Signatures of generalized ALP interactions in SM decays of mesons

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Introduction

Strong CP problem

Cosmology

Dark Matter
Signatures in CMB & LSS
Astrophysical Signatures

Axion

String Theory

Phenomenology

Hierarchy Problem
Baryogenesis
Flavor problem
$(g-2)_\mu$

QCD Axion

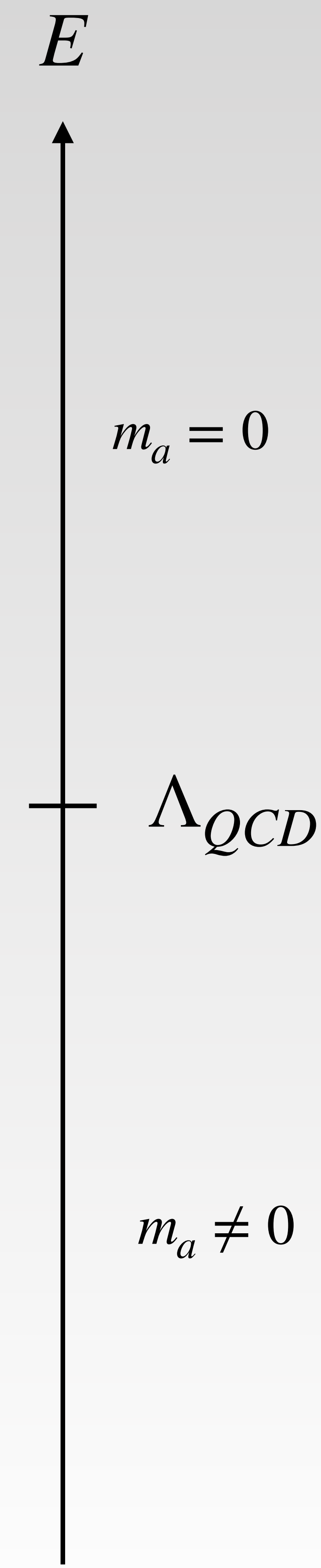
$$\mathscr{L} \supset \frac{\alpha_s}{4\pi}\left(\bar{\theta} + \frac{a}{f_a}\right) G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}$$

Shift symmetry: $a \to a + \text{const}$

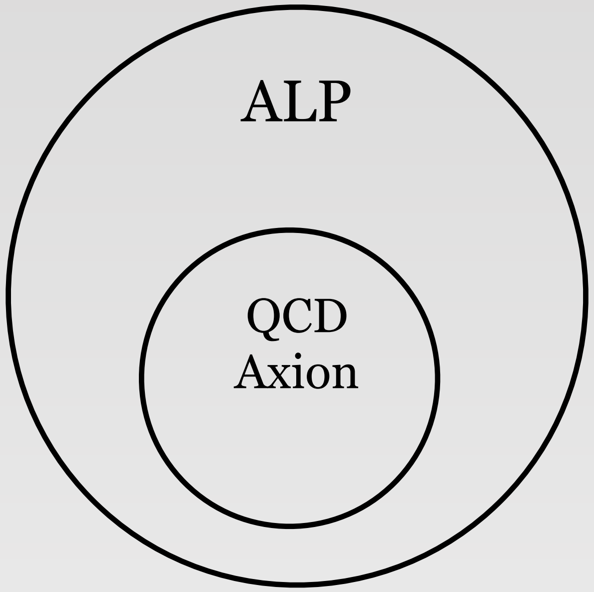
Periodic Symmetry: $a \to a + 2\pi n f_a$

QCD breaks the shift symmetry and generate potential (mass) for axion

$$V_{\text{eff}} \sim \cos\left(\bar{\theta} + \frac{a}{f_a}\right) \qquad m_a^2 \sim m_\pi^2 \frac{f_\pi^2}{f_a^2}$$



Axion Like Particles (ALP)



CP odd

Tree level mass term at UV

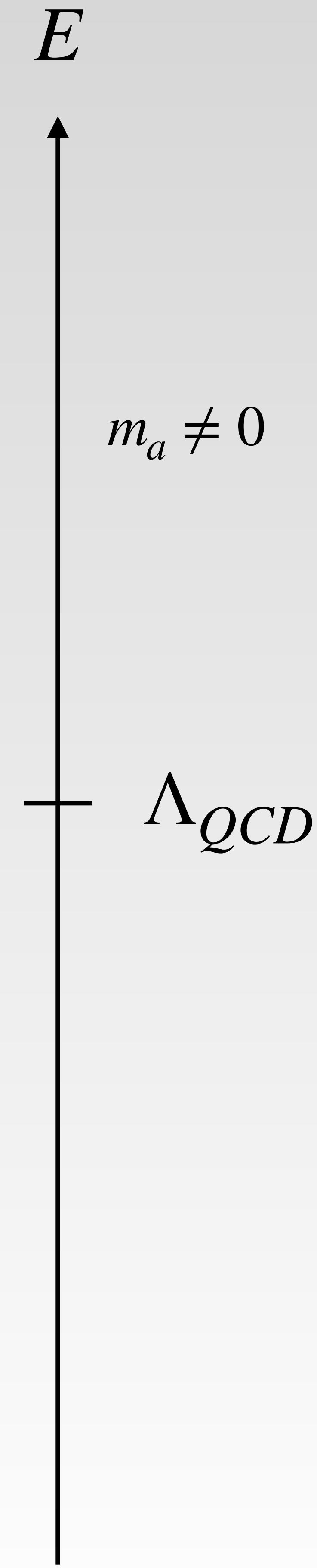
$$\mathscr{L} \supset \frac{\alpha_s}{4\pi}\frac{a}{f_a}G^a_{\mu\nu}\tilde{G}^{a,\mu\nu} + \frac{1}{2}\tilde{m}_a^2 a^2$$

~~Shift symmetry: $a \to a + \text{const}$~~

Periodic Symmetry: $a \to a + 2\pi f_a$

Because of the pNGB nature of the ALP
mass is generated by hard breaking of Goldstone symmetry

The mass (m_a) and the decay constant (f_a) are decoupled



Axion Like Particles (ALP) : Operator with quarks

Quark Couplings

$$c_d\left(\frac{\partial_\mu a}{f_a}\bar{q}\gamma^\mu\gamma^5 q\right) \longrightarrow \text{Axion-Pion kinetic Mixing}$$

Low energy
χEFT

$$\exp\left[c_m\frac{ia}{f_a}\right]\ \bar{q}_L m_q q_R \longrightarrow \text{Axion-Pion mass Mixing}$$

Chiral symmetry is used to set axion gluon coupling to Zero

Other operators are $\frac{a}{f_a}F_{\mu\nu}\tilde{F}^{\mu\nu}$, operators with W, Z, leptons

Focus on operator with quarks and finding their signatures

ALP : Generalized quark Lagrangian

Working in a manifestly EW basis + 3 quark flavor

$$\sum_{i=0}^{8}\left(C_L^i\mathcal{O}_L^i + C_R^i\mathcal{O}_R^i + C_{LR}^i\mathcal{O}_{LR}^i\right) + C_W\mathcal{O}_W + C_Z\mathcal{O}_Z$$

familiar ALP operators

$\mathcal{O}_L^i$	$\frac{1}{f_a}\partial_\mu a\, \bar{q}_L t^i \gamma^\mu q_L$	$i=0,8$: Allowed $i=1,2,4,5$: break EM $i=1(1)7$: break $SU(2)_W$ $i=6,7$: tree FCNC
$\mathcal{O}_R^i$	$\frac{1}{f_a}\partial_\mu a\, \bar{q}_R t^i \gamma^\mu q_R$	$i=0,3,8$: Allowed $i=1,2,4,5$: break EM $i=6,7$: tree FCNC
$\mathcal{O}_{LR}^i$	$\frac{a}{f_a}\, \bar{q}_L t^i M q_R$	
$\mathcal{O}_W$	$-\frac{a}{f_a}\, \bar{q}_L Q^W \not{j}_\pm q_L$	
$\mathcal{O}_Z$	$-\frac{a}{f_a}\, \left(\bar{q}_L Q_L^Z \not{j}_Z q_L + \bar{q}_R Q_R^Z \not{j}_Z q_R\right)$	

New ALP operators

$j_\pm^\mu$ and j_Z are the current replacements of $W_\pm^\mu$ and Z_μ with lepton bilinear : $g_W W_\pm^\mu \to 4G_F j_\pm^\mu$

Triparno Bandyopadhyay, SG, Tuhin S. Roy: arXiv:2112.13147

ALP : Matching → Generalized Meson Lagrangian

$$\mathcal{L} \supset \frac{f_\pi^2}{4}\,\mathrm{Tr}\Big[\,|\partial_\mu U_\pi - i(L_\mu U_\pi - U_\pi R_\mu)|^2\Big] \;+\; \frac{\Lambda f_\pi^2}{2}\,\mathrm{Tr}\Big[\overline{M}U_\pi^\dagger\Big] \;+\; \mathrm{h.c.} \;+\; \cdots .$$

$$L^\mu = QA^\mu + \left(1 + C_Z \frac{a}{f_a}\right) Q_L^Z j_Z^\mu + \left(1 + C_W \frac{a}{f_a}\right) Q^W j_\pm^\mu + \frac{\partial^\mu a}{f_a} C_L^8 t_8 \,,$$

$$R^\mu = QA^\mu + \left(1 + C_Z \frac{a}{f_a}\right) Q_R^Z j_Z^\mu + \frac{\partial_\mu a}{f_a} \sum_{i=3,8} C_R^i t^i \,,$$

$$\overline{M} = \sum_i^{0,3,8} \left(1 + i C_{LR}^i \frac{a}{f_a} t^i + \cdots\right) M.$$

$$U_\pi \equiv \exp\left(\frac{2i\pi^a t^a}{f_\pi}\right)$$

Kinetic Mixing

$$-i\frac{f_\pi^2}{4}\mathrm{Tr}\left[\partial^\mu U_\pi^\dagger L_\mu U_\pi\right] + \mathrm{h.c.} = -\frac{1}{2}\frac{f_\pi}{f_a} C_L^8 \partial^\mu a\, \partial_\mu \eta + \cdots \,,$$

$$i\frac{f_\pi^2}{4}\mathrm{Tr}\left[\partial^\mu U_\pi^\dagger U_\pi R_\mu\right] + \mathrm{h.c.} = \frac{1}{2}\frac{f_\pi}{f_a} C_R^3 \partial^\mu a\, \partial_\mu \pi^0 + \frac{1}{2}\frac{f_\pi}{f_a} C_R^8 \partial^\mu a\, \partial_\mu \eta + \cdots \,,$$

Mass Mixing

$$\frac{\Lambda f_\pi^2}{2}\,\mathrm{Tr}\left[\overline{M}U_\pi^\dagger\right] + \mathrm{h.c.} = \frac{B_0 f_\pi}{f_a}\left[\left(\frac{C_{LR}^8}{\sqrt{3}} m_\Delta - C_{LR}^3 \hat{m}\right) a\pi^0 - \frac{1}{\sqrt{3}}\left(C_{LR}^3 m_\Delta + \frac{C_{LR}^8}{\sqrt{3}}\hat{m} + \frac{2}{\sqrt{3}} C_{LR}^8 m_s\right) a\eta + \cdots\right],$$

where $m_\Delta = \dfrac{m_u - m_d}{2}$, $\hat{m} = \dfrac{m_u + m_d}{2}$, and $B_0 = \Lambda$.

Non Unitary rotation to remove kinetic mixing and an orthogonal rotation to remove mass missing

Triparno Bandyopadhyay, SG, Tuhin S. Roy: arXiv:2112.13147

*messy

Finding ALP: Produce ALP in decays

We perform the diagonalization at $\mathcal{O}(\xi^2)$

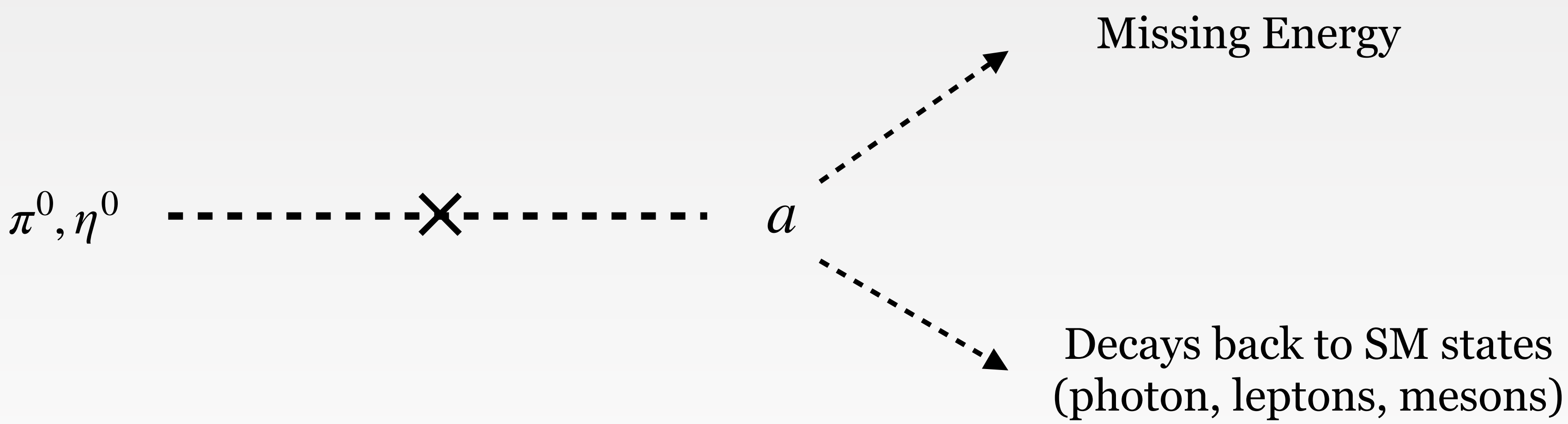
$$\boxed{\xi = \frac{f_\pi}{f_a}}$$

Assumptions: $\dfrac{m_a^2}{\Lambda m_\pi} \lesssim \dfrac{m_\pi^2}{m_\eta^2} \sim \epsilon \sim 10^{-2}$

Old basis → New basis

$$\boxed{\begin{aligned} \pi^0 &\to \epsilon_{\pi a}\, a + \epsilon_{\pi\pi}\, \pi^0 + \epsilon_{\pi\eta}\, \eta^0 \\ \eta^0 &\to \epsilon_{\eta a}\, a + \epsilon_{\eta\pi}\, \pi^0 + \epsilon_{\eta\eta}\, \eta^0 \\ a &\to \epsilon_{aa}\, a + \epsilon_{a\pi}\, \pi^0 + \epsilon_{a\eta}\, \eta^0 \end{aligned}}$$

Produce ALP in the final state in mesonic processes



Signal (bound) depends on ALP decay length and branching ratios

For review see: Bauer et. al., 2110.10698

Finding ALP: Look into the SM channels

$$\pi^0 \to \epsilon_{\pi a}\, a + \epsilon_{\pi\pi}\, \pi^0 + \epsilon_{\pi\eta}\, \eta^0$$
$$\eta^0 \to \epsilon_{\eta a}\, a + \epsilon_{\eta\pi}\, \pi^0 + \epsilon_{\eta\eta}\, \eta^0$$
$$a \to \epsilon_{aa}\, a + \epsilon_{a\pi}\, \pi^0 + \epsilon_{a\eta}\, \eta^0$$

Changes the rates for the SM processes
(Modifications of Form Factors)

Pros : Robust bounds

Bounds do not depend on ALP decay length/branching ratios
Bounds do not explicitly depends on ALP mass (implicit dependence through mixing parameters)

Cons : Theoretical estimate the SM rate

The theoretical calculations of SM meson decays often have large hadronic uncertainties : Large theory error

Modifications of the Form Factor (FF)

FF for the $K^+ \to \pi^0 \ell^+ \nu_\ell$ and $K^0 \to \pi^- \ell^+ \nu_\ell$ processes

$$\langle \pi^0(p_\pi)|\bar{s}\gamma_\mu u|K^+(p_K)\rangle \equiv \frac{1}{\sqrt{2}}\left[f_{+,\,\mathrm{SM}}^{K^+\pi^0}(q^2)\,Q_\mu + f_{-,\,\mathrm{SM}}^{K^+\pi^0}(q^2)\,q_\mu\right],$$
$$\langle \pi^+(p_\pi)|\bar{s}\gamma_\mu u|K^0(p_K)\rangle \equiv f_{+,\,\mathrm{SM}}^{K^0\pi^-}(q^2)\,Q_\mu + f_{-,\,\mathrm{SM}}^{K^0\pi^-}(q^2)\,q_\mu,$$

Matching quark operator to meson current

$$\bar{s}\gamma_\mu u = -f_\pi^2 \mathrm{Tr}\left[U_\pi^\dagger\left(t^4 - it^5\right)\partial_\mu U_\pi\right]$$
$$\supset \left(K^0\partial_\mu\pi^+ - \partial_\mu K^0\pi^+\right) + \frac{1}{\sqrt{2}}\left[K^+\partial_\mu\left(\pi_0 + \sqrt{3}\eta\right) - \partial_\mu K^+\left(\pi_0 + \sqrt{3}\eta\right)\right],$$

Not affected by ALP interaction

Affected by ALP interaction

$$\frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} = 1 - \sqrt{3}\,\epsilon - \xi^2\frac{C_3}{8}\left[C_A^3 + C_{LR}^3 + 2\sqrt{3}C_{LR}^8\right].$$

$$C_{A/V}^8 = C_R^8 \mp C_L^8$$
$$C_A^3 \equiv C_R^3$$

Iso-spin breaking effects

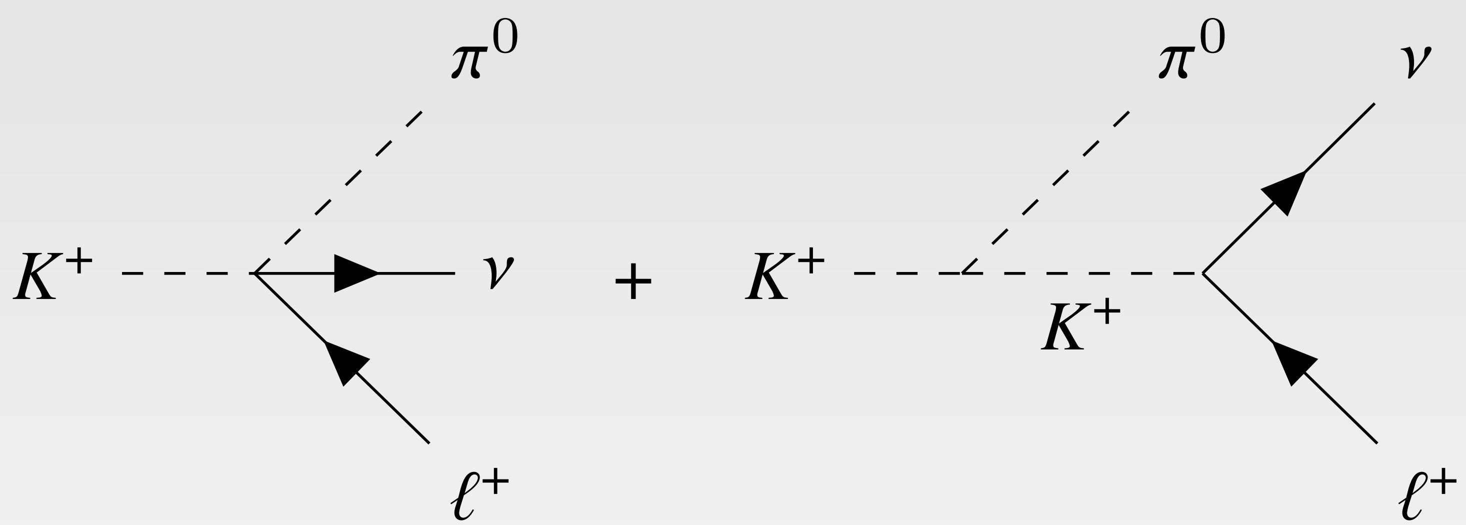
$$C_3 = C_{LR}^3 - C_R^3;\; C_8 = C_{LR}^8 - C_A^8$$

$$Q^\mu = p_K^\mu + p_\pi^\mu;\quad q_\mu = p_K^\mu - p_\pi^\mu.$$

Modifications of the Form Factor (FF) at $\mathcal{O}(p^2)$ for $K_{\ell 3}$

$$\tilde{f}_+^{K^+\pi^0}(0) = \alpha^{(0)}_{K^+\pi_0} + \xi^2 \left(\alpha^{(2)}_{K^+\pi_0} + i\tilde{\alpha}^{(2)}_{K^+\pi_0}\right)$$

$$\tilde{f}_-^{K^+\pi^0}(0) = \beta^{(0)}_{K^+\pi_0} + \xi^2 \left(\beta^{(2)}_{K^+\pi_0} + i\tilde{\beta}^{(2)}_{K^+\pi_0}\right)$$



Operator in original basis that contributes to $K_{\ell 3}$

Operator $\left(\mathcal{O}^i_{K^+_{\ell_3}}\right)$
$\mathcal{O}^0_{K^+_{\ell_3}} = [K^+\partial_\mu(\pi_0+\sqrt{3}\eta) - \partial_\mu K^+(\pi_0+\sqrt{3}\eta)]j^\mu_{-,\ell}$
$\mathcal{O}^1_{K^+_{\ell_3}} = (K^+\partial_\mu a - \partial_\mu K^+ a)\, j^\mu_{-,\ell}$
$\mathcal{O}^2_{K^+_{\ell_3}} = (K^+\partial_\mu a + \partial_\mu K^+ a)\, j^\mu_{-,\ell}$
$\mathcal{O}^3_{K^+_{\ell_3}} = \partial^\mu a\ (\partial_\mu K^+ K^- - K^+\partial_\mu K^-)$
$\mathcal{O}^4_{K^+_{\ell_3}} = \partial_\mu K^+ j^\mu_-$

$$\alpha^{(0)}_{K^+\pi_0} = 1-\sqrt{3}\,\epsilon,\ \alpha^{(2)}_{K^+\pi_0} = -\frac{C_3}{8}(C^3_{LR} - C^3_R + 2\sqrt{3}(C^8_{LR} - C^8_R)),\ \tilde{\alpha}^{(2)}_{K^+\pi_0} = -\frac{1}{2}C_3 C_W$$

$$\beta^{(0)}_{K^+\pi_0} = 0,\ \beta^{(2)}_{K^+\pi_0} = -\frac{\sqrt{3}}{4}C_3 C^8_L,\ \tilde{\beta}^{(2)}_{K^+\pi_0} = \frac{1}{2}C_3 C_W$$

Triparno Bandyopadhyay, SG, Tuhin S. Roy: arXiv:2112.13147

Effect of $f_-(q^2)$ & $f_+(q^2)$ modifications

$$\langle \pi^+(p_\pi)|\bar{s}\gamma_\mu u|K^0(p_K)\rangle \equiv f_+^{K^0\pi^-}(q^2)\, Q_\mu + f_-^{K^0\pi^-}(q^2)\, q_\mu,$$

Dominant effect

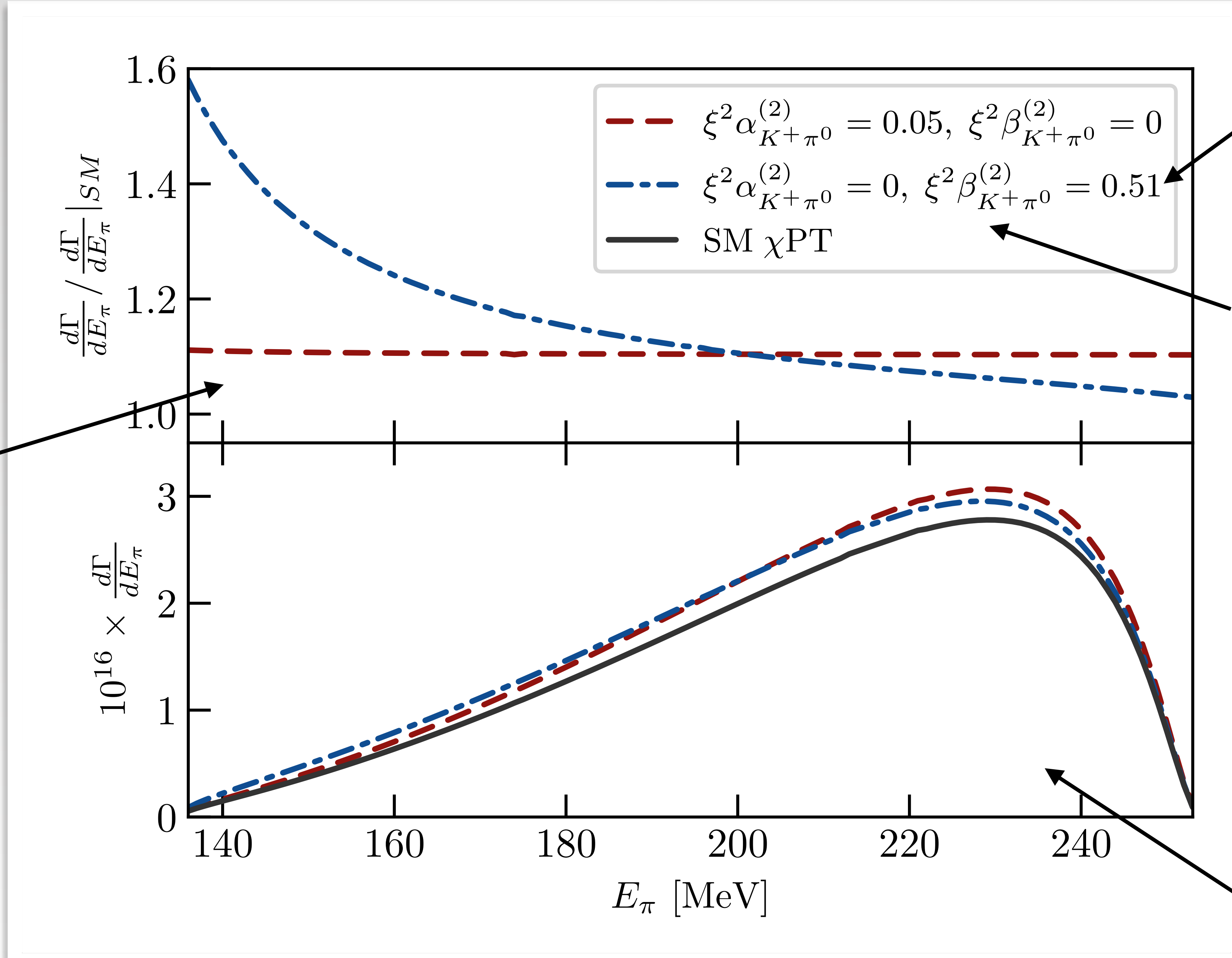
Effect is suppressed by the final state lepton mass
Bigger effect in $K_{\mu 3}$ decays than K_{e3}

$$q_\mu \left(\bar{\nu}\gamma^\mu P_L \ell\right) = (p_\nu^\mu + p_\ell^\mu)\left(\bar{\nu}\gamma_\mu P_L \ell\right) = m_l \left(\bar{\nu} P_R \ell\right)$$

Due to the different momentum dependences of the two FFs —
the total rate and the differential rate are both modified

Effect on the decay spectrum (of $K_{\mu3}$)

$$\xi = \frac{f_\pi}{f_a}$$



Large value needed to surpass lepton mass suppression

Chosen such a way that total decay rates of NP are same

Comparison with SM

Differential rate

Triparno Bandyopadhyay, SG, Tuhin S. Roy: arXiv:2112.13147

Comparison with $K_{\ell 3}$ Data

$$f_{+/0,\,\mathrm{SM}}^{K^+\pi^0}(t) = f_{+/0,\,\mathrm{SM}}^{K^+\pi^0}(0)\left[1 + \lambda_{K^+\pi^0}^{+/0,\,(0)}\frac{t}{M_\pi^2} + \frac{1}{2}\lambda_{K^+\pi^0}^{\prime\,+/0,\,(0)}\frac{t^2}{M_\pi^4}\right] + \cdots\,,$$

$$f_{-,\,\mathrm{SM}}^{K^+\pi^0}(t) = \left[f_{0,\,\mathrm{SM}}^{K^+\pi^0}(t) - f_{+,\,\mathrm{SM}}^{K^+\pi^0}(t)\right]\frac{M_K^2 - M_\pi^2}{t}\,.$$

Experimental collaboration/lattice give results in terms $f_0(q^2)$ instead of $f_-(q^2)$

The theoretical computation of the FFs are taken from European twisted mass collaboration

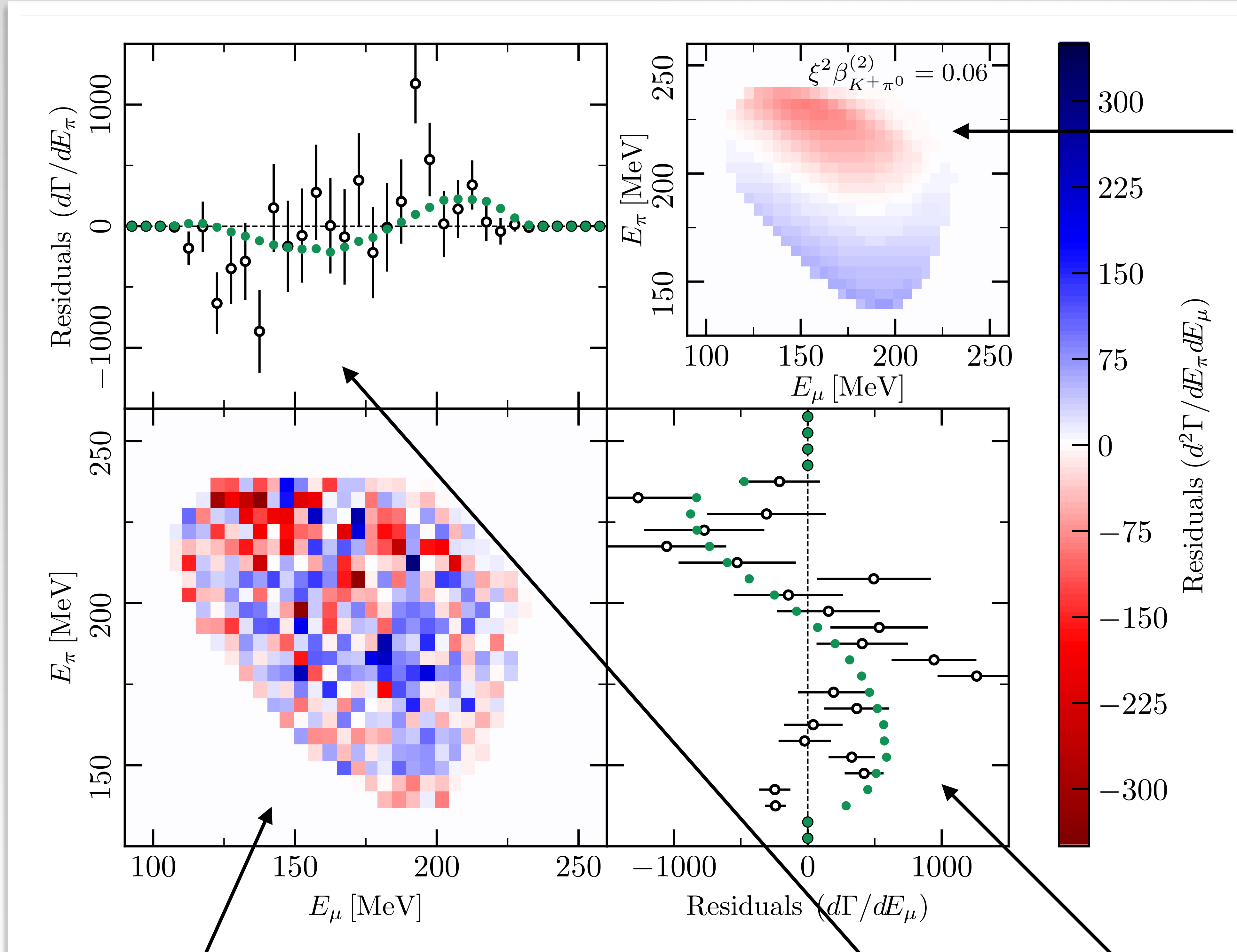
Phys. Rev. D 93 (2016) 114512 [1602.04113].

We simulate the total NP signal for the NA 48/2 experiment and compare it against the data

JHEP 10 (2018) 150 [1808.09041]

Comparison with Data : NA 48/2

$$\xi = \frac{f_\pi}{f_a}$$



2D residual events from NP

2D residual (Data - Theory) event distribution

Marginalized residual signal compared with fluctuation in data (error bar is experimental error)

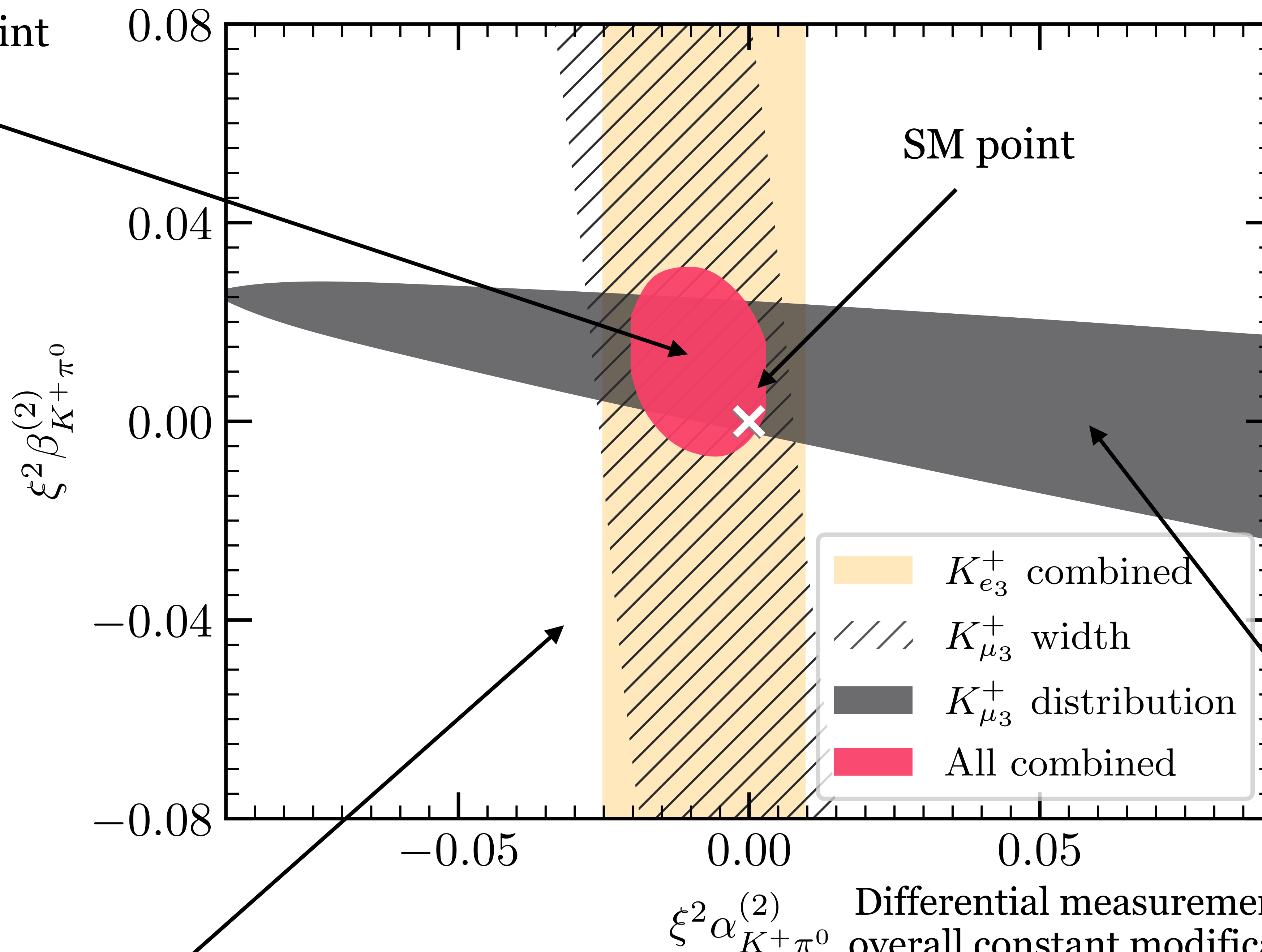
Triparno Bandyopadhyay, SG, Tuhin S. Roy: arXiv:2112.13147

Constraints at 95% C.L: NA48/2 + total decay width

With both theory & experimental error

Combined constraint

$$\xi = \frac{f_\pi}{f_a}$$



Differential measurement is insensitive to overall constant modification of decay rate

Total decay width measurement is mostly sensitive on α

$$\overline{|\mathcal{A}|}^2_{K_{l3}} = 2G_F^2 |V_{\bar{s}u}|^2 C_{\rm cor} \left[1 + 2\xi^2 \frac{\alpha^{(2)}_{K^+\pi^0}}{\alpha^{(0)}_{K^+\pi^0}}\right] (2H\cdot p_\ell \, H\cdot p_{\nu_\ell} - H^2 p_\ell \cdot p_{\nu_\ell}),$$

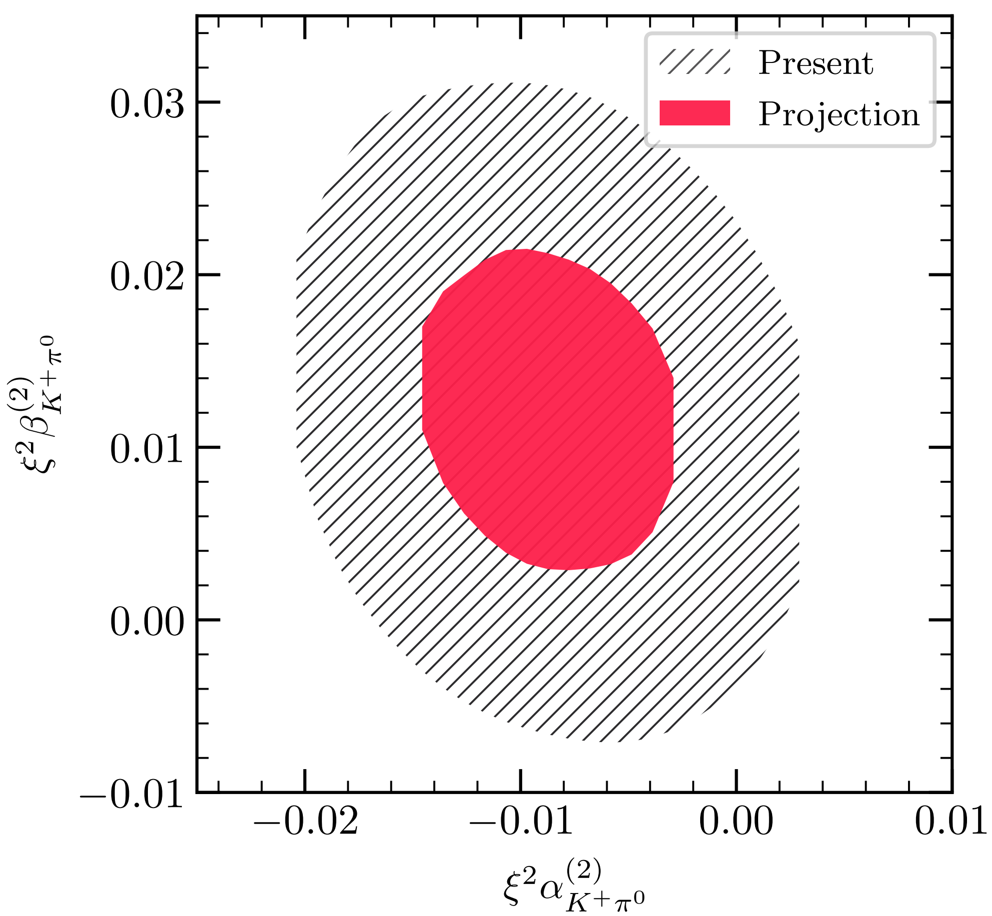
$$\text{where } H_\mu \equiv f^{K^+\pi^0}_{+,\rm SM}(t)\, Q_\mu + \left[1 + \xi^2 \left(\frac{\beta^{(2)}_{K^+\pi^0}}{\delta\beta^{(0)}_{K^+\pi^0}} - \frac{\alpha^{(2)}_{K^+\pi^0}}{\alpha^{(0)}_{K^+\pi^0}}\right)\right] f^{K^+\pi^0}_{-,\rm SM}(t)\, q_\mu.$$

Triparno Bandyopadhyay, SG, Tuhin S. Roy: arXiv:2112.13147

Constraints at 95% C.L: NA48/2 + total decay width

$$\xi = \frac{f_\pi}{f_a}$$

	$\xi^2\beta^{(2)}_{K^+\pi^0}\ \left(\xi^2\alpha^{(2)}_{K^+\pi^0}=0\right)$	$\xi^2\alpha^{(2)}_{K^+\pi^0}\ \left(\xi^2\beta^{(2)}_{K^+\pi^0}=0\right)$
$K^+_{\mu 3}$	$-0.006 < \xi^2\beta^{(2)}_{K^+\pi^0} < 0.026$	$-0.021 < \xi^2\alpha^{(2)}_{K^+\pi^0} < 0.007$
$K^+_{\mu 3} + K^+_{e3}$	$-0.006 < \xi^2\beta^{(2)}_{K^+\pi^0} < 0.026$	$-0.018 < \xi^2\alpha^{(2)}_{K^+\pi^0} < 0.003$



Future projection of the bound with experimental and theoretical error reduced to 50%

Triparno Bandyopadhyay, SG, Tuhin S. Roy: arXiv:2112.13147

Direct vis á vis indirect detection

Indirect Channel		Direct Channel
$K^+ \to \pi^0 \ell^+ \nu_\ell$		$K^+ \to a \ell^+ \nu_\ell$
$\mathscr{A}_{\text{dir}} \neq 0$ (In some limits)	⟷	Pion Phobia limit: $\mathscr{A}_{\text{dir}} \to 0$

A limit exists where direct rate is zero but indirect rate is nonzero

$$C_W = 0, \quad C^3_{LR} = C^3_R, \quad \text{and} \quad C^8_{LR} = C^8_R,$$

Indirect detection removes blind spots

Triparno Bandyopadhyay, SG, Tuhin S. Roy: arXiv:2112.13147

Sum Rules

Identifying nature of ALP coupling from observable

Distinguish between ALP interactions via mixing only vs interaction with weak current

SM sum:

$$\frac{1}{4}\left|f_{+,\,\mathrm{SM}}^{K^+\pi^0}(0)\right|^2 + \frac{3}{4}\left|f_{+,\,\mathrm{SM}}^{K^+\eta}(0)\right|^2 = 1.$$

← Completeness of basis

Sum in presence of ALP:

$$\frac{1}{4}\left|\tilde{f}_{+}^{K^+\pi^0}(0)\right|^2 + \frac{3}{4}\left|\tilde{f}_{+}^{K^+\eta}(0)\right|^2 = 1 - \frac{\xi^2}{16}\left(C_{LR}^3 - C_R^3 + \sqrt{3}(C_{LR}^8 - C_R^8)\right)^2 + \xi^2\frac{3}{16}(C_L^8)^2$$

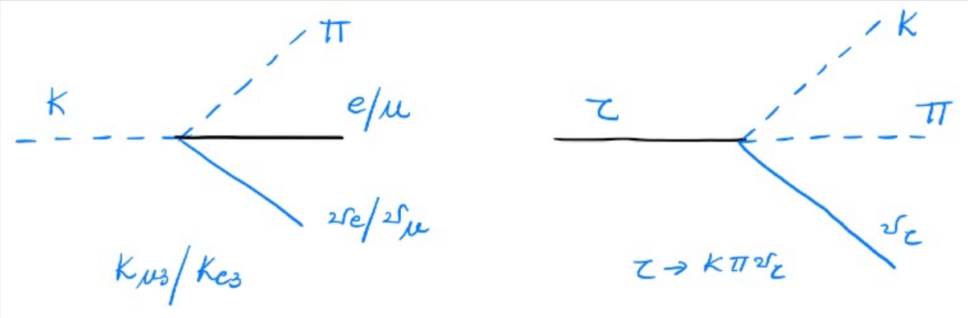
The sum can be greater than 1 which is a tale-tell signature of $\mathcal{O}_L^8$ operator

Triparno Bandyopadhyay, SG, Tuhin S. Roy: arXiv:2112.13147

Future Direction

Large theoretical Form Factor uncertainties ☹️
Measure Form Factor from “clean” channels 🙂

Effect of $\beta^{(2)}$ is surpassed by Lepton mass ☹️
Look in strange decays τ of tau lepton 🙂



$$f_+^{K^-\pi^0}(s)/f_+^{\bar{K}^0\pi^-}(s) = \left(1+\sqrt{3}\,\epsilon\right)\left(1+\tilde{g}\frac{m_K^2}{(4\pi F_\pi)^2}\frac{s}{m_{K^*}^2}\,\epsilon\right)$$

Antonelli et. al., arXiv:1304.8134

Measurement of FFs from $\tau^- \to \bar{K}^0\pi^-\nu_\tau$
(NOT affected by ALP)

Predict the theory FF for $\tau^- \to K^-\pi^0\nu_\tau$
(Affected by ALP)

Compare against observed distribution to constrain ALP

*Distribution data from BABAR
*Need more precise measurement from Belle

N.B: Implications for V_{us} anomalies

Triparno Bandyopadhyay, SG, Girish Kumar: arXiv:22xx.xxxxx

Conclusion

- Modification of SM decays (Indirect search) is a viable avenue to look for ALP signatures (Precise estimate of theory FFs are very essential)
- Indirect searches complement the direct searches for the ALP signatures and eliminate certain blind spots (pion phobia)
- Precision measurements of decay distributions of SM processes are vital to this program
- Similar exercises for B and D systems can provide significant ALP constraints

Post ~~Credit~~ Conclusion

QCD Axion

Smallness is not protected by any symmetry

$$\mathscr{L} \supset \frac{\alpha_s}{4\pi} \bar{\theta} \, G^a_{\mu\nu} \tilde{G}^{a,\mu\nu}$$

Measurement of Neutron EDM : $\bar{\theta} < 10^{-10}$

Goldstone Boson

$$\mathscr{L} \supset \frac{\alpha_s}{4\pi} \left(\bar{\theta} + \frac{a}{f_a} \right) G^a_{\mu\nu} \tilde{G}^{a,\mu\nu}$$

Axion shift symmetry is anomalous under the QCD → solves the Strong CP problem

Strong QCD generates potential ⟶ $V_{\rm eff} \sim \cos\left(\bar{\theta} + \frac{a}{f_a} \right)$

↓

$$\langle a \rangle = -\bar{\theta} f_a$$

$$a_{\rm phys} = a - \langle a \rangle$$

Peccei & Quinn, Phys. Rev. Lett 38, 1440 (1977)

ALP: Chiral Symmetry

$$\mathscr{L} \supset c_G \frac{\alpha_s}{4\pi} \frac{a}{f_a} G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} + \frac{1}{2} \tilde{m}_a^2 a^2$$

$$+ c_d \left(\frac{\partial_\mu a}{f_a} \bar{q} \gamma^\mu \gamma^5 q \right) + \exp\left[c_m \frac{ia}{f_a} \right] \bar{q}_L m_q q_R$$

<u>Not all couplings are independent</u>

Under axion dependent chiral rotation : $q \to \exp\left[i\alpha\gamma_5 \frac{a}{f_a} \right] q$

$$\boxed{\begin{array}{c} c_G \to c_G + 2\alpha \\ c_d \to c_d - \alpha \\ c_m \to c_m + 2\alpha \end{array}}$$

This freedom can be used to set $c_G \to 0$

* convenient for Chiral matching

Origin story of $\mathcal{O}_W$ and $\mathcal{O}_Z$

$$V(H,a) = -\mu^2(a)\, H^\dagger H + \frac{1}{2}\lambda(a)\left(H^\dagger H\right)^2 .$$

Axion dependent Higgs potential

CP even state

$$v^2 \to v^2(a) = \frac{\mu^2(a)}{\lambda(a)} \equiv v^2\left(1 - C_v \frac{a}{f_a} + \dots\right)$$

$$\implies G_F \to G_F(a) = G_F\left(1 + C_v \frac{a}{f_a} + \dots\right).$$

$$\boxed{C_W = C_Z = C_v}$$

$\mathcal{O}_W$	$-\frac{a}{f_a}\,\bar{q}_L Q^W \not{J}_\pm q_L$
$\mathcal{O}_Z$	$-\frac{a}{f_a}\left(\bar{q}_L Q_L^Z \not{J}_Z q_L + \bar{q}_R Q_R^Z \not{J}_Z q_R\right)$

~~ALP shift symmetry~~

ALP periodic symmetry

$$\frac{a}{f_a} \subset \sin\left(\frac{a}{f_a}\right)$$

ALP : Matching → Generalized Meson Lagrangian

Deriving low energy Axion pion interactions : SU(3) Chiral Lagrangian

$$\mathcal{L} \supset \frac{f_\pi^2}{4}\,\mathrm{Tr}\Big[\,|\partial_\mu U_\pi - i(L_\mu U_\pi - U_\pi R_\mu)|^2\Big] + \frac{\Lambda f_\pi^2}{2}\,\mathrm{Tr}\Big[\overline{M} U_\pi^\dagger\Big] + \text{h.c.} + \cdots .$$

$$U_\pi \equiv \exp\left(\frac{2i\pi^a t^a}{f_\pi}\right)$$

$SU(3)$ pion matrix

$$L^\mu = QA^\mu + \left(1 + C_Z\frac{a}{f_a}\right)Q_L^Z j_Z^\mu + \left(1 + C_W\frac{a}{f_a}\right)Q^W j_\pm^\mu + \frac{\partial^\mu a}{f_a}C_L^8 t_8\,,$$

$$R^\mu = QA^\mu + \left(1 + C_Z\frac{a}{f_a}\right)Q_R^Z j_Z^\mu + \frac{\partial_\mu a}{f_a}\sum_{i=3,8} C_R^i t^i\,,$$

$$\overline{M} = \sum_i^{0,3,8}\left(1 + iC_{LR}^i\frac{a}{f_a}t^i + \cdots\right)M.$$

Quark mass matrix: diag(m_u, m_d, m_s)

Modifications of the Form Factor (FF) [including Higher order effect]

$$
\begin{aligned}
\mathrm{Re}\left(\tilde{f}_{+}^{K^+\pi^0}(t)\right) = &\left(\alpha^{(0)}_{K^+\pi^0} + \xi^2\alpha^{(2)}_{K^+\pi^0} + \underline{\delta\alpha^{(0)}_{K^+\pi^0}} + \underline{\xi^2\delta\alpha^{(2)}_{K^+\pi^0}}\right) \\
&\times\left[1+\left(\lambda^{+,(0)}_{K^+\pi^0} + \underline{\xi^2\lambda^{+,(2)}_{K^+\pi^0}}\right)\frac{t}{M_\pi^2} + \left(\lambda'^{+,(0)}_{K^+\pi^0} + \underline{\xi^2\lambda'^{+,(2)}_{K^+\pi^0}}\right)\frac{t^2}{2M_\pi^4}\right] + \cdots \\
\simeq &\left[1+\xi^2\frac{\alpha^{(2)}_{K^+\pi^0}}{\alpha^{(0)}_{K^+\pi^0}}\right] f_{+,\,\mathrm{SM}}^{K^+\pi^0}(t)\,,
\end{aligned}
$$

<u>Amplitude square for $K_{\ell 3}$</u>

$$
\overline{|\mathcal{A}|}^2_{K_{l3}} = 2G_F^2|V_{\bar{s}u}|^2 C_{\mathrm{cor}}\left[1+2\,\xi^2\frac{\alpha^{(2)}_{K^+\pi^0}}{\alpha^{(0)}_{K^+\pi^0}}\right](2H\cdot p_\ell\, H\cdot p_{\nu_\ell} - H^2 p_\ell\cdot p_{\nu_\ell}),
$$

$$
\text{where } H_\mu \equiv f_{+,\mathrm{SM}}^{K^+\pi^0}(t)\,Q_\mu + \left[1+\xi^2\left(\frac{\beta^{(2)}_{K^+\pi^0}}{\delta\beta^{(0)}_{K^+\pi^0}} - \frac{\alpha^{(2)}_{K^+\pi^0}}{\alpha^{(0)}_{K^+\pi^0}}\right)\right] f_{-,\mathrm{SM}}^{K^+\pi^0}(t)\,q_\mu.
$$

Lattice computation

$$\lambda_{K^+\pi^0}^{+\,(0)} = \Lambda_+ \;;$$

$$\lambda_{K^+\pi^0}^{\prime +\,(0)} = \left(\lambda_{K^+\pi^0}^{+\,(0)}\right)^2 + 5.79(97) \times 10^{-4} \;;$$

$$\lambda_{K^+\pi^0}^{0\,(0)} = \frac{M_\pi^2}{t_{CT}} \left[\log(C) - 0.0398(44)\right] \;;$$

$$\lambda_{K^+\pi^0}^{\prime 0\,(0)} = \left(\lambda_{K^+\pi^0}^{0\,(0)}\right)^2 + 4.16(56) \times 10^{-4}.$$

Parameter	Correlation
$\Lambda_+ = 24.22(1.16) \times 10^{-3}$	$\rho\,[\Lambda_+, \log(C)] = 0.376$
$\log(C) = 0.1998(138)$	$\rho\left[f_{+/0,\,\mathrm{SM}}^{K^+\pi^0}(0), \log(C)\right] = -0.719$
$f_{+/0,\,\mathrm{SM}}^{K^+\pi^0}(0) = 0.9709(46)$	$\rho\left[f_{+/0,\,\mathrm{SM}}^{K^+\pi^0}(0), \Lambda_+\right] = -0.228$

Phys. Rev. D 93 (2016) 114512 [1602.04113].

Signal estimate for NA 48/2

$$\frac{d\Gamma}{dE_\pi dE_\mu}(E_\pi, E_\mu)|_{\rm final} \equiv \mathcal{R}(E_\pi, E\mu) \otimes \frac{d\Gamma}{dE_\pi dE_\mu}(E_\pi, E_\mu),$$

Bin by Bin theory convoluted with acceptance and selection

Bin by Bin ratio of theory MC and SM signal (from NA 48/2 collab. data) [MC takes into account real emission And radiative correction]

Direct vis á vis indirect detection

Lagrangian and Amplitude squared for $K^+ \to a\ell^+\nu_\ell$

$$\mathcal{L}_{a\ell^+\nu} \supset iG_F V_{\bar{s}u}\,\xi\Big[\left(\alpha^{(1)}_{K^+a} + i\tilde{\alpha}^{(1)}_{K^+a}\right)\left(K^+\partial_\mu \hat{a} - \partial_\mu K^+ \hat{a}\right) + \left(\beta^{(1)}_{K^+a} + i\tilde{\beta}^{(1)}_{K^+a}\right)\partial_\mu\left(K^+\hat{a}\right)\Big] j^\mu_{-,\ell}$$

$$|\mathcal{A}|^2_{K^+\to a\ell^+\nu} \propto \xi^2\left(\left|\alpha^{(1)}_{K^+a}\right|^2 + \left|\tilde{\alpha}^{(1)}_{K^+a}\right|^2\right) \propto \xi^2\left[\left(C^3_{LR} - C^3_R + \sqrt{3}(C^8_{LR} - C^8_R)\right)^2 + (2C_W)^2\right].$$

Direct rate

*Assuming $m_\ell \to 0$ for simplification

Pion Phobia $\left(|\mathscr{A}|^2_{K^+\to a\ell^+\nu_\ell} \to 0\right)$ limit does not exists if $C_W \neq 0$

In the limit $C_W = 0$

Indirect rate

$$|\mathscr{A}|^2_{K^+\to\pi^0\ell^+\nu_\ell} \propto \xi^2\,|\alpha^{(2)}_{K^+\pi^0}|^2$$

$$\alpha^{(2)}_{K^+\pi^0} = \frac{C_3}{2}\left(\alpha^{(1)}_{K^+a} + \frac{C_3}{4}\right),$$

A limit exists where direct rate is zero but indirect rate is nonzero

$$C_W = 0, \quad C^3_{LR} = C^3_R, \text{ and } C^8_{LR} = C^8_R,$$

Triparno Bandyopadhyay, SG, Tuhin S. Roy: arXiv:2112.13147

Direct vis á vis indirect detection

The situation is slightly different in case of pions

In the limit $C_W = 0$

Direct rate : $|\mathscr{A}|^2_{\pi^+ \to a \ell^+ \nu_\ell} \to$

$$\left|\alpha^{(1)}_{\pi^+ a}\right|^2 \propto \frac{(C_3)^2}{4},$$

Indirect rate : $|\mathscr{A}|^2_{\pi^+ \to \pi^0 \ell^+ \nu_\ell} \to$

$$\left|\alpha^{(2)}_{\pi^+ \pi^0}\right|^2 \propto \frac{(C_3)^2}{4} \left(\alpha^{(1)}_{\pi^+ a} + \frac{C_3}{4}\right)^2.$$

$C_3 = 0$ sets both rate to zero

Reason: Absence of $\mathscr{O}^3_L$ operator due to EW symmetry

Future Direction

Direct channel

Missing energy

$$\tau^- \to K^- a \nu_\tau$$

$$K^- \to \mu^- a \nu_\tau$$

Missing energy

background for

$$\tau^- \to K^- \nu_\tau$$

$$K^- \to \mu^- \nu_\tau$$

Constraint on ALP parameters

For best constraints need more precise/updated measurements

Triparno Bandyopadhyay, SG, Girish Kumar: arXiv:22xx.xxxxx