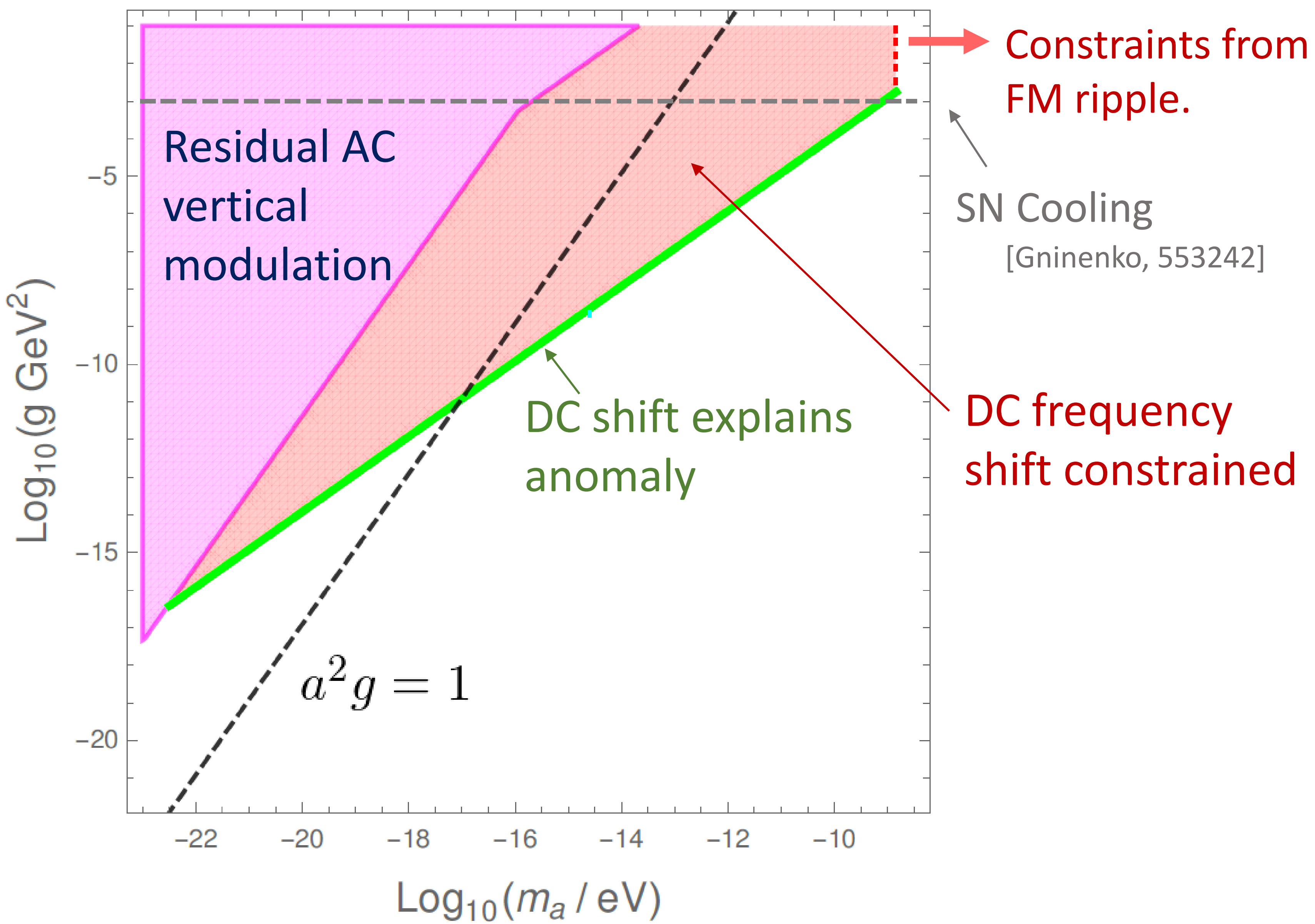
Static FM is a ρ_{dm}–dependent apparent contribution to a_μ.

AM of vertical count will average away in the stacked EDM measurement – we may place constraint from the residual of that averaging.

Mass Tuning: Muon-ALP EDM Coupling



Limits on ALP DM-Muon EDM Coupling



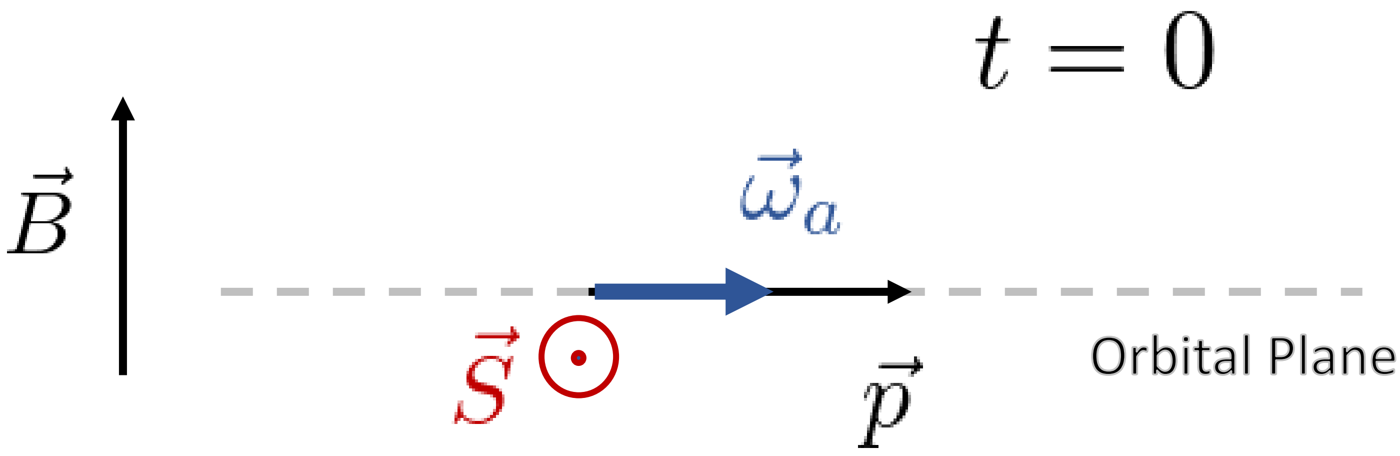
Frozen Spin

A dedicated EDM search would do best by minimizing $\omega_{\rm sm}$.

Choose laboratory EM fields to set $\omega_{\rm sm} = 0$.

[Adelmann et al, 0606034]

(Rotated Rest Frame)



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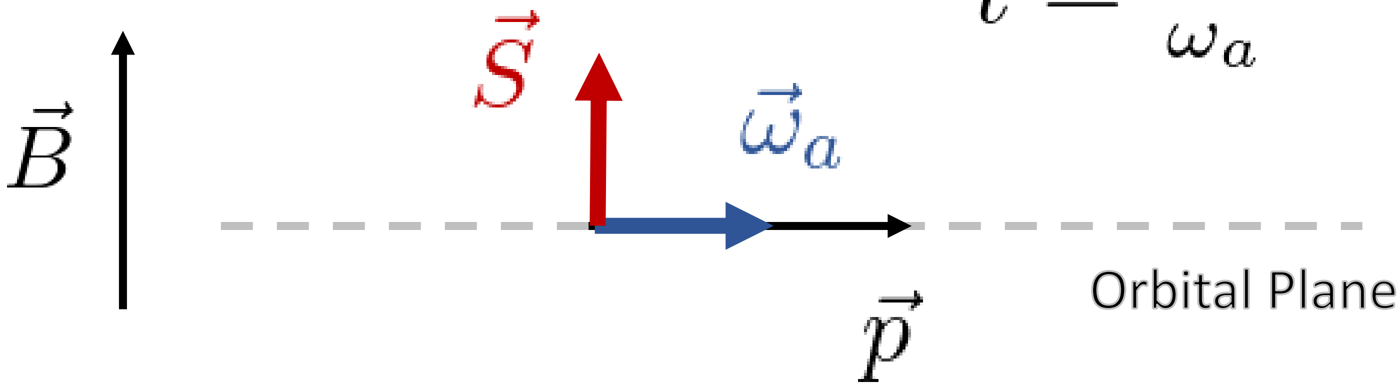
[Adelmann et al, 0606034]

(Rotated Rest Frame)

$$t = \frac{\pi}{\omega_a}$$



Vertical Count: $\vec{S} \cdot \hat{B} = S_0 \cos\left(\omega_{\rm edm} t\right) \approx S_0\, \omega_{\rm edm} t$