Theory review for hadronic corrections to g-2

Maarten Golterman

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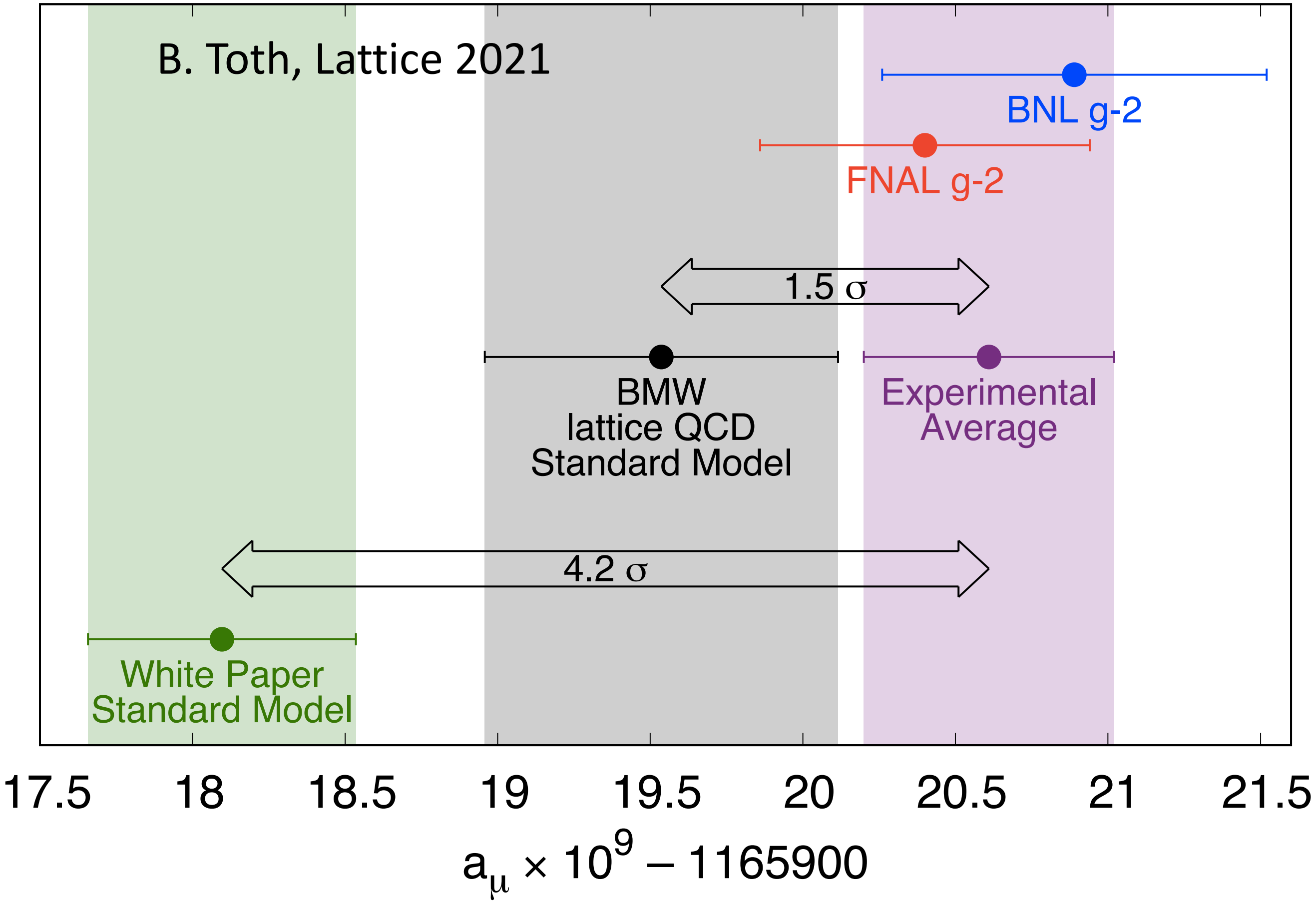
Thanks to Christopher Aubin, Tom Blum, Diogo Boito, Aida El-Khadra, Gregorio Herdoiza, Martin Hoferichter, Alex Keshavarzi, Christoph Lehner, Kim Maltman, Santi Peris & others

20th Conference on Flavor Physics and CP Violation, May 23-27, 2022

University of Mississippi

Current status



Recent history:

- White paper 2020 & BNL g-2 2006: 3.7σ
- BMW 2020: shift SM value up by 1.44×10^{-9}, i.e., $\sim 2\sigma$
- FNAL 2021 (6% of data!) & White paper: 3.3σ
- BNL and FNAL consistent $\Longrightarrow$ can average: 4.2σ
- Work for theorists: Reconcile WP and BMW 2020!

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The anomalous magnetic moment of the muon in the Standard Model

T. Aoyama [1,2,3], N. Asmussen [4], M. Benayoun [5], J. Bijnens [6], T. Blum [7,8],
M. Bruno [9], I. Caprini [10], C.M. Carloni Calame [11], M. Cè [9,12,13], G. Colangelo [14,*],
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J. Wilhelm [12], R. Williams [71], A.S. Zhevlakov [78]

White paper (g-2 Theory Initiative):

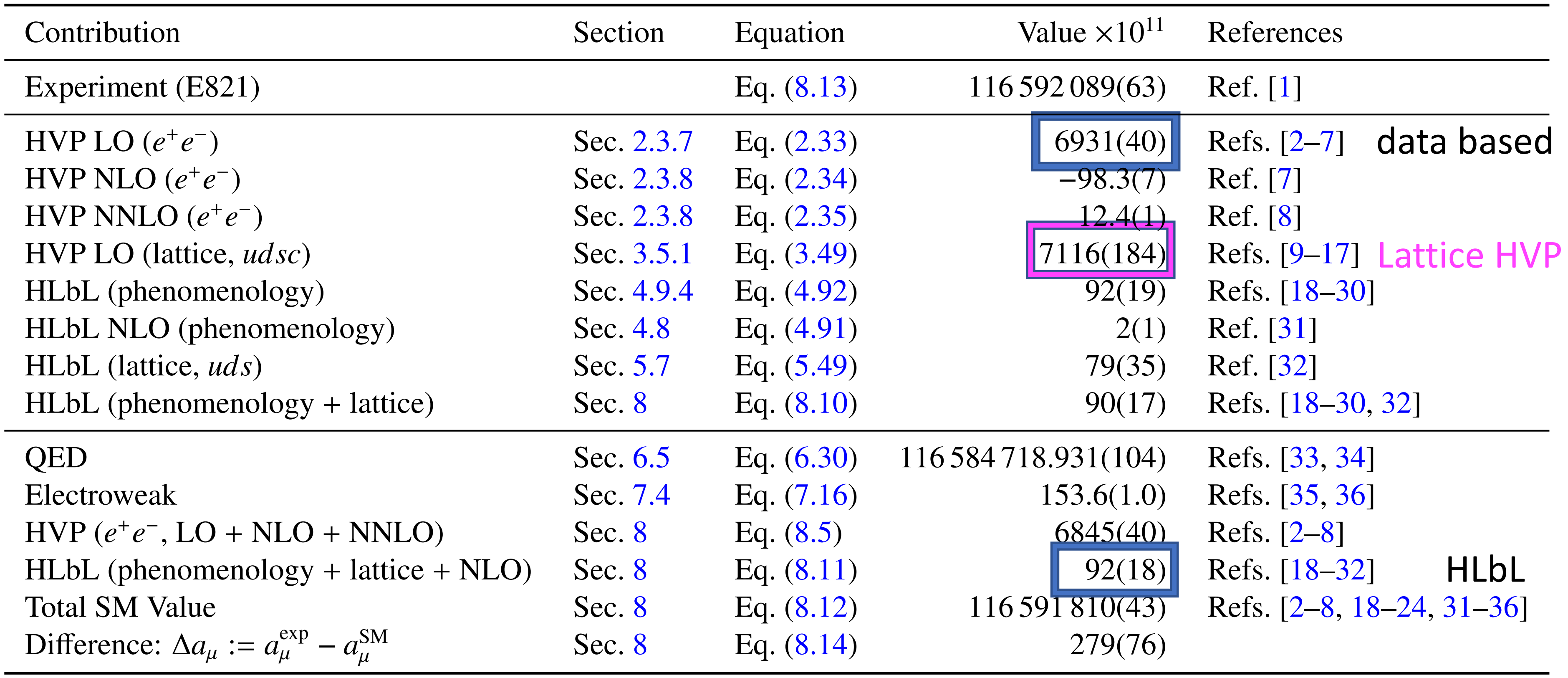
Community effort of theorists
(with help from experimentalists!)

Base “SM world average” for muon g-2
on existing SM calculations
with reliable error

Appeared in June 2020
too early to review and include BMW 2020

Before announcement of new FNAL result

White paper Table 1 & BMW'20: contributions to SM value of $a_\mu = (g-2)/2$



Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, *udsc*)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, *uds*)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18–30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18–32]
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\rm exp} - a_\mu^{\rm SM}$	Sec. 8	Eq. (8.14)	279(76)	

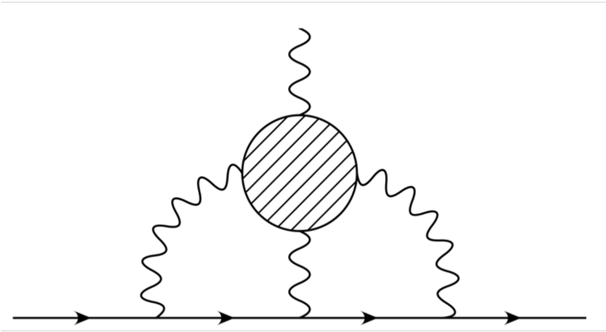
data based

Lattice HVP

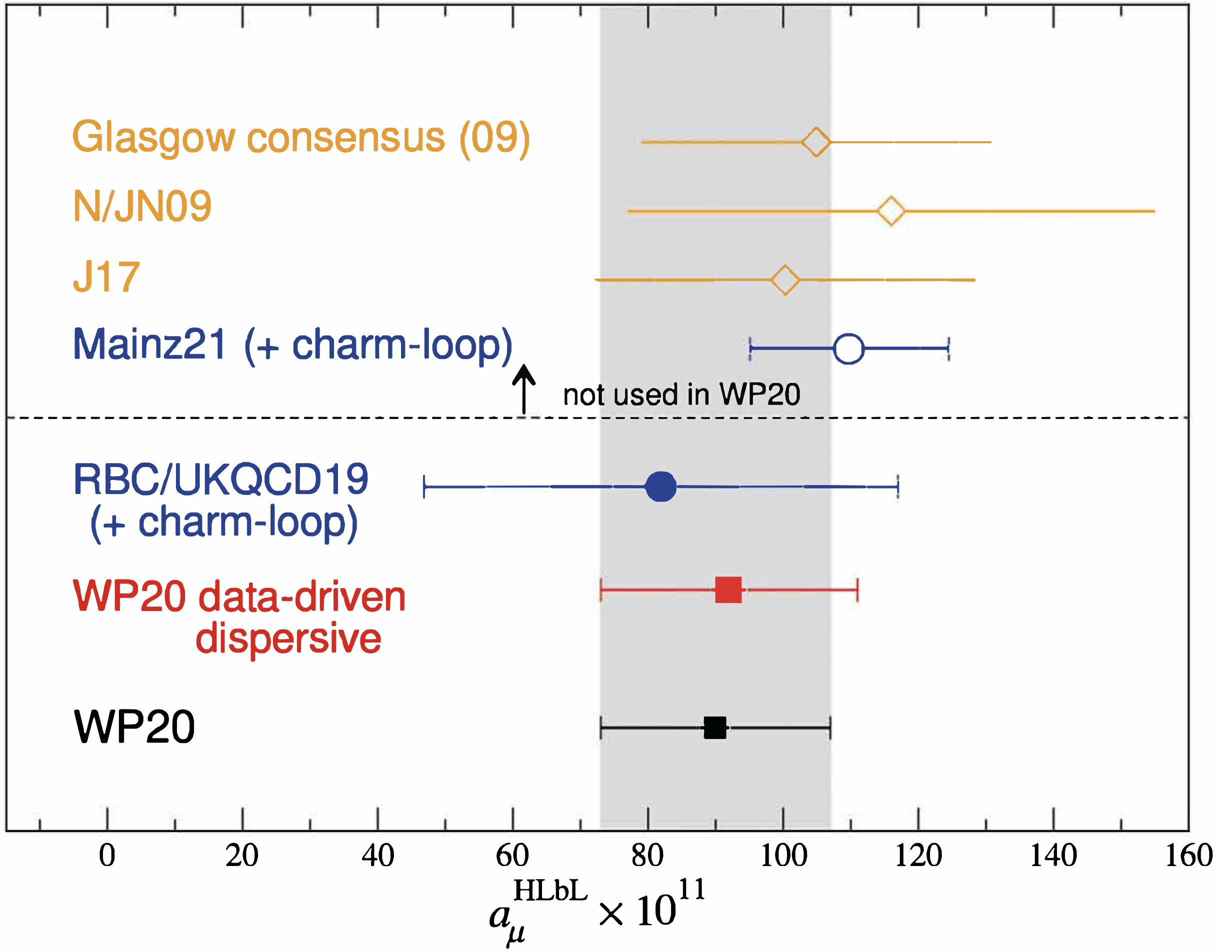
HLbL

BMW'20: HVP LO (lattice, *udsc*) = $7075(55) \times 10^{-11}$, 2.1σ higher than HVP LO (e^+e^-)

Hadronic Light by Light (HLbL): the $90(17) \times 10^{-11}$



Credit: A. El-Khadra, Lattice 2021



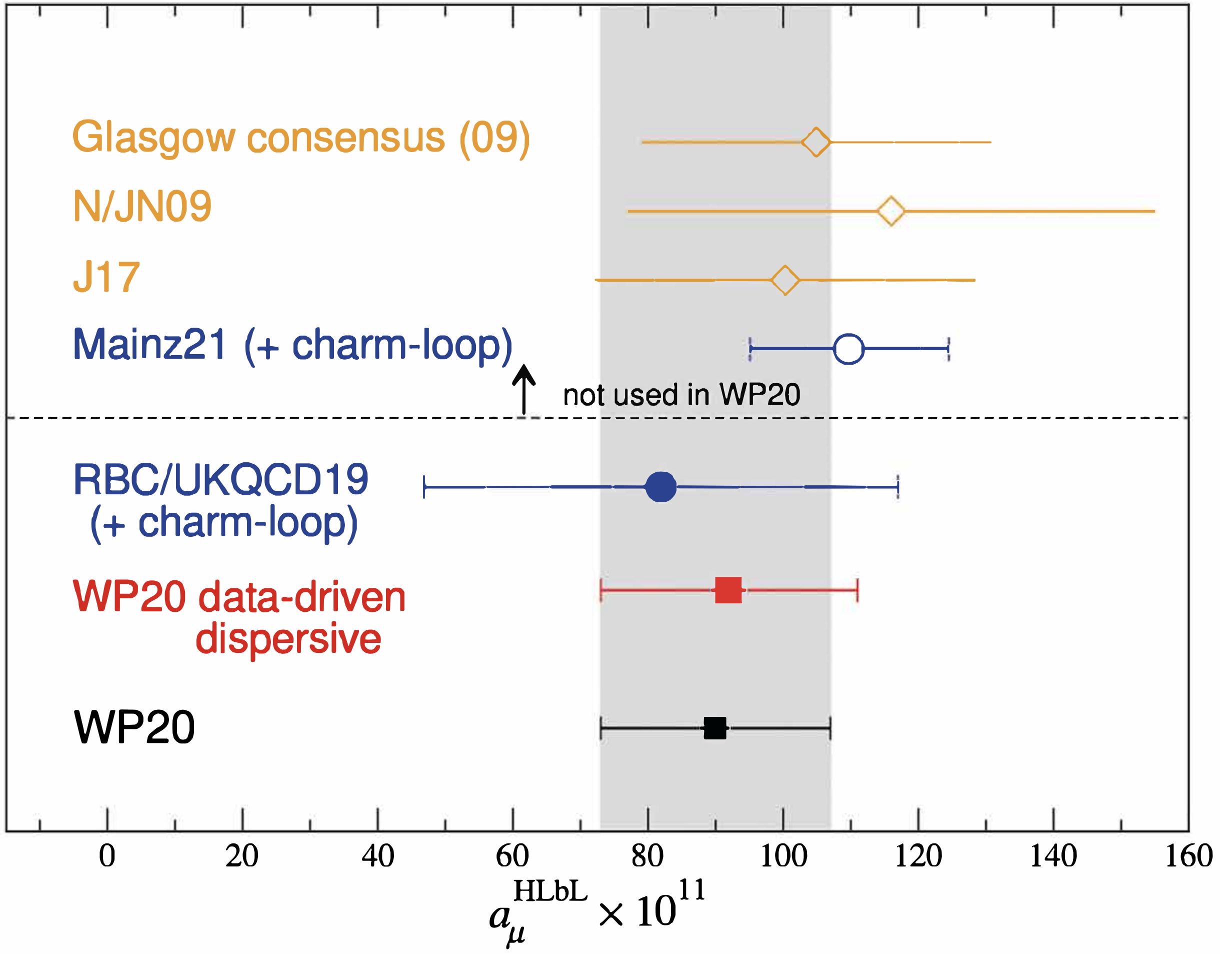
models – no reliable error

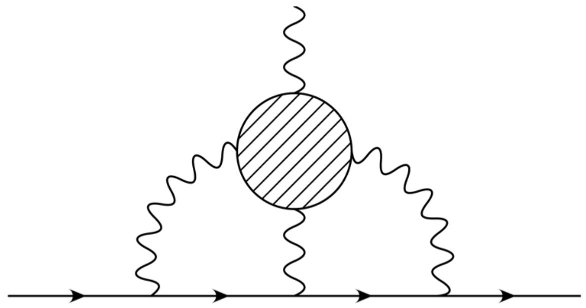
too recent for WP Mainz 2021

lattice RBC/UKQCD 2020

systematic dispersive approach
(Colangelo, Hoferichter, Procura, Stoffer and many many others)
conservative error treatment

Hadronic Light by Light (HLbL): the $90(17) \times 10^{-11}$


Credit: A. El-Khadra, Lattice 2021



Combine Mainz21 and WP20:

HLbL (phenomenology+lattice) =

$100(11) \times 10^{-11}$ (my estimate)

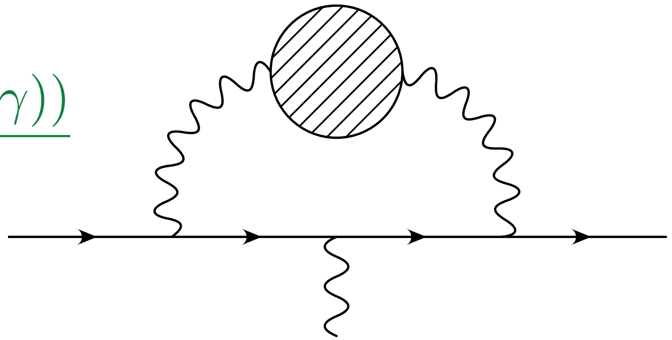
negligible impact on discrepancy

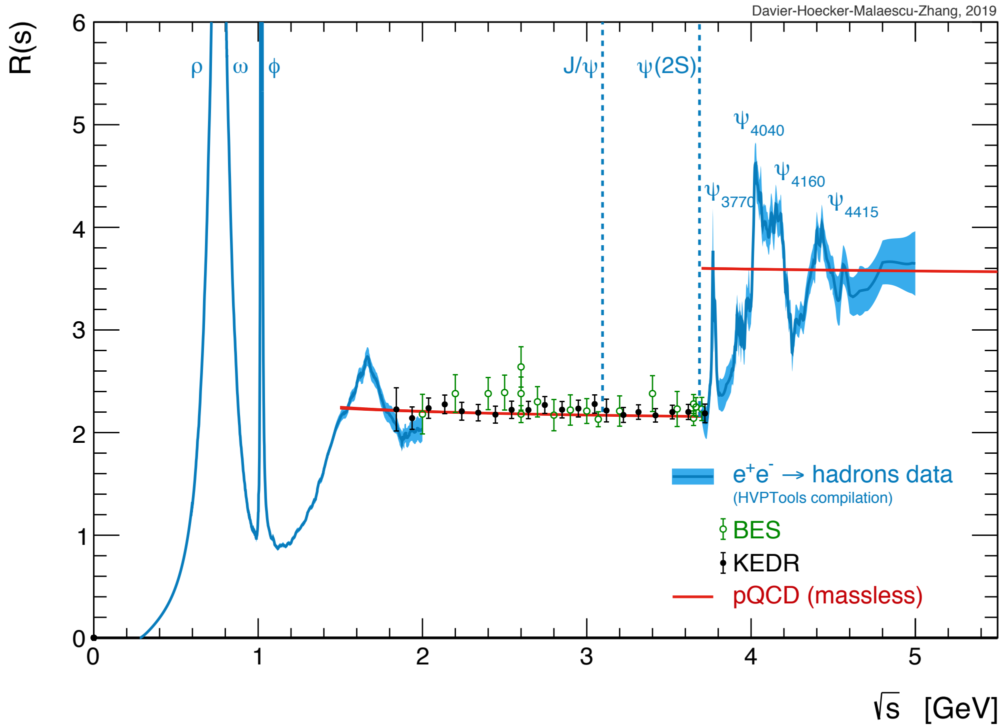
Detailed comparisons between lattice and data-driven (e.g. π^0-pole); very recent: charm contribution from lattice (Mainz 2022)

IN VERY GOOD SHAPE, continuing progress!

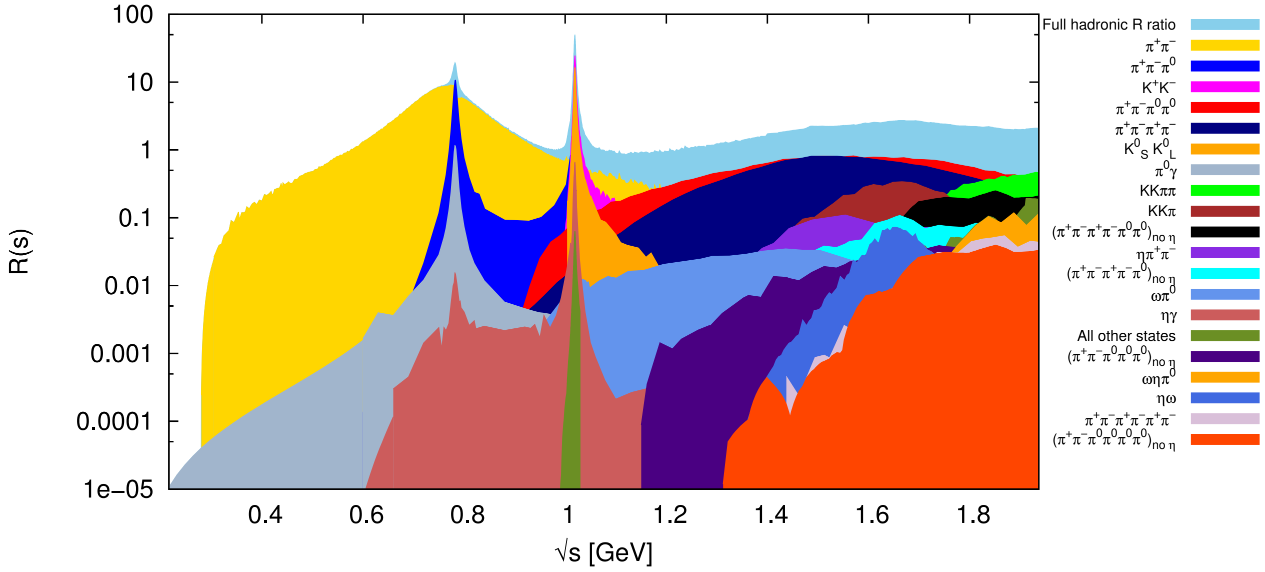
Hadronic vacuum polarization: optical theorem & data for e^+e^- → hadrons(+γ)

$$a_\mu^{\mathrm{HVP}} = \frac{\alpha^2 m_\mu^2}{9\pi^2}\int_{M_\pi^2}^{\infty} \frac{\hat{K}(s)}{s^2}\, R(s)\, ds \;, \qquad R(s) = \frac{\sigma^0(e^+e^- \to \mathrm{hadrons}(\gamma))}{4\pi\alpha^2/(3s)}$$





DHMZ 2019



KNT 2018/9

Hadronic vacuum polarization (LO): optical theorem & data for $e^+e^- \rightarrow$ hadrons(+γ)

White paper: combine DHMZ'19 and KNT'19 (add'l input from CHHKS for $\pi^+\pi^-$ and $\pi^+\pi^-\pi^0$ (78% of total))

$$a_\mu^{\rm HVP} = 693.1(2.8)_{\rm exp}(2.8)_{\rm sys}(0.7)_{\rm DV+QCD} \times 10^{-10} = 693.1(4.0) \times 10^{-10}$$

- BaBar-KLOE discrepancy: taken into account in the WP systematic error (dominant component)
- New data in inclusive region from BES III – tension with QCD pert. theory? Unlikely to affect $a_\mu^{\rm HVP}$
- No τ-based data used: insufficient control of isospin breaking – Lattice can help! (Bruno *et al*. 2018)
- Potential reduction of error by factor 2 based on (see g-2 TI Snowmass arXiv:2203.15810)
  analysis of full BaBar data set, new data from SND, CMD, BESIII, Belle II
  It remains to be seen whether new data will resolve 2π BaBar-KLOE discrepancy → role for lattice?

Hadronic vacuum polarization (LO): optical theorem & data for $e^+e^- \rightarrow$ hadrons(+γ)

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$$a_\mu^{\mathrm{HVP}} = 693.1(2.8)_{\mathrm{exp}}(2.8)_{\mathrm{sys}}(0.7)_{\mathrm{DV+QCD}} \times 10^{-10} = 693.1(4.0) \times 10^{-10}$$

Very nice result! Thanks to DHMZ, KNT & g-2 Theory Initiative

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- New data in inclusive region from BES III – tension with QCD pert. theory? Unlikely to affect a_μ^{HVP}
- No τ-based data used: insufficient control of isospin breaking – Lattice can help! (Bruno *et al*. 2018)
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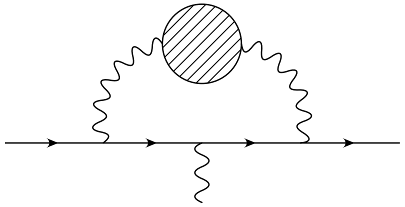
Hadronic vacuum polarization (LO): Lattice QCD

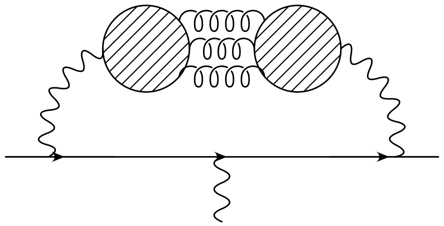
Breakdown into contributions based on quark-picture basis:

- Light quark (*u*, *d*) connected part (about 90% of total)

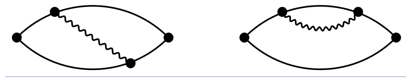


FOCUS OF THIS TALK

- Strange and charm contributions (bottom & high-energy perturbative very small)
- Quark-disconnected part: small (~2%) but not negligible – and expensive!



- Order-α^3 QED contributions (data-driven "leading-order (LO)" includes final-state radiation!)



plus many other diagrams

- Strong isospin breaking effects

Hadronic vacuum polarization (LO): Lattice QCD

- Compute $a_\mu^{\rm HVP} = \int_{-\infty}^{\infty} dt\, w(t)\, C(t)\ ,\qquad C(t) = \frac{1}{3}\sum_i \int d^3x \langle j_i^{\rm EM}(t,\vec{x}) j_i^{\rm EM}(0,\vec{0})\rangle$

  $w(t)$ known weight Bernecker&Meyer 2011

  Need sub-percent precision: this means good control over systematic errors!

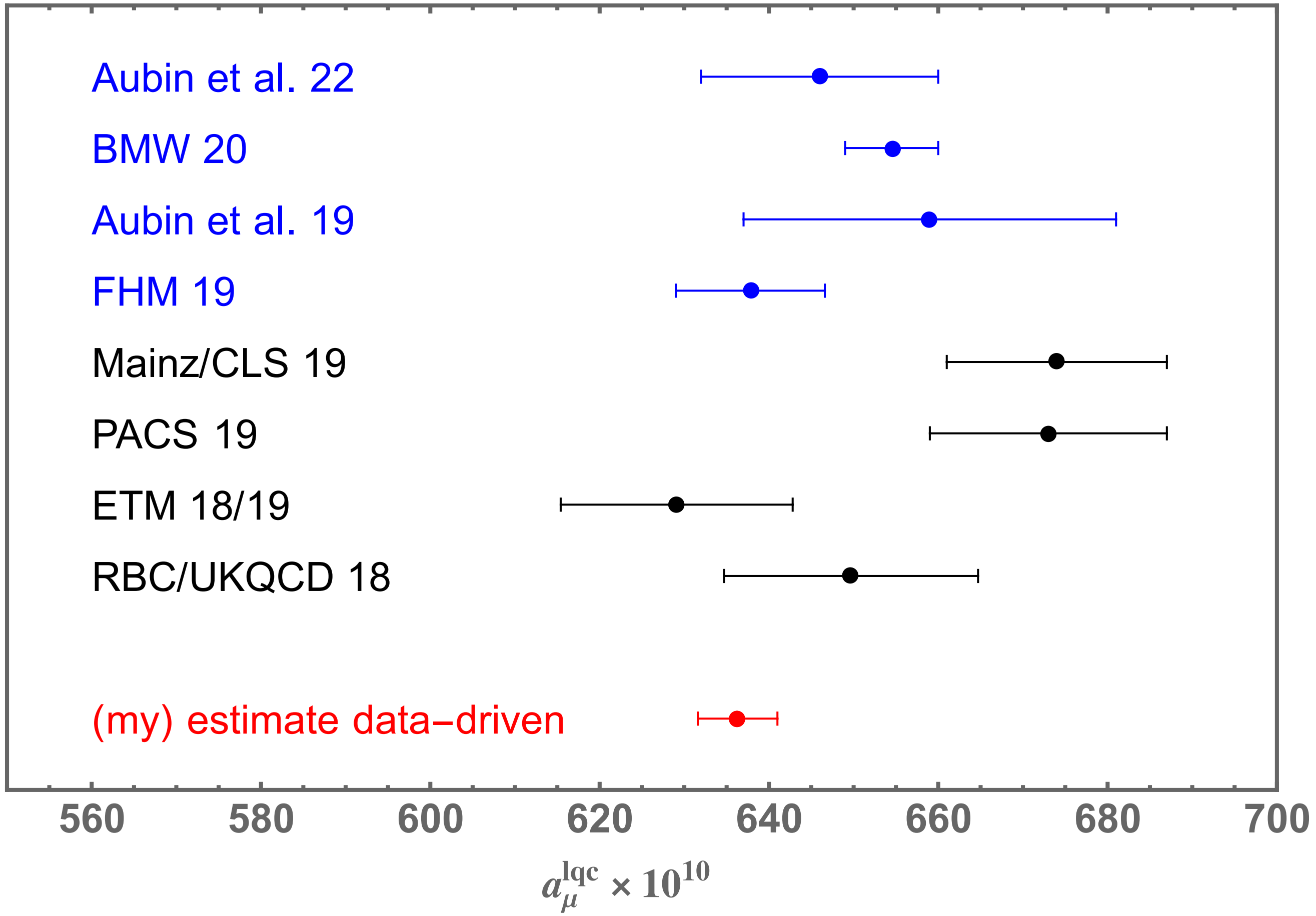
- Large-time behavior: $C(t)$ becomes very noisy at large t -- need to control this

  bounding method (RBC/ '18, ABGP '19, BMW '20); dedicated reconstruction of tail (FHM, RBC/, Mainz, …)
- Finite volume effects: a typical volume $L \sim 5-6\ {\rm fm}$ leads to 3-4% FV effects

  most collaborations use NNLO chiral perturbation theory or physical models
- Scale setting: ratio of hadronic scale to muon mass; 1% error in $a \Longrightarrow$ 1.8% error in $a_\mu^{\rm HVP}$ (Mainz 2017)

- Continuum limit: this talk (for many other aspects, see A. El-Khadra's talk at Lattice 2021)

Light-quark connected part



Blue: staggered fermions
flavor symmetry broken
Black: flavor-symmetric fermions
Red: my estimate correcting full
data-driven result subtracting:
strange+disc. (Boito et al. 2022)
charm (white paper)
QED+SIB (BMW 2020/
James et al. 2021)

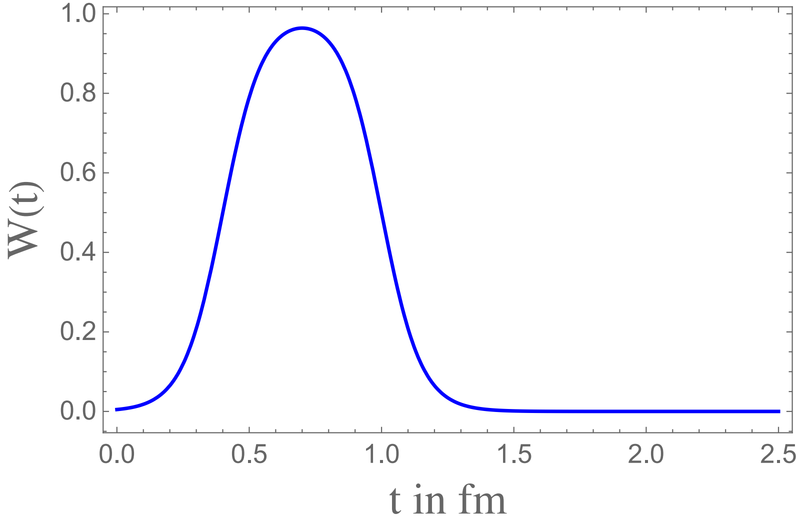
Note discrepancy between BMW and data-driven! (18×10^{-10})

Otherwise errors typically large

An intermediate, more precise quantity: the “window”

Introduce the “window” quantity $a_\mu^{\rm W} = 2\int_0^\infty dt\, W(t)\, w(t)\, C(t)$

(RBC/UKQCD 2018)



- advantages:

  - cuts out short distance (lattice spacing artifacts)

  - cuts out long distance (large-time tail, finite-volume effects)

  - can be computed very precisely on the lattice – lattice computations have to agree!

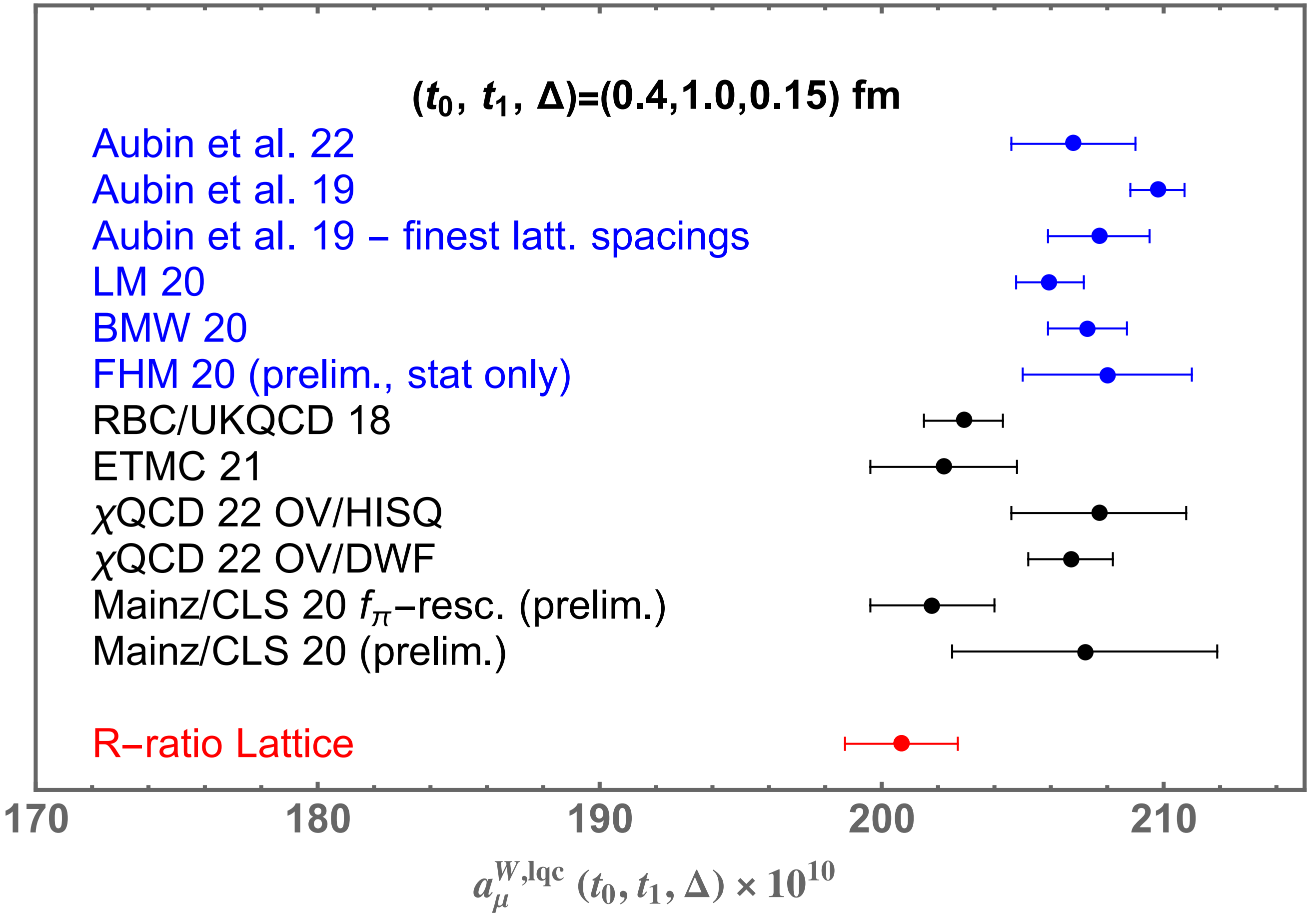
  all lattice collaborations are now computing this quantity

- caveat: systematic effects much smaller, but not negligible!

  intermediate distance not accessible to ChPT to correct for finite-volume *etc.* effects

  need to resort to models (at least at present) (ABGP 2022)

An intermediate, more precise quantity: the “window”



Results for light-quark connected window

- Clear disagreements – good!
  Discrepancies of order ½ times total difference between BMW 20 and data-driven
- Staggered (blue) values high
  (also overlap (OV) valence quarks on DWF/HISQ sea quarks)
- Issue with continuum limit?
  Focus on staggered, because that’s where the puzzle is

(“R-ratio Lattice” courtesy of C. Lehner)

Staggered fermions & taste breaking

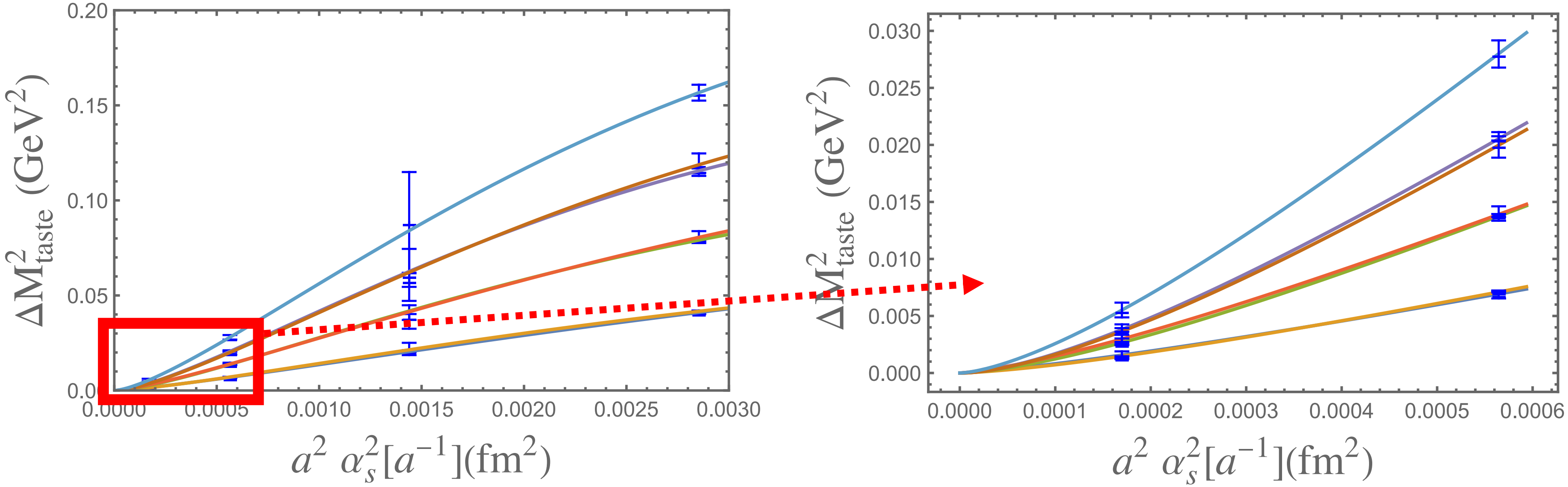
- Lattice fermions with exact chiral symmetry have “species doublers” (Karsten&Smit, Nielsen&Ninomiya, 1981): “naïve fermions” have 16 doublers
- Staggered fermions minimize the number of doublers by “spreading out spin” over the lattice: 1-component fermion $\Longrightarrow$ 16 components in continuum = (4 spin) x (4 “tastes” = flavors), 16 components on corners of hypercube: hence all symmetries broken like rotational symmetry
- only discrete subgroup of $SU(4)_{\text{“taste”}} \times SO(4)_{\text{rot}}$ remains

⇒ 16 charged pions made of 4 up and 4 down quarks split into 8 non-degenerate multiplets;

only one exact Nambu—Goldstone (NG) pion (one exact chiral symmetry)

- Even if the NG pion is physical, the other 15 pions are heavier; lattice spacing artifact: disappears in the continuum limit
- To reduce number of quarks on loops: take 4th root of determinant – another talk!

Taste splittings (on “HISQ” staggered ensembles – courtesy MILC collaboration)

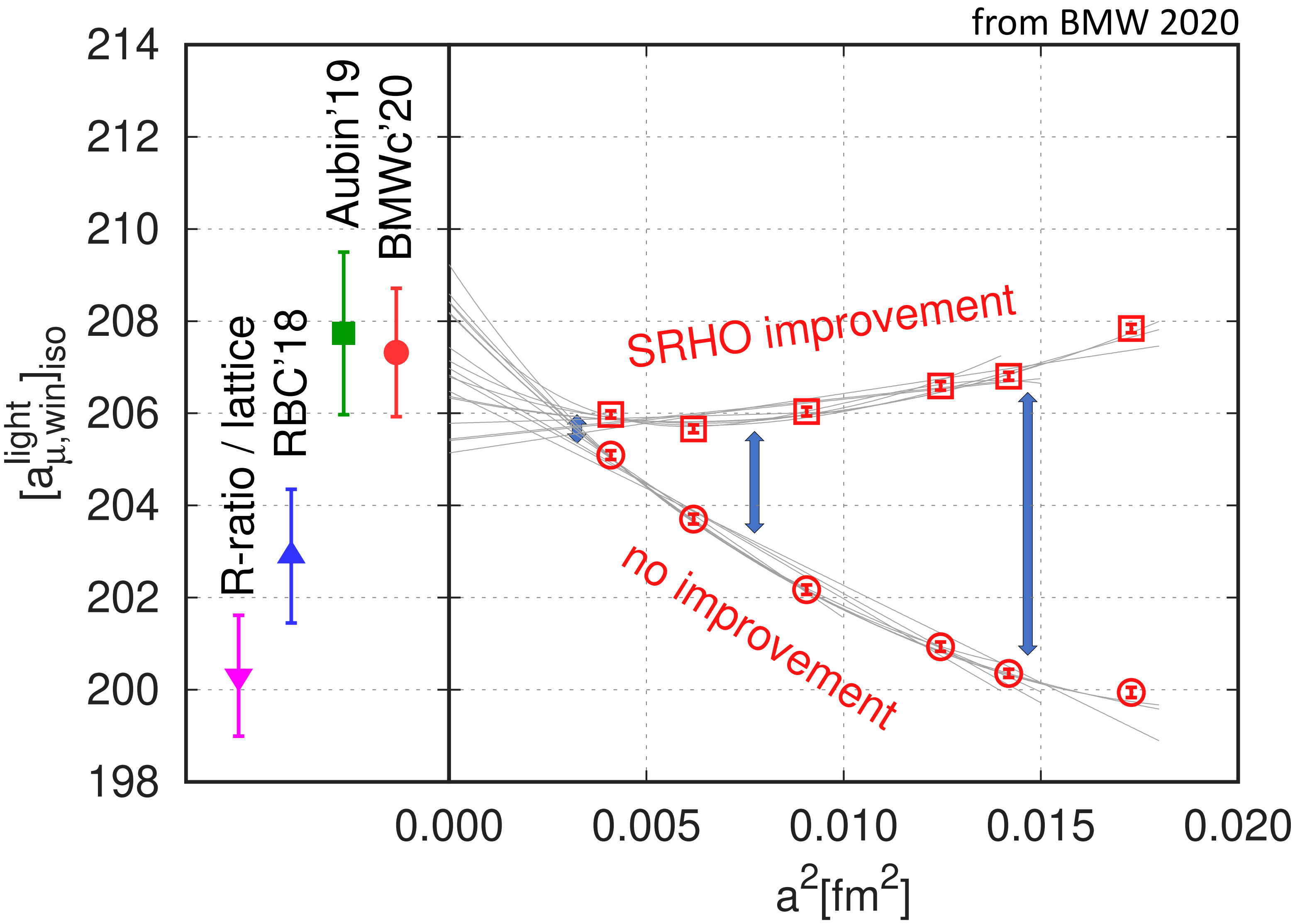


ABGP 2022

Taste splittings as a function of a^2 for $a = (0.057,\ 0.088,\ 0.12,\ 0.15)\ \mathrm{fm}$
Heaviest pion has mass of $153,\quad 212,\quad 326,\quad 418\ \mathrm{MeV}$ (lightest pion physical)

- Serious lattice artifact, quite non-linear in a^2 -- makes continuum extrapolation difficult! Can correct for taste splittings using NNLO ChPT, if they are small enough: ChPT predicts a^2 behavior at leading order – not seen in these fits (which are $a^4 + a^6$)!

Window with staggered fermions – continuum limit

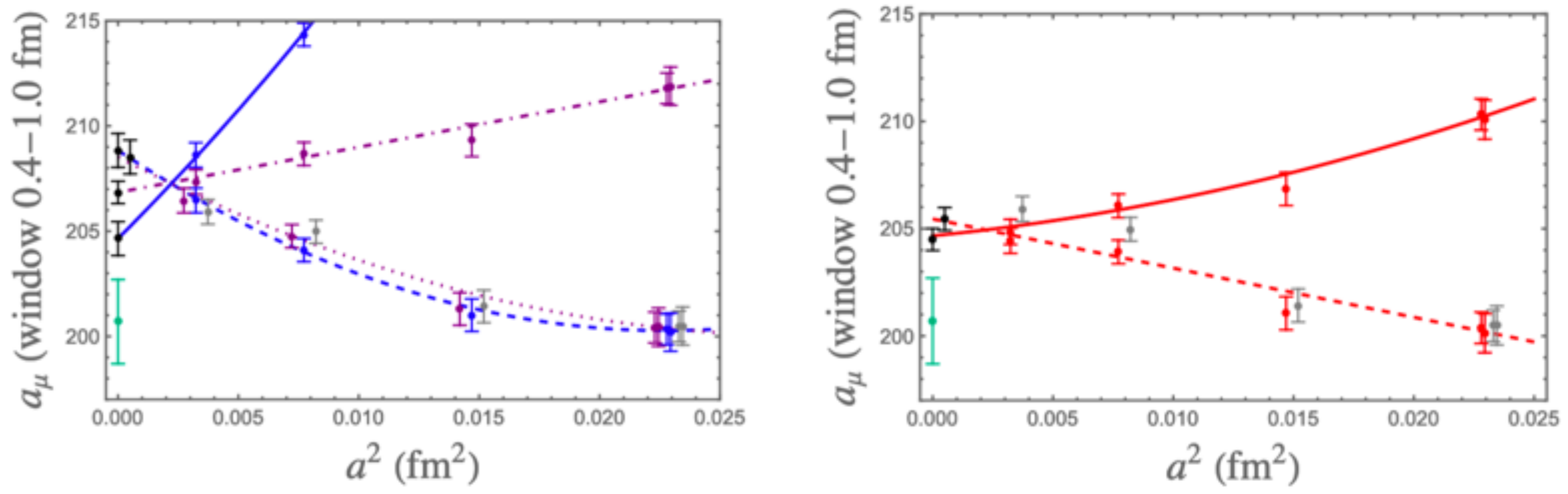


Taste-breaking effect as a function of a^2 (BMW 2020) computed with "SRHO" model (NLO ChPT plus $\rho,\ \gamma$)

Superimposed arrows:
Same for ABGP 2022

- Very similar taste-breaking effects, despite different lattice actions!
- Continuum extrapolation very non-linear (in a^2)

Window with staggered fermions – continuum limit

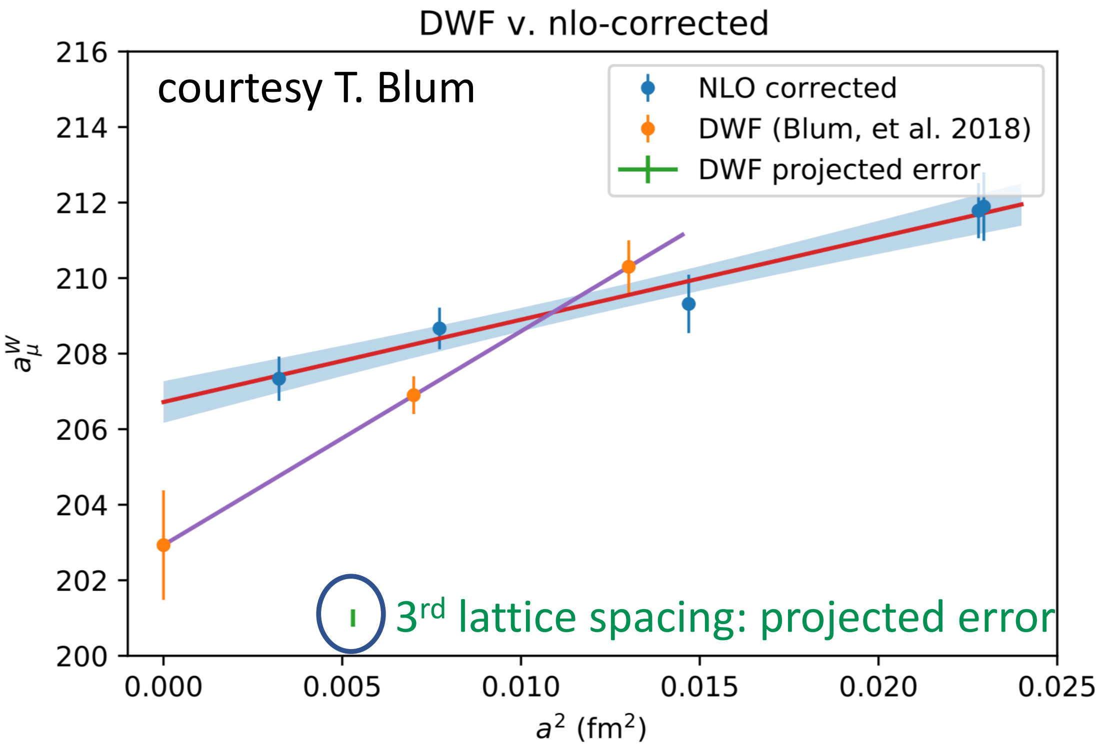


Taste **B**reaking effect as a function of a^2 (ABGP 2022)

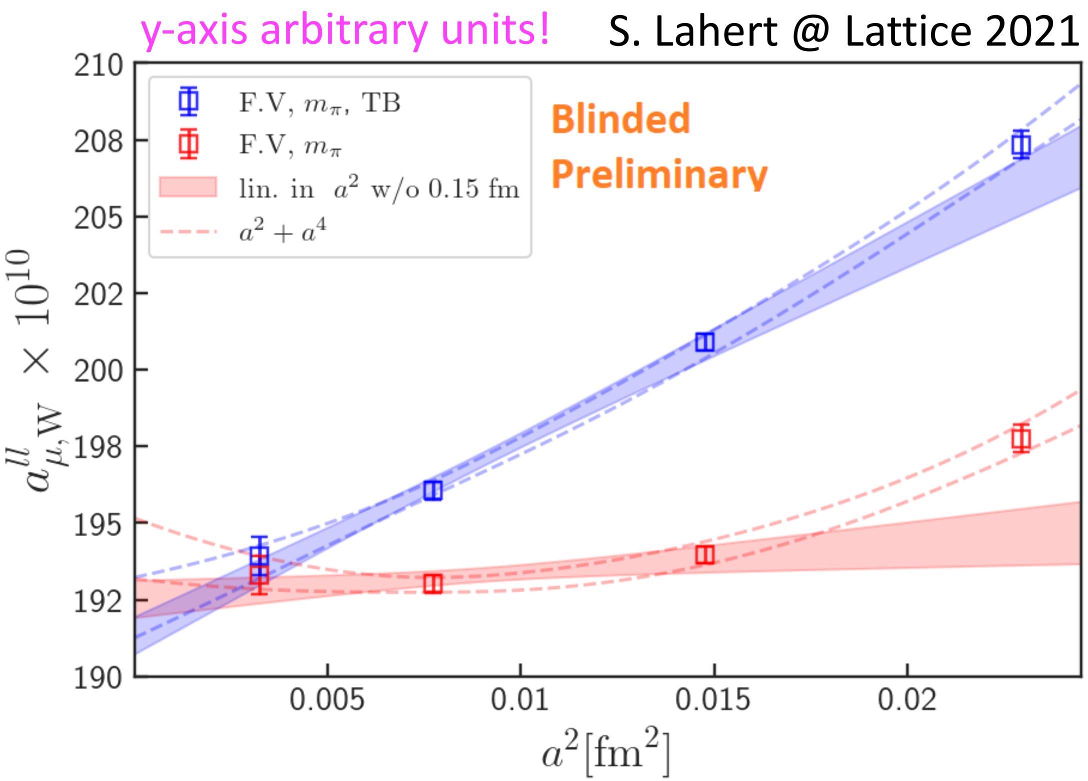
Blue: NNLO ChPT
Purple: NLO ChPT
Red: SRHO model
Lower: **TB**-uncorrected
Upper: **TB**-corrected
Green: R-ratio based

- NNLO ChPT no good (no surprise!) but hard to tell whether NLO ChPT or SRHO model is better model, based on fits (see backup slide)
  Note taste-breaking effects are not the ***only*** lattice-spacing artifact!
  All extrapolations should agree in the continuum limit
- Need data at smaller lattice spacing to see linear behavior in a^2
  (HISQ ensembles with $a^2 = 0.0018\ \mathrm{fm}^2$ available – will be used, but big project!)

Window -- taste breaking is not the only lattice artifact!

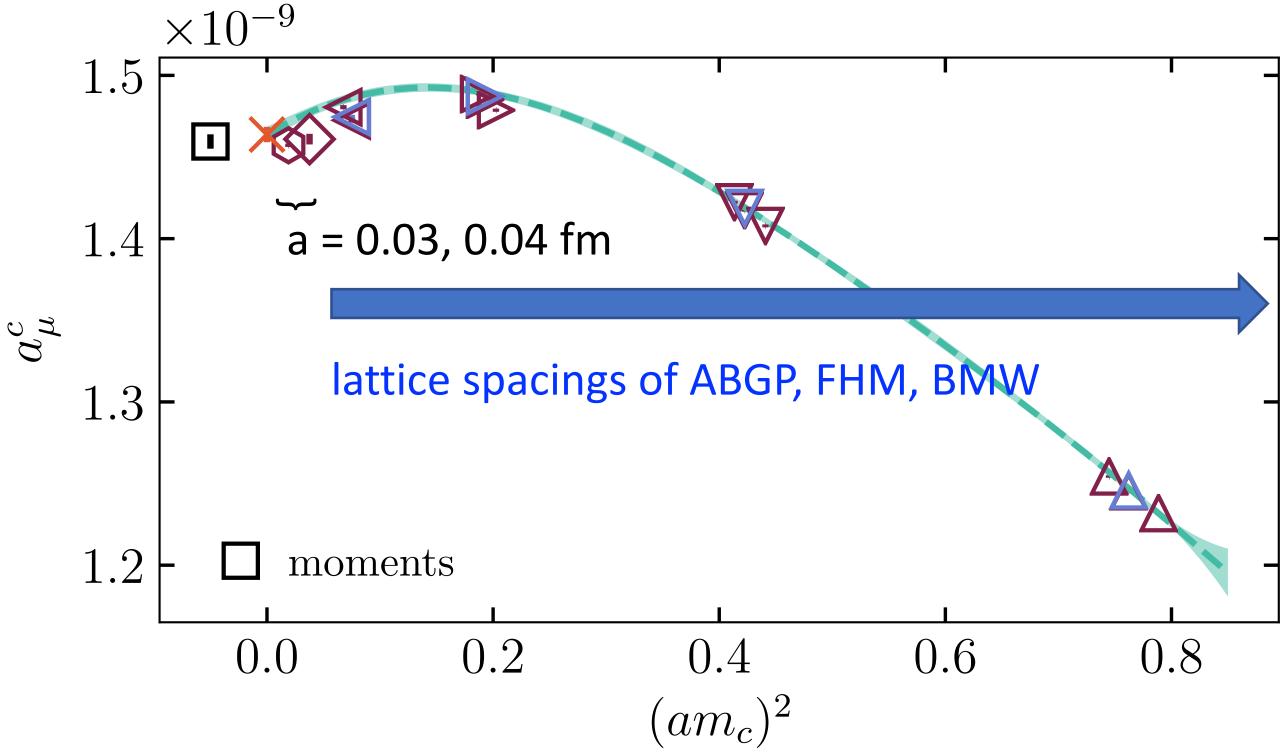


Comparison of DWF window (RBC 18) with
NLO-ChPT-corrected staggered
RBC/UKQCD will have 3rd (smaller) lattice spacing soon



FNAL-MILC-HPQCD window
same latt. spacings as ABGP but local currents
Are the 3 lowest a^2 points in the linear regime?

Need smaller lattice spacings for staggered!   Another example:



a = 0.03, 0.04 fm

lattice spacings of ABGP, FHM, BMW

HPQCD 2020:   charm contribution to muon magn. moment: Note bending at smaller lattice spacings!

(may be worse for charm)

Conclusions

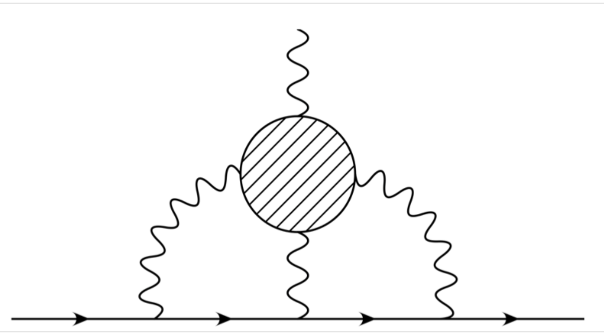
- Theory Initiative: community effort to produce a SM estimate for a_μ with reliable errors in particular for the hadronic contributions.
- HLbL is already in very good shape, and will continue to be improved.
- HVP: discrepancy between data-driven and BMW-lattice estimates (2.1σ).
- For HVP the work by all lattice collaborations (RBC/UKQCD, BMW, ETMC, Mainz, ABGP, χQCD, FHM, PACS) is important for understanding the systematics; they will get there!

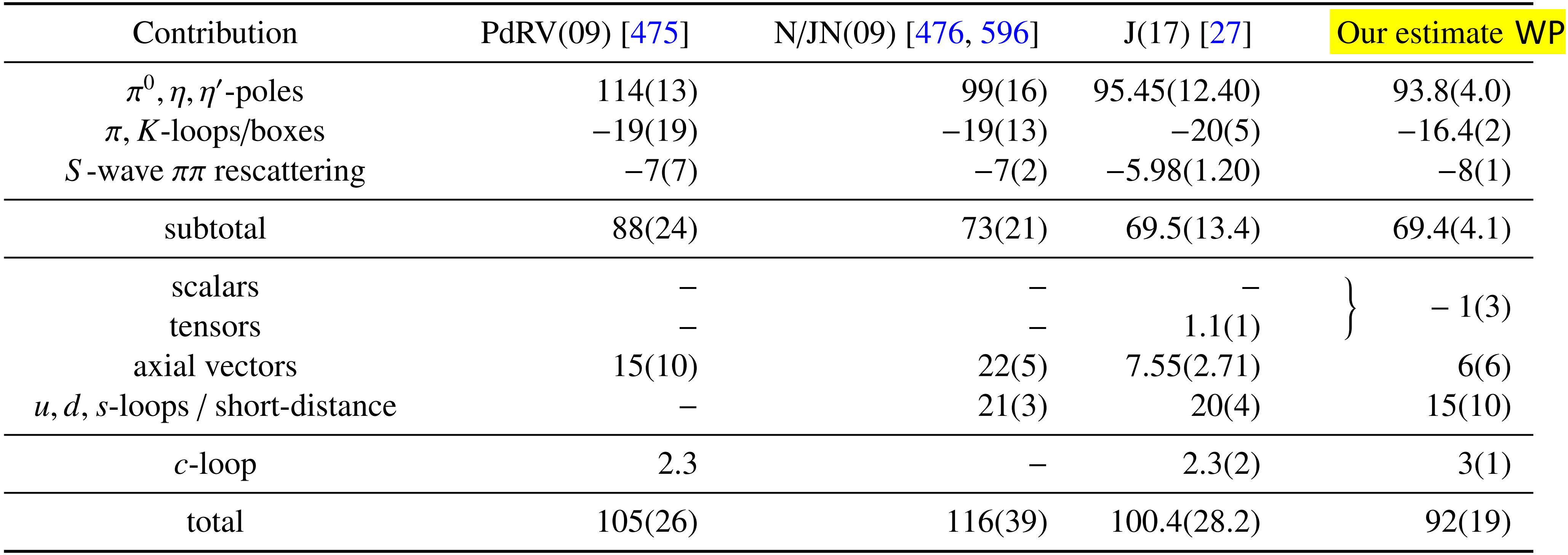
HVP:

- Major systematic error: continuum extrapolation! Need smaller lattice spacings
- Window quantities very useful for comparisons with very small statistical errors – highlight systematics
- "Standard window": 0.4-1 fm Warning: need models to correct finite volume and pion mass – no EFT method available and these corrections are small but not negligible!
- Consider longer-distance windows? (E.g. 1.5-1.9 fm proposed in ABGP 2022)

BACK-UP SLIDES

Hadronic Light by Light (HLbL): the $90(17) \times 10^{-11}$

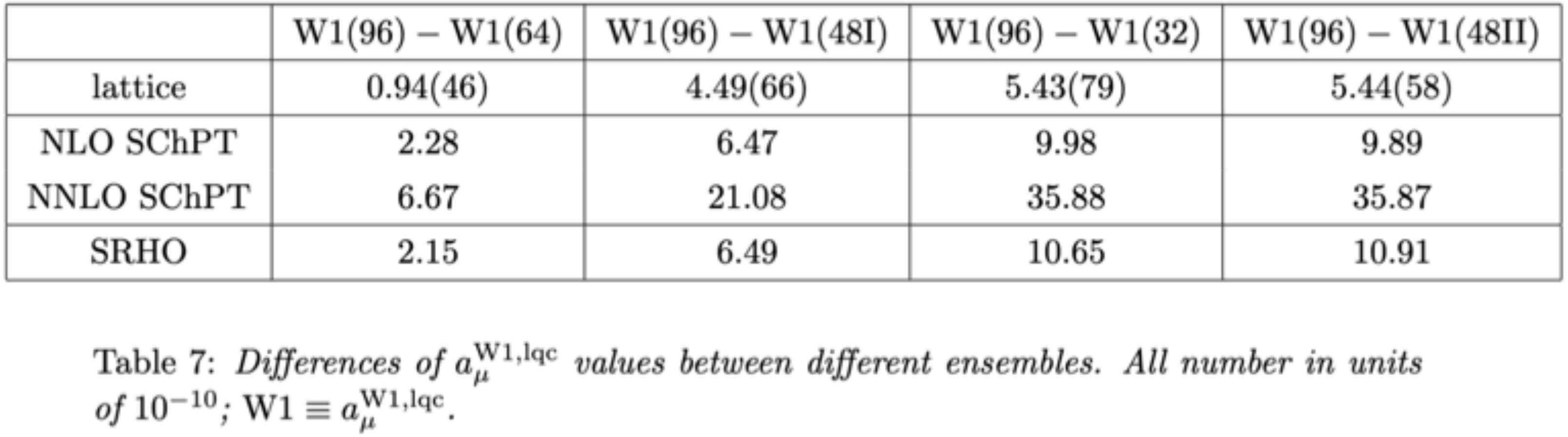




Contribution	PdRV(09) [475]	N/JN(09) [476, 596]	J(17) [27]	Our estimate WP
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	} − 1(3)
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

Table 15 of WP

Comparing S(taggered)ChPT and the SRHO model for the window quantity

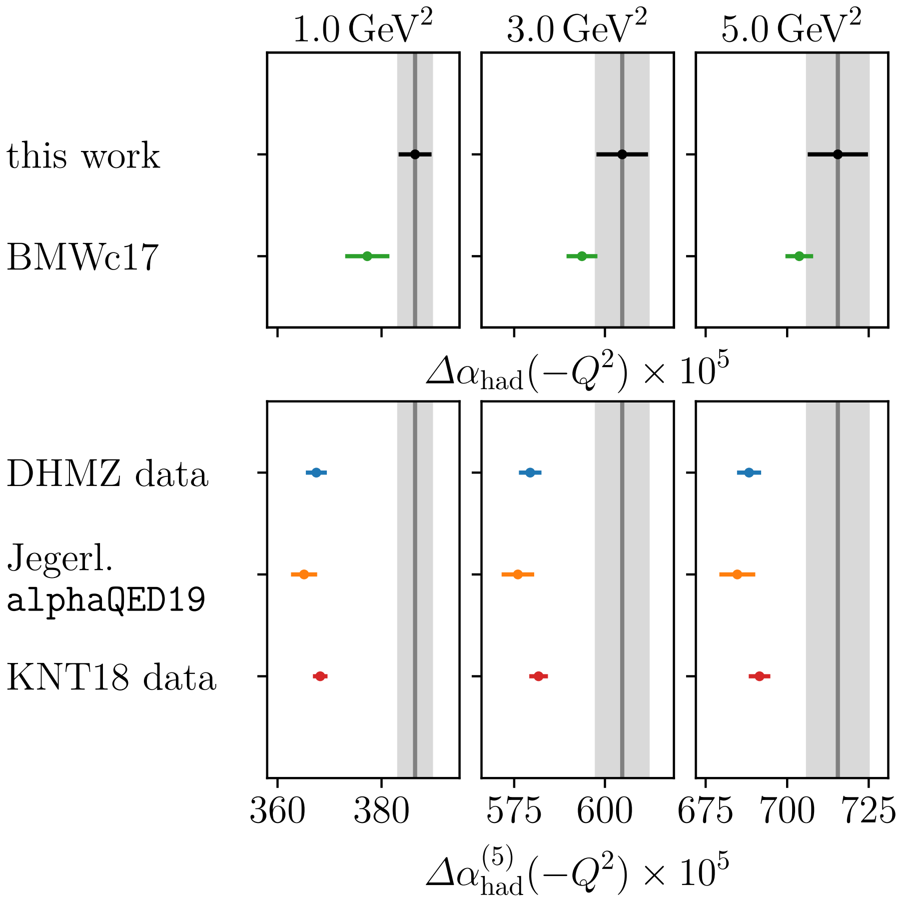


	W1(96) – W1(64)	W1(96) – W1(48I)	W1(96) – W1(32)	W1(96) – W1(48II)
lattice	0.94(46)	4.49(66)	5.43(79)	5.44(58)
NLO SChPT	2.28	6.47	9.98	9.89
NNLO SChPT	6.67	21.08	35.88	35.87
SRHO	2.15	6.49	10.65	10.91

Table 7: *Differences of* $a_\mu^{\mathrm{W1,lqc}}$ *values between different ensembles. All number in units of* 10^{-10}*;* $\mathrm{W1} \equiv a_\mu^{\mathrm{W1,lqc}}$.

From ABGP 2022
“W1” is window quantity
differences between ensembles with different lattice spacings
See BMW 2020 “Extended Data Figure 2” for similar comparison

New lattice results for $\Delta\alpha(-Q^2) = 4\pi\alpha(\Pi(-Q^2) - \Pi(0))$ (Mainz 2022)



Finds very high values at euclidean momenta!

(Note: these values of Q^2 give very small contribution to a_μ^{HVP})

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