Massive Right-handed Neutrinos in B Decay

In collaboration with Alakabha Datta, Danny Marfatia
[arXiv:2204.01818]

Hongkai Liu

liu.hongkai@campus.technion.ac.il

Technion

FPCP 2022
May 25th, 2022

Motivation

Hints of new physics in the semileptonic B decays $b \to c\ell^- \bar{\nu}$

$$R^{\tau/\ell}_{D^{(*)}} \equiv \mathrm{Br}(\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau)/\mathrm{Br}(\bar{B} \to D^{(*)}\ell^-\bar{\nu}_\ell)$$

Larger than the SM value by $\sim 3.4\sigma$

$$R_{J/\psi} \equiv \mathrm{Br}(B_c^+ \to J/\psi\tau^+\nu_\tau)/\mathrm{Br}(B_c^+ \to J/\psi\mu^+\nu_\mu) \quad \sim 1.7\sigma$$

$$\Delta A_{\mathrm{FB}} \equiv A^{\mu}_{\mathrm{FB}} - A^{e}_{\mathrm{FB}} \quad \sim 4\sigma$$

Bobeth, Bordone, Gubernari, Jung, Dyk, 2104.02094

Possible new physics explanations have been extensively studied based on an EFT framework including both left-handed and right-handed neutrinos (RHNs).

RHNs are singlet under the SM gauge group, can evade strong constraints from charged leptons.

In those previous studies, the neutrinos are all treated as massless particles.

Motivation

- Massive right-handed neutrinos (RHNs) can be naturally introduced to explain neutrino mass.

- Massive RHN is a viable dark matter candidate.

Motivation

- Massive right-handed neutrinos (RHNs) can be naturally introduced to explain neutrino mass.
- Massive RHN is a viable dark matter candidate.
- Massive RHN is essential to explain the anomaly in forward-backward asymmetry.

SMNEFT = SMEFT + N

16 new SMNEFT operators $\Delta B = \Delta L = 0$

$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$		$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$	
$\mathcal{O}_{nd}$	$(\bar{n}_p\gamma_\mu n_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{qn}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{n}_s\gamma^\mu n_t)$	$\mathcal{O}_{\ell n\ell e}$	$(\bar{\ell}_p^j n_r)\epsilon_{jk}(\bar{\ell}_s^k e_t)$
$\mathcal{O}_{nu}$	$(\bar{n}_p\gamma_\mu n_r)(\bar{u}_s\gamma^\mu u_t)$	$\mathcal{O}_{\ell n}$	$(\bar{\ell}_p\gamma_\mu \ell_r)(\bar{n}_s\gamma^\mu n_t)$	$\mathcal{O}_{\ell nqd}^{(1)}$	$(\bar{\ell}_p^j n_r)\epsilon_{jk}(\bar{q}_s^k d_t)$
$\mathcal{O}_{ne}$	$(\bar{n}_p\gamma_\mu n_r)(\bar{e}_s\gamma^\mu e_t)$			$\mathcal{O}_{\ell nqd}^{(3)}$	$(\bar{\ell}_p^j \sigma_{\mu\nu} n_r)\epsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} d_t)$
$\mathcal{O}_{nn}$	$(\bar{n}_p\gamma_\mu n_r)(\bar{n}_s\gamma^\mu n_t)$			$\mathcal{O}_{\ell nuq}$	$(\bar{\ell}_p^j n_r)(\bar{u}_s q_t^j)$
$\mathcal{O}_{nedu}$	$(\bar{n}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu u_t)$				
$\psi^2\phi^3$		$\psi^2\phi^2 D$		$\psi^2 X\phi$	
$\mathcal{O}_{n\phi}$	$(\phi^\dagger\phi)(\bar{l}_p n_r\tilde{\phi})$	$\mathcal{O}_{\phi n}$	$i(\phi^\dagger \overleftrightarrow{D}_\mu \phi)(\bar{n}_p\gamma^\mu n_r)$	$\mathcal{O}_{nW}$	$(\bar{\ell}_p\sigma^{\mu\nu} n_r)\tau^I\tilde{\phi}W_{\mu\nu}^I$
		$\mathcal{O}_{\phi ne}$	$i(\tilde{\phi}^\dagger D_\mu \phi)(\bar{n}_p\gamma^\mu e_r)$	$\mathcal{O}_{nB}$	$(\bar{\ell}_p\sigma^{\mu\nu} n_r)\tilde{\phi}B_{\mu\nu}$

11 fermionic operators + 5 bosonic operators

N production operator

- We assume N can talk to B quark and is at sub GeV scale
- N can be produced via B meson decay

$$-\mathcal{L}_{\rm eff} = \frac{4G_F V_{cb}}{\sqrt{2}} \left(O^V_{LL} + \sum_{\substack{X=S,V,T \\ \alpha,\beta=L,R}} C^X_{\alpha\beta}\; O^X_{\alpha\beta} \right)$$

$$O^V_{\alpha\beta} \equiv (\bar{c}\gamma^\mu P_\alpha b)(\bar{\ell}\gamma^\mu P_\beta \nu)\,,$$

$$O^S_{\alpha\beta} \equiv (\bar{c} P_\alpha b)(\bar{\ell} P_\beta \nu)\,,$$

$$O^T_{\alpha\beta} \equiv \delta_{\alpha\beta}(\bar{c}\sigma^{\mu\nu} P_\alpha b)(\bar{\ell}\sigma_{\mu\nu} P_\beta \nu)\,.$$

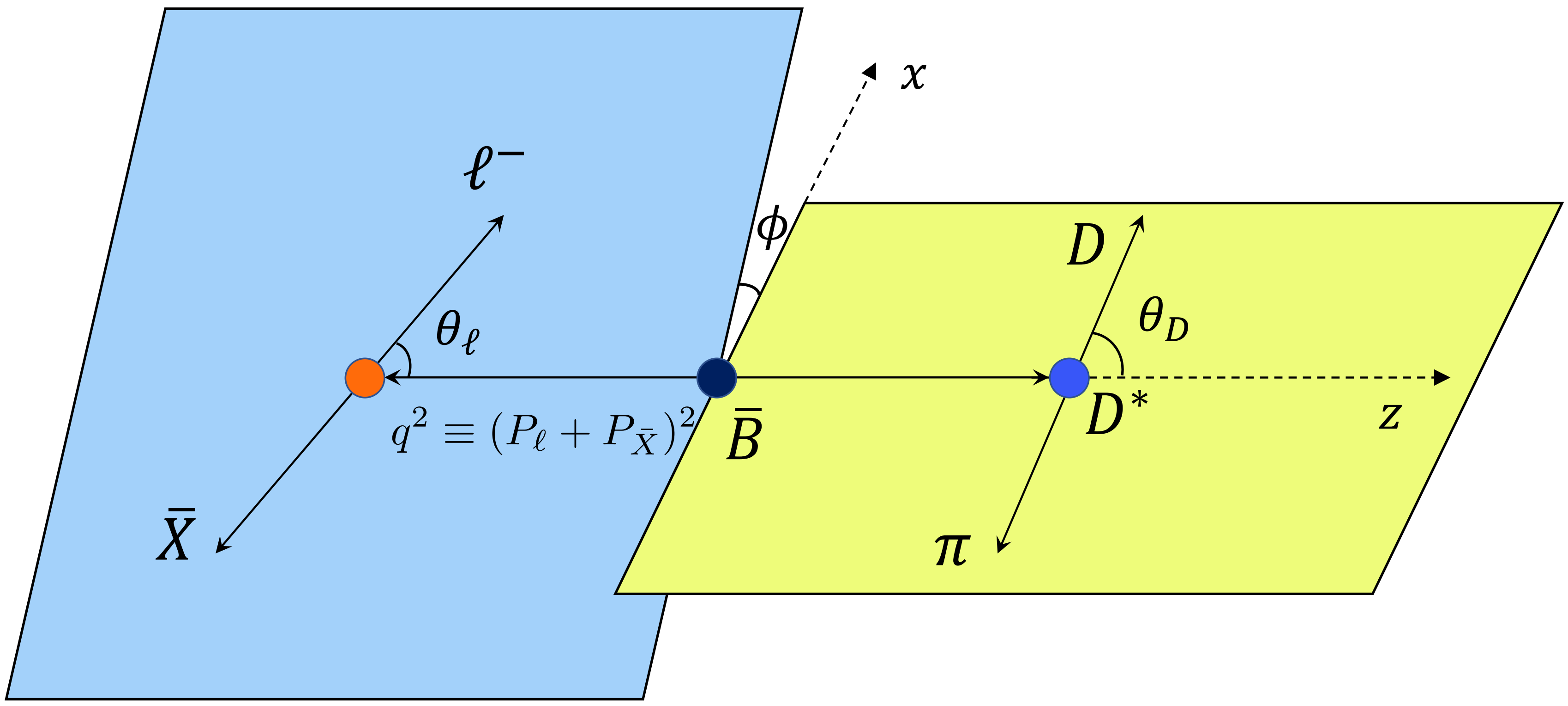
$$\mathcal{O}_{nedu} \to O^V_{RR}$$

$$\mathcal{O}_{\ell nuq} \to O^S_{LR}$$

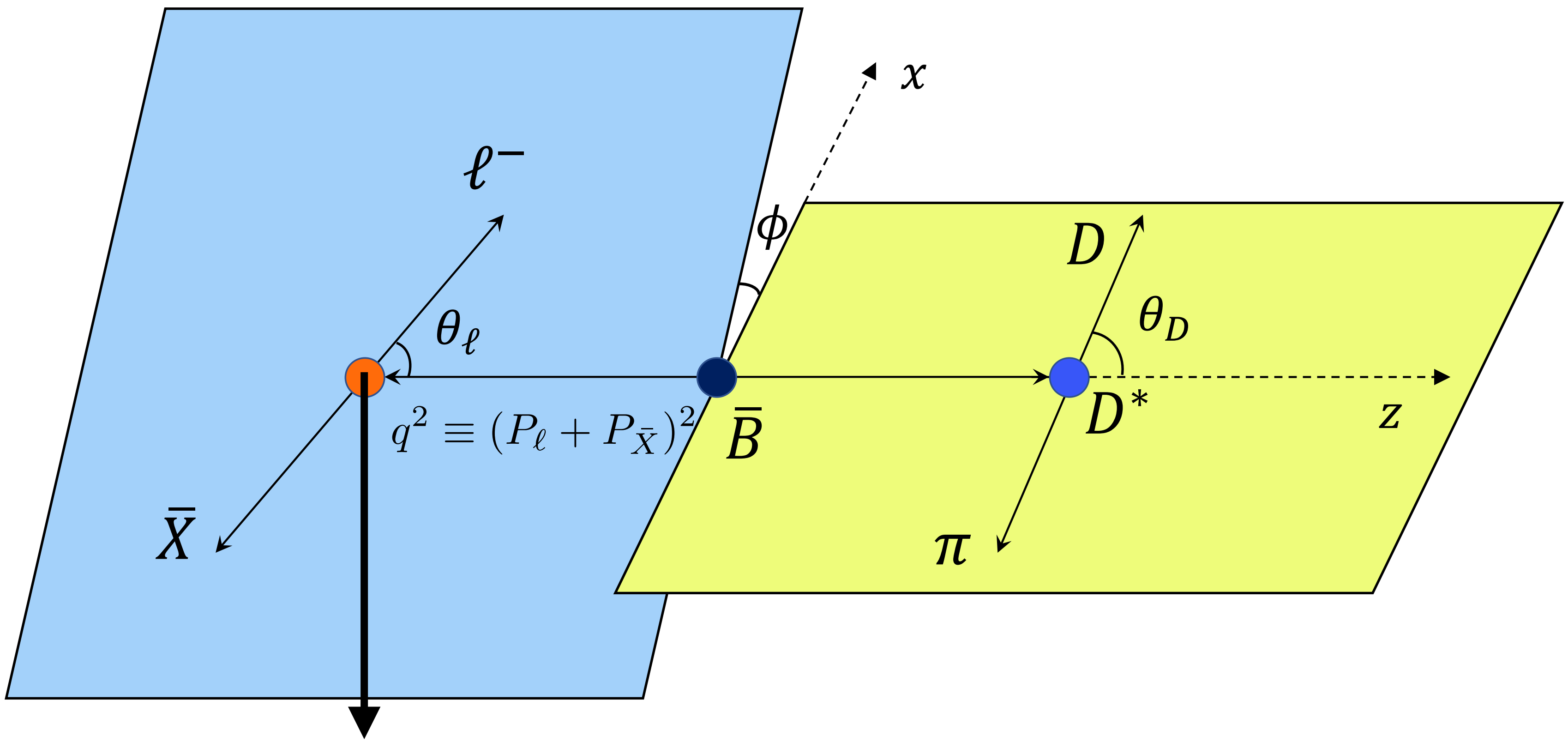
$$\mathcal{O}^{(1)}_{\ell nqd} \to O^S_{RR}$$

$$\mathcal{O}^{(3)}_{\ell nqd} \to O^T_{RR}$$

N production from B meson decay $\bar{B} \to D^{(*)} \ell \bar{X}$



N production from B meson decay $\bar{B} \to D^{(*)} \ell \bar{X}$



$$L^{V,L/R}_{\lambda_\ell,\lambda_{\bar{X}},\lambda} = \epsilon_\mu(\lambda)\langle \ell(\lambda_\ell)\bar{\nu}(\lambda_{\bar{X}})|\bar{\ell}\gamma^\mu(1\mp\gamma_5)\nu|0\rangle\,,$$

$$L^{S,L/R}_{\lambda_\ell,\lambda_{\bar{X}}} = \langle \ell(\lambda_\ell)\bar{\nu}(\lambda_{\bar{X}})|\bar{\ell}(1\mp\gamma_5)\nu|0\rangle\,,$$

$$L^{T,L/R}_{\lambda_\ell,\lambda_{\bar{X}},\lambda\lambda'} = -L^{T,L/R}_{\lambda_\ell,\lambda_{\bar{X}},\lambda'\lambda} = -i\epsilon_\mu(\lambda)\epsilon_\nu(\lambda')\langle \ell(\lambda_\ell)\bar{\nu}(\lambda_{\bar{X}})|\bar{\ell}\sigma^{\mu\nu}(1\mp\gamma_5)\nu|0\rangle\,.$$

$$\cdot\ L \equiv L\ (q^2, m_\ell, m_N, \theta_\ell, \phi)$$

$\bar{B} \to D\ell\bar{X}$

Three-body decay
$$\frac{d^2\Gamma_D}{dq^2 d\cos\theta_\ell} = \mathcal{J}_0(q^2) + \mathcal{J}_1(q^2)\cos\theta_\ell + \mathcal{J}_2(q^2)\cos^2\theta_\ell$$

Angular functions
$$A_{FB}^D(q^2) \equiv -\frac{\mathcal{J}_1(q^2)}{d\Gamma_D/dq^2}$$

$$\langle O \rangle \equiv \frac{1}{\Gamma_{\rm tot}^{D^{(*)}}} \int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 O(q^2) \frac{d\Gamma^{D^{(*)}}}{dq^2}$$

$\bar{B} \to D^*(\to D\pi)\ell\bar{X}$

Four-body decay

$$\frac{8\pi}{3}\frac{d^4\Gamma_{D^*}}{dq^2 d\cos\theta_\ell d\cos\theta_D d\phi} = (\mathcal{I}_{1s} + \mathcal{I}_{2s}\cos 2\theta_\ell + \mathcal{I}_{6s}\cos\theta_\ell)\sin^2\theta_D$$
$$+ (\mathcal{I}_{1c} + \mathcal{I}_{2c}\cos 2\theta_\ell + \mathcal{I}_{6c}\cos\theta_\ell)\cos^2\theta_D$$
$$+ (\mathcal{I}_3\cos 2\phi + \mathcal{I}_9\sin 2\phi)\sin^2\theta_D\sin^2\theta_\ell$$
$$+ (\mathcal{I}_4\cos\phi + \mathcal{I}_8\sin\phi)\sin 2\theta_D\sin 2\theta_\ell$$
$$+ (\mathcal{I}_5\cos\phi + \mathcal{I}_7\sin\phi)\sin 2\theta_D\sin\theta_\ell\ ,$$

Angular functions

$$A_{FB}^{D^*}(q^2) = -\frac{\mathcal{I}_{6s}(q^2) + \frac{1}{2}\mathcal{I}_{6c}(q^2)}{\Gamma_f^{D^*}(q^2)} \qquad F_L(q^2) = \frac{\mathcal{I}_{1c}(q^2) - \frac{1}{3}\mathcal{I}_{2c}(q^2)}{\Gamma_f^{D^*}(q^2)}$$

$$\tilde{F}_L(q^2) = \frac{1}{3} - \frac{8}{9}\frac{2\mathcal{I}_{2s}(q^2) + \mathcal{I}_{2c}(q^2)}{\Gamma_f^{D^*}(q^2)}$$

$$S_i(q^2) = \frac{\mathcal{I}_i(q^2)}{\Gamma_f^{D^*}(q^2)}\,, \quad i = \{3,4,5,7,8,9\}$$

Measurements

Observable	Measurement
$\Delta\langle A_{\mathrm{FB}}^{D^*}\rangle$	0.0349 ± 0.0089
$\Delta\langle F_L\rangle$	-0.0065 ± 0.0059
$\Delta\langle \tilde{F}_L\rangle$	-0.0107 ± 0.0142
$\Delta\langle S_3\rangle$	-0.0127 ± 0.0109
$R_D^{\mu/e}$	$0.995 \pm 0.022 \pm 0.039$
$R_{D^*}^{\mu/e}$	$0.99 \pm 0.01 \pm 0.03$

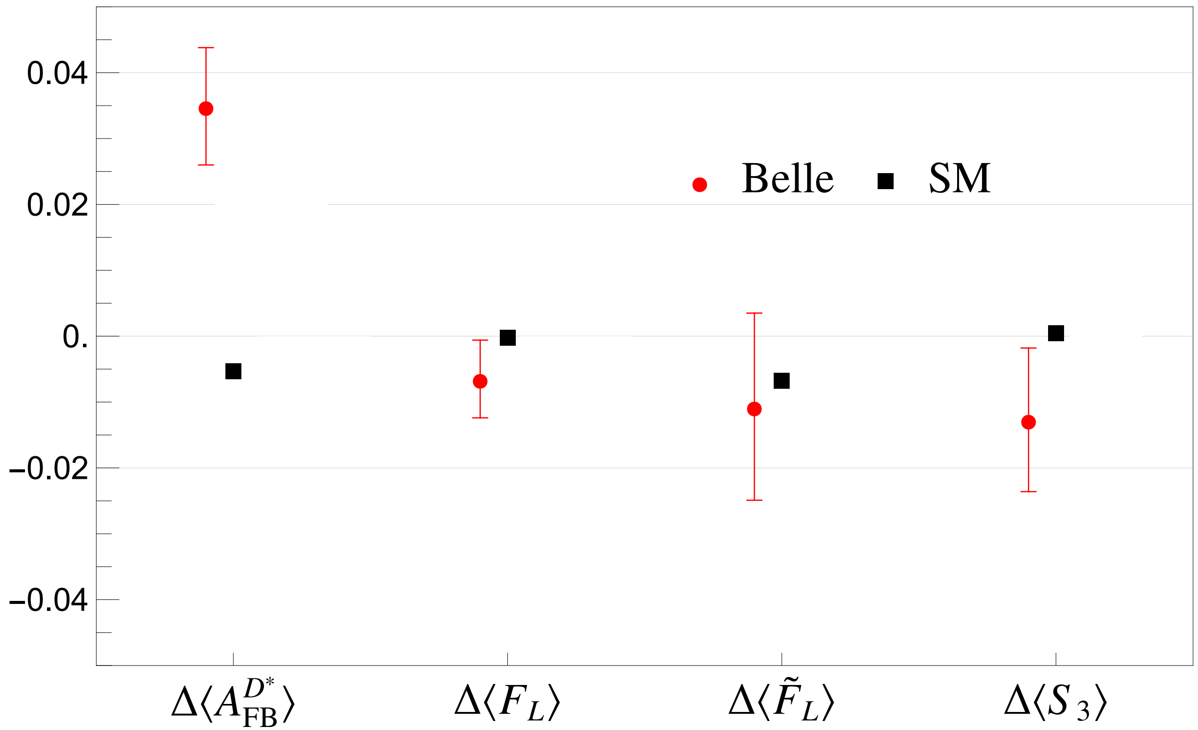
The difference and ratio between observables of muon and electron channel have little form factor sensitivities

$$\Delta\langle O\rangle \equiv \langle O^\mu\rangle - \langle O^e\rangle$$

$$R_{D^{(*)}}^{\mu/e} \equiv \mathrm{Br}(\bar{B} \to D^{(*)}\mu^- \bar{X})/\mathrm{Br}(\bar{B} \to D^{(*)}e^- \bar{X})$$

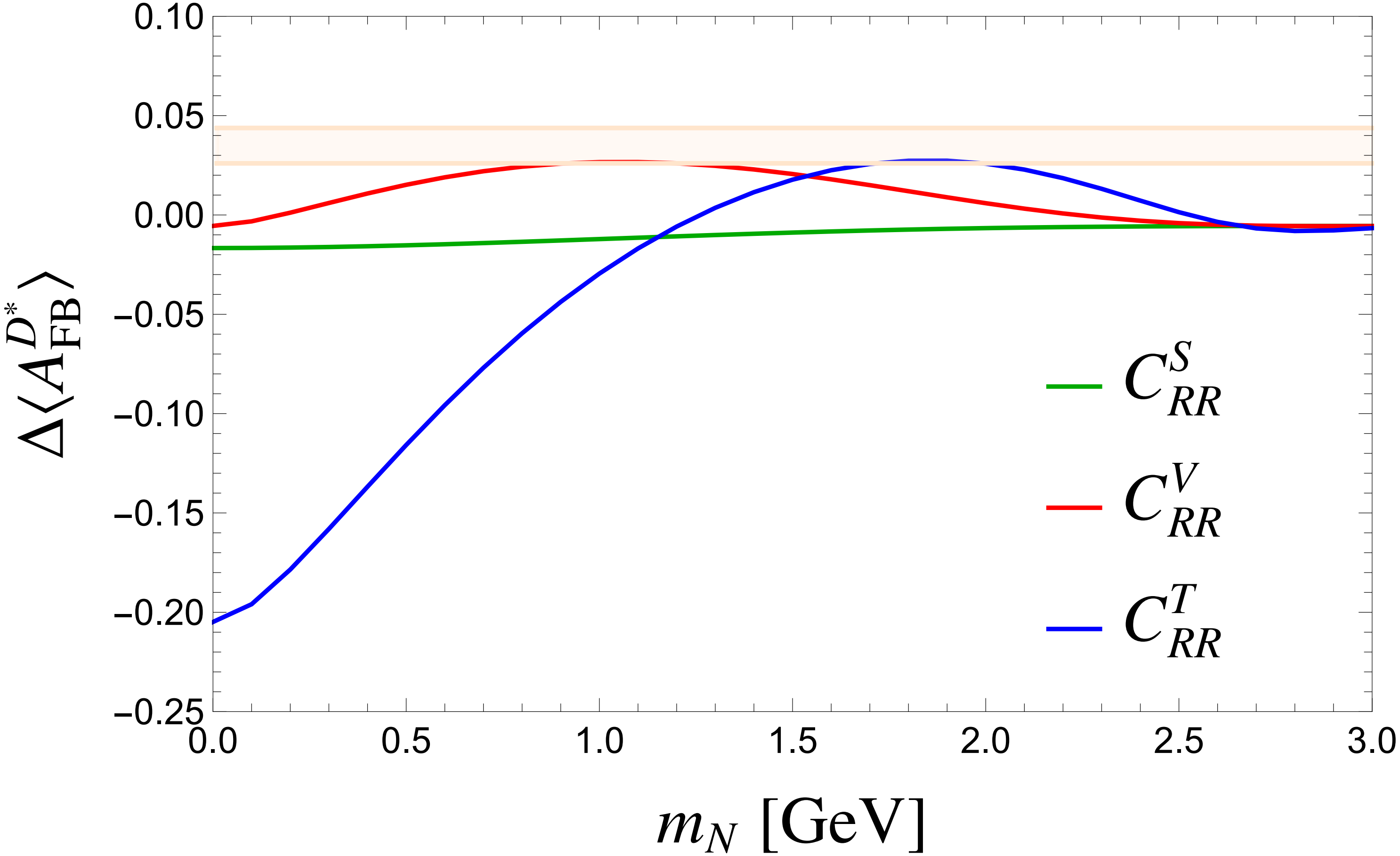
Measurements

Observable	Measurement
$\Delta\langle A_{\rm FB}^{D^*}\rangle$	0.0349 ± 0.0089
$\Delta\langle F_L\rangle$	-0.0065 ± 0.0059
$\Delta\langle \tilde{F}_L\rangle$	-0.0107 ± 0.0142
$\Delta\langle S_3\rangle$	-0.0127 ± 0.0109
$R_D^{\mu/e}$	$0.995 \pm 0.022 \pm 0.039$
$R_{D^*}^{\mu/e}$	$0.99 \pm 0.01 \pm 0.03$



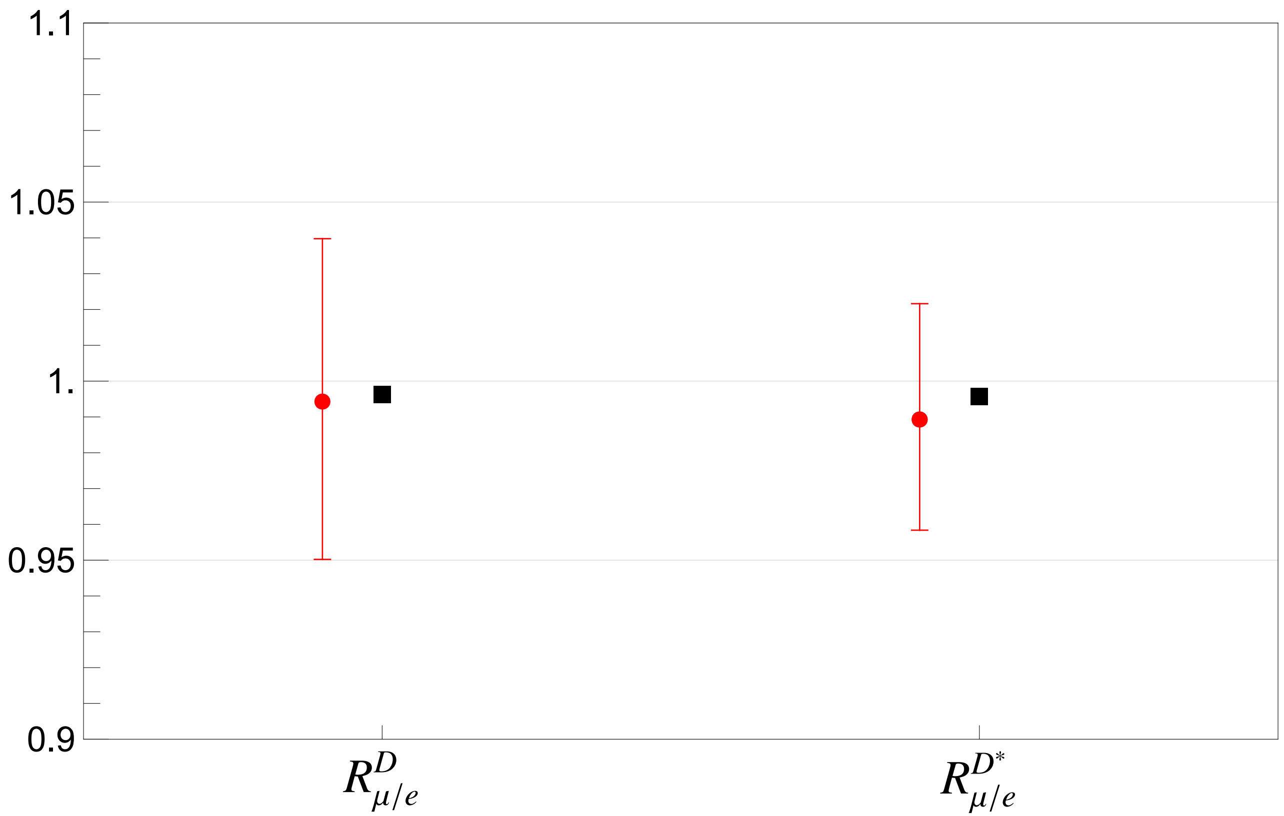
Anomaly

We assume the new physics is only in the muon sector



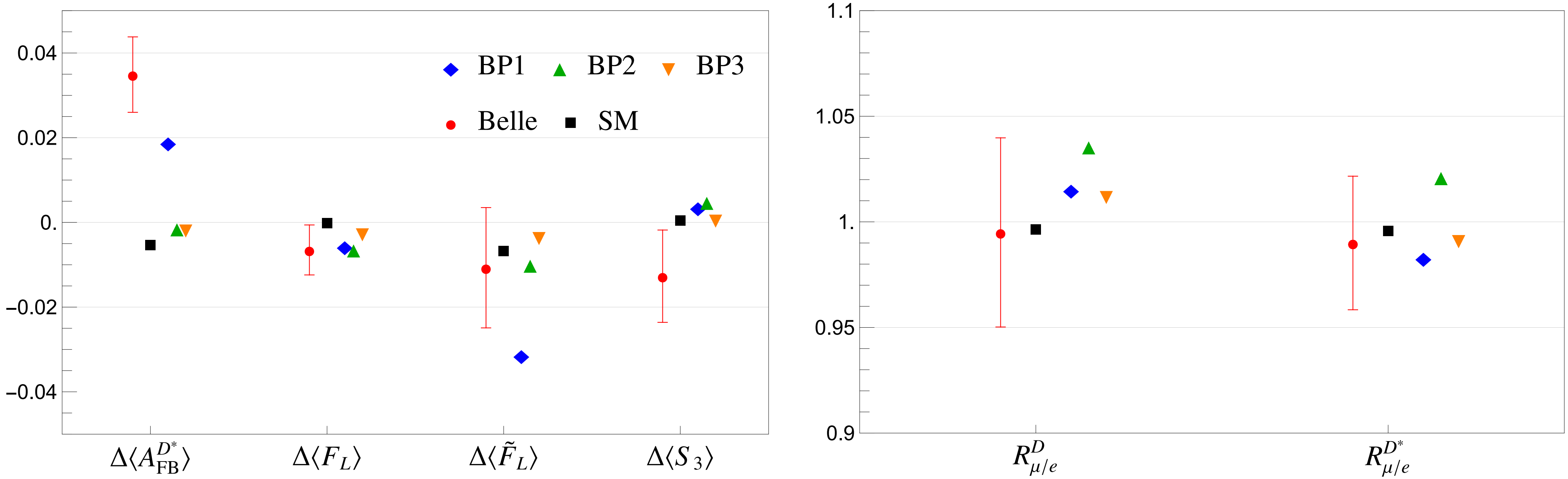
Measurements

Observable	Measurement
$\Delta\langle A_{\rm FB}^{D^*}\rangle$	0.0349 ± 0.0089
$\Delta\langle F_L\rangle$	-0.0065 ± 0.0059
$\Delta\langle \tilde{F}_L\rangle$	-0.0107 ± 0.0142
$\Delta\langle S_3\rangle$	-0.0127 ± 0.0109
$R_D^{\mu/e}$	$0.995 \pm 0.022 \pm 0.039$
$R_{D^*}^{\mu/e}$	$0.99 \pm 0.01 \pm 0.03$

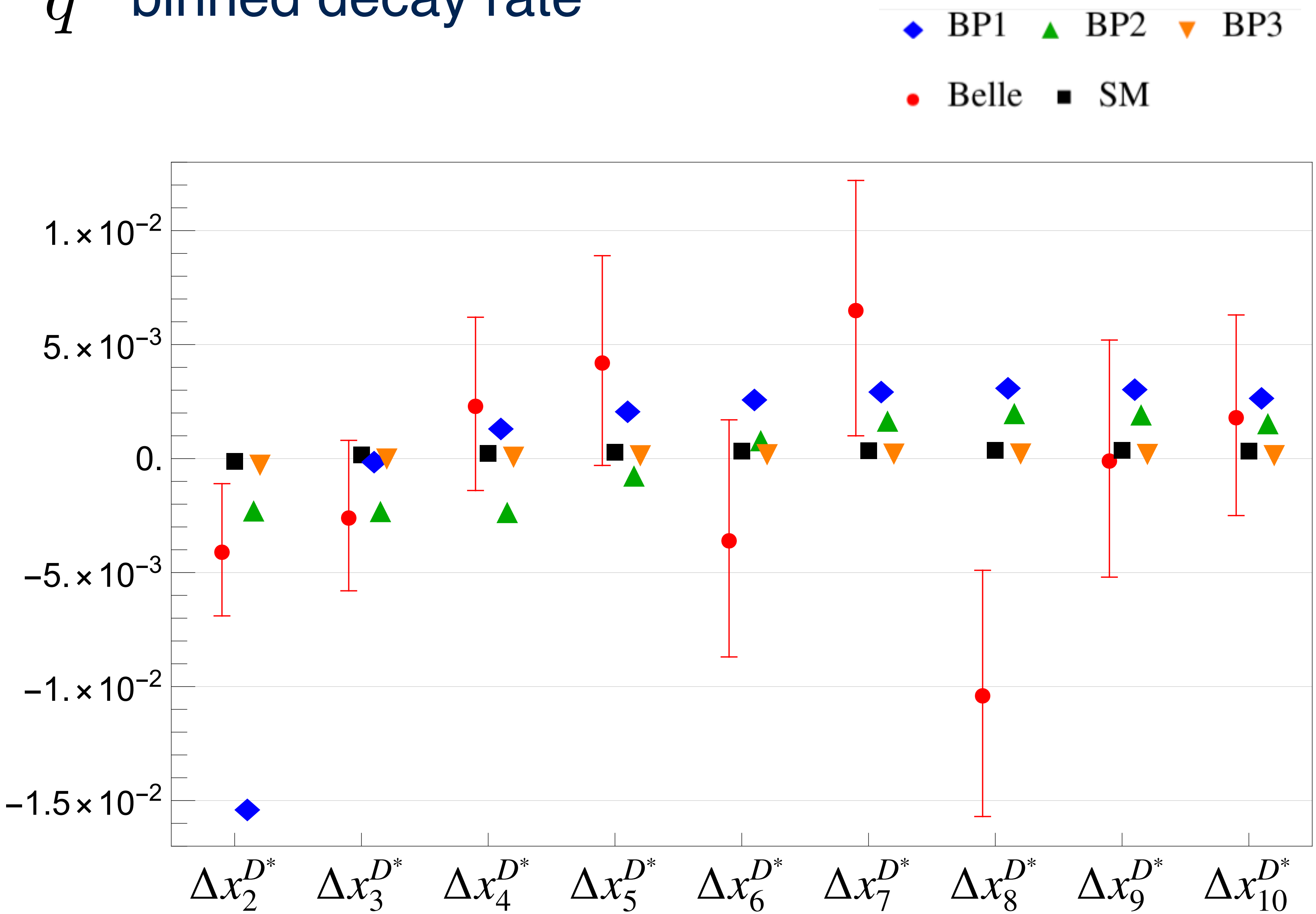


Benchmark points

	m_N (GeV)	C^V_{RR}	C^S_{RR}	C^T_{RR}	C^V_{LL}	C^S_{LL}	C^T_{LL}
BP1	0.4	0.82	0.1	0.02	-0.4	0	0
BP2	1.6	0.15	-0.3	0.06	0	0	0
BP3	0	0	0	0	0	0.06	0.02

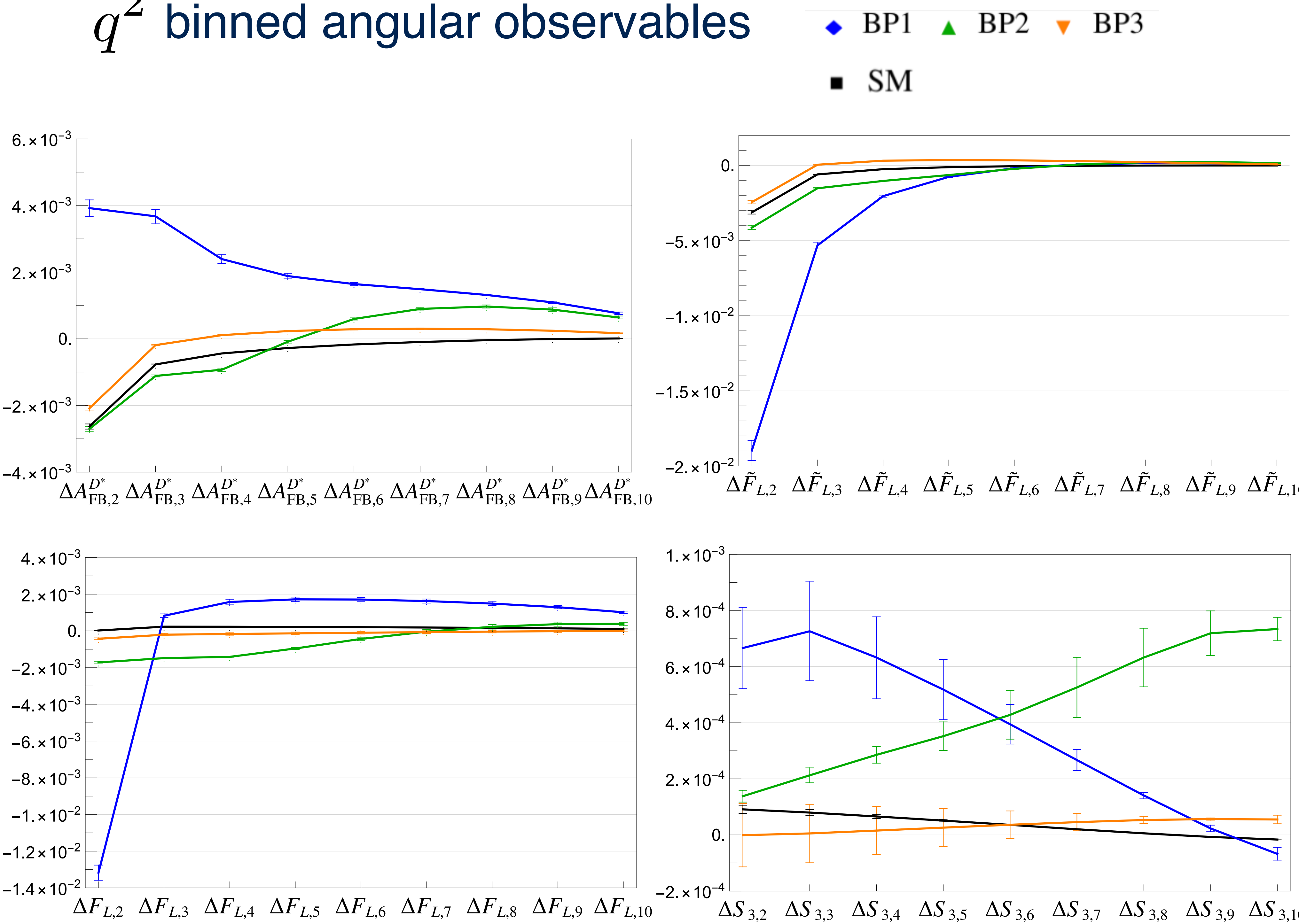


q^2 binned decay rate



$$q_i^2 \equiv m_B^2 + m_{D^{(*)}}^2 - 2m_B m_{D^{(*)}}(1 + i/20)\,, \quad i = 1 \text{ to } 10$$

q^2 binned angular observables



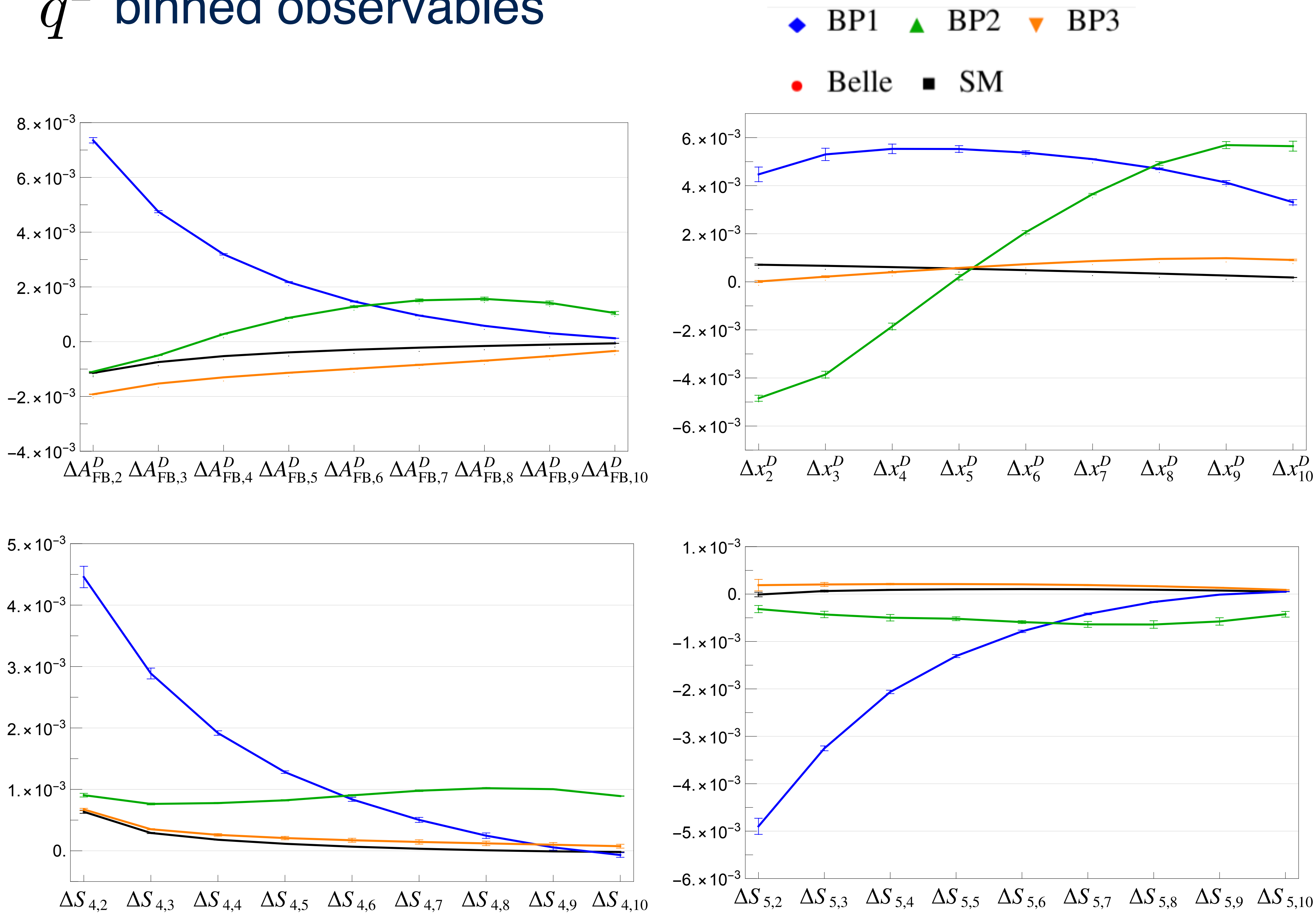
Summary

- We calculate the RHN mass effect on the decay channel $\bar{B} \to D^{(*)} \ell \bar{X}$
- Nonzero RHN mass produced significant effects in the angular observables which may explain the 4σ tension in $\Delta \langle A_{\rm FB}^{D^*} \rangle$
- Large effects may appear in the q^2-binned observables, even though the effects on the q^2-averaged ones are small.

Thanks!
Questions to liu.hongkai@campus.technion.ac.il

Back-up slides

q^2 binned observables



General Neutrino Interactions

j	$\overset{(\sim)}{\epsilon}_j$	$\mathcal{O}_j$	$\mathcal{O}'_j$
1	ϵ_L	$\gamma_\mu(\mathbb{1}-\gamma^5)$	$\gamma^\mu(\mathbb{1}-\gamma^5)$
2	$\tilde{\epsilon}_L$	$\gamma_\mu(\mathbb{1}+\gamma^5)$	$\gamma^\mu(\mathbb{1}-\gamma^5)$
3	ϵ_R	$\gamma_\mu(\mathbb{1}-\gamma^5)$	$\gamma^\mu(\mathbb{1}+\gamma^5)$
4	$\tilde{\epsilon}_R$	$\gamma_\mu(\mathbb{1}+\gamma^5)$	$\gamma^\mu(\mathbb{1}+\gamma^5)$
5	ϵ_S	$(\mathbb{1}-\gamma^5)$	$\mathbb{1}$
6	$\tilde{\epsilon}_S$	$(\mathbb{1}+\gamma^5)$	$\mathbb{1}$
7	$-\epsilon_P$	$(\mathbb{1}-\gamma^5)$	γ^5
8	$-\tilde{\epsilon}_P$	$(\mathbb{1}+\gamma^5)$	γ^5
9	ϵ_T	$\sigma_{\mu\nu}(\mathbb{1}-\gamma^5)$	$\sigma^{\mu\nu}(\mathbb{1}-\gamma^5)$
10	$\tilde{\epsilon}_T$	$\sigma_{\mu\nu}(\mathbb{1}+\gamma^5)$	$\sigma^{\mu\nu}(\mathbb{1}+\gamma^5)$

$$L_{\rm GNI}^{\rm NC} = -\frac{G_F}{\sqrt{2}} \sum_{j=1}^{10} (\overset{(\sim)}{\epsilon}_{j,f})^{\alpha\beta\gamma\delta} (\overline{\nu}_\alpha O_j \nu_\beta)(\overline{f}_\gamma O'_j f_\delta)$$

$$L_{\rm GNI}^{\rm CC} = -\frac{G_F V_{\gamma\delta}}{\sqrt{2}} \sum_{j=1}^{10} (\overset{(\sim)}{\epsilon}_{j,ff'})^{\alpha\beta\gamma\delta} (\overline{\ell}_\alpha O_j \nu_\beta)(\overline{f}_\gamma O'_j f'_\delta) + \text{h.c.}$$

SMEFT

GNI

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu \tau^I q_r)(\bar{q}_s\gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu \tau^I l_r)(\bar{q}_s\gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$		

[arXiv:1008.4884]

j	$\overset{(\sim)}{\epsilon}_j$	$\mathcal{O}_j$	$\mathcal{O}'_j$
1	ϵ_L	$\gamma_\mu(\mathbb{1}-\gamma^5)$	$\gamma^\mu(\mathbb{1}-\gamma^5)$
2	$\tilde{\epsilon}_L$	$\gamma_\mu(\mathbb{1}+\gamma^5)$	$\gamma^\mu(\mathbb{1}-\gamma^5)$
3	ϵ_R	$\gamma_\mu(\mathbb{1}-\gamma^5)$	$\gamma^\mu(\mathbb{1}+\gamma^5)$
4	$\tilde{\epsilon}_R$	$\gamma_\mu(\mathbb{1}+\gamma^5)$	$\gamma^\mu(\mathbb{1}+\gamma^5)$
5	ϵ_S	$(\mathbb{1}-\gamma^5)$	$\mathbb{1}$
6	$\tilde{\epsilon}_S$	$(\mathbb{1}+\gamma^5)$	$\mathbb{1}$
7	$-\epsilon_P$	$(\mathbb{1}-\gamma^5)$	γ^5
8	$-\tilde{\epsilon}_P$	$(\mathbb{1}+\gamma^5)$	γ^5
9	ϵ_T	$\sigma_{\mu\nu}(\mathbb{1}-\gamma^5)$	$\sigma^{\mu\nu}(\mathbb{1}-\gamma^5)$
10	$\tilde{\epsilon}_T$	$\sigma_{\mu\nu}(\mathbb{1}+\gamma^5)$	$\sigma^{\mu\nu}(\mathbb{1}+\gamma^5)$

$$L_{\rm GNI}^{\rm NC} = -\frac{G_F}{\sqrt{2}}\sum_{j=1}^{10}(\overset{(\sim)}{\epsilon}_{j,q})^{\alpha\beta\gamma\delta}(\overline{\nu}_\alpha O_j \nu_\beta)(\overline{q}_\gamma O'_j q_\delta)$$

j = 1, 3, 5, 7, 9

In SMEFT

$$L_{\rm GNI}^{\rm CC} = -\frac{G_F V_{\gamma\delta}}{\sqrt{2}}\sum_{j=1}^{10}(\overset{(\sim)}{\epsilon}_{j,ud})^{\alpha\beta\gamma\delta}(\overline{\ell}_\alpha O_j \nu_\beta)(\overline{u}_\gamma O'_j d_\delta) + \text{h.c.}$$

j =1, 5, 7, 9

SMNEFT = SMEFT + N

16 new SMNEFT operators $\Delta B = \Delta L = 0$

$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$		$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$	
$\mathcal{O}_{nd}$	$(\bar{n}_p\gamma_\mu n_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{qn}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{n}_s\gamma^\mu n_t)$	$\mathcal{O}_{\ell n\ell e}$	$(\bar{\ell}_p^j n_r)\epsilon_{jk}(\bar{\ell}_s^k e_t)$
$\mathcal{O}_{nu}$	$(\bar{n}_p\gamma_\mu n_r)(\bar{u}_s\gamma^\mu u_t)$	$\mathcal{O}_{\ell n}$	$(\bar{\ell}_p\gamma_\mu \ell_r)(\bar{n}_s\gamma^\mu n_t)$	$\mathcal{O}_{\ell nqd}^{(1)}$	$(\bar{\ell}_p^j n_r)\epsilon_{jk}(\bar{q}_s^k d_t)$
$\mathcal{O}_{ne}$	$(\bar{n}_p\gamma_\mu n_r)(\bar{e}_s\gamma^\mu e_t)$			$\mathcal{O}_{\ell nqd}^{(3)}$	$(\bar{\ell}_p^j \sigma_{\mu\nu} n_r)\epsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} d_t)$
$\mathcal{O}_{nn}$	$(\bar{n}_p\gamma_\mu n_r)(\bar{n}_s\gamma^\mu n_t)$			$\mathcal{O}_{\ell nuq}$	$(\bar{\ell}_p^j n_r)(\bar{u}_s q_t^j)$
$\mathcal{O}_{nedu}$	$(\bar{n}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu u_t)$				
$\psi^2\phi^3$		$\psi^2\phi^2 D$		$\psi^2 X\phi$	
$\mathcal{O}_{n\phi}$	$(\phi^\dagger\phi)(\bar{l}_p n_r\tilde{\phi})$	$\mathcal{O}_{\phi n}$	$i(\phi^\dagger \overleftrightarrow{D}_\mu\phi)(\bar{n}_p\gamma^\mu n_r)$	$\mathcal{O}_{nW}$	$(\bar{\ell}_p\sigma^{\mu\nu} n_r)\tau^I\tilde{\phi}W_{\mu\nu}^I$
		$\mathcal{O}_{\phi ne}$	$i(\tilde{\phi}^\dagger D_\mu\phi)(\bar{n}_p\gamma^\mu e_r)$	$\mathcal{O}_{nB}$	$(\bar{\ell}_p\sigma^{\mu\nu} n_r)\tilde{\phi}B_{\mu\nu}$

11 fermionic operators + 5 bosonic operators

SMNEFT

$(\bar{L}L)(\bar{L}L)$ and $(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$		$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$	
$\mathcal{O}_{ll}$	$(\bar{l}_\alpha\gamma_\mu l_\beta)(\bar{l}_\gamma\gamma^\mu l_\delta)$	$\mathcal{O}_{le}$	$(\bar{l}_\alpha\gamma_\mu l_\beta)(\bar{e}_\gamma\gamma^\mu e_\delta)$	$\mathcal{O}_{elqd}$	$(\bar{e}_\alpha l^j_\beta)(\bar{q}^j_\gamma d_\delta)$
$\mathcal{O}^{(1)}_{lq}$	$(\bar{l}_\alpha\gamma_\mu l_\beta)(\bar{q}_\gamma\gamma^\mu q_\delta)$	$\mathcal{O}_{lu}$	$(\bar{l}_\alpha\gamma_\mu l_\beta)(\bar{u}_\gamma\gamma^\mu u_\delta)$	$\mathcal{O}_{eluq}$	$(\bar{e}_\alpha l^j_\beta)\epsilon_{jk}(\bar{u}_\gamma q^k_\delta)$
$\mathcal{O}^{(3)}_{lq}$	$(\bar{l}_\alpha\gamma_\mu\tau^I l_\beta)(\bar{q}_\gamma\gamma^\mu\tau^I q_\delta)$	$\mathcal{O}_{ld}$	$(\bar{l}_\alpha\gamma_\mu l_\beta)(\bar{d}_\gamma\gamma^\mu d_\delta)$	$\mathcal{O}'_{eluq}$	$(\bar{e}_\alpha\sigma_{\mu\nu}l^j_\beta)\epsilon_{jk}(\bar{u}_\gamma\sigma^{\mu\nu}q^k_\delta)$
$\mathcal{O}_{Ne}$	$(\overline{N}_\alpha\gamma_\mu N_\beta)(\bar{e}_\gamma\gamma^\mu e_\delta)$	$\mathcal{O}_{Nl}$	$(\overline{N}_\alpha\gamma_\mu N_\beta)(\bar{l}_\gamma\gamma^\mu l_\delta)$	$\mathcal{O}_{Nlel}$	$(\overline{N}_\alpha l^j_\beta)\epsilon_{jk}(\bar{e}_\gamma l^k_\delta)$
$\mathcal{O}_{Nu}$	$(\overline{N}_\alpha\gamma_\mu N_\beta)(\bar{u}_\gamma\gamma^\mu u_\delta)$	$\mathcal{O}_{Nq}$	$(\overline{N}_\alpha\gamma_\mu N_\beta)(\bar{q}_\gamma\gamma^\mu q_\delta)$	$\mathcal{O}_{lNqd}$	$(\bar{l}^j_\alpha N_\beta)\epsilon_{jk}(\bar{q}^k_\gamma d_\delta)$
$\mathcal{O}_{Nd}$	$(\overline{N}_\alpha\gamma_\mu N_\beta)(\bar{d}_\gamma\gamma^\mu d_\delta)$			$\mathcal{O}'_{lNqd}$	$(\bar{l}^j_\alpha\sigma_{\mu\nu}N_\beta)\epsilon_{jk}(\bar{q}^k_\gamma\sigma^{\mu\nu}d_\delta)$
$\mathcal{O}_{eNud}$	$(\bar{e}_\alpha\gamma_\mu N_\beta)(\bar{u}_\gamma\gamma^\mu d_\delta)$			$\mathcal{O}_{lNuq}$	$(\bar{l}^j_\alpha N_\beta)(\bar{u}_\gamma q^j_\delta)$

[arXiv:1905.08699]

GNI

j	$\overset{(\sim)}{\epsilon}_j$	$\mathcal{O}_j$	$\mathcal{O}'_j$
1	ϵ_L	$\gamma_\mu(\mathbb{1}-\gamma^5)$	$\gamma^\mu(\mathbb{1}-\gamma^5)$
2	$\tilde{\epsilon}_L$	$\gamma_\mu(\mathbb{1}+\gamma^5)$	$\gamma^\mu(\mathbb{1}-\gamma^5)$
3	ϵ_R	$\gamma_\mu(\mathbb{1}-\gamma^5)$	$\gamma^\mu(\mathbb{1}+\gamma^5)$
4	$\tilde{\epsilon}_R$	$\gamma_\mu(\mathbb{1}+\gamma^5)$	$\gamma^\mu(\mathbb{1}+\gamma^5)$
5	ϵ_S	$(\mathbb{1}-\gamma^5)$	$\mathbb{1}$
6	$\tilde{\epsilon}_S$	$(\mathbb{1}+\gamma^5)$	$\mathbb{1}$
7	$-\epsilon_P$	$(\mathbb{1}-\gamma^5)$	γ^5
8	$-\tilde{\epsilon}_P$	$(\mathbb{1}+\gamma^5)$	γ^5
9	ϵ_T	$\sigma_{\mu\nu}(\mathbb{1}-\gamma^5)$	$\sigma^{\mu\nu}(\mathbb{1}-\gamma^5)$
10	$\tilde{\epsilon}_T$	$\sigma_{\mu\nu}(\mathbb{1}+\gamma^5)$	$\sigma^{\mu\nu}(\mathbb{1}+\gamma^5)$

$$L_{\text{SMNEFT}} \supset 2\sqrt{2}G_F \sum_i C_i O_i \,,$$

$$L^{\text{NC}}_{\text{GNI}} = -\frac{G_F}{\sqrt{2}} \sum_{j=1}^{10} (\overset{(\sim)}{\epsilon}_{j,q})^{\alpha\beta\gamma\delta} (\overline{\nu}_\alpha O_j \nu_\beta)(\overline{q}_\gamma O'_j q_\delta)$$

j = 1 to 10

$$L^{\text{CC}}_{\text{GNI}} = -\frac{G_F V_{\gamma\delta}}{\sqrt{2}} \sum_{j=1}^{10} (\overset{(\sim)}{\epsilon}_{j,ud})^{\alpha\beta\gamma\delta} (\bar{\ell}_\alpha O_j \nu_\beta)(\overline{u}_\gamma O'_j d_\delta) + \text{h.c.}$$

j != 2, 3

Leptonic amplitudes

- $L \equiv L\ (q^2, m_\ell, m_N, \theta_\ell, \phi)$

Mass dependence $$K_{\pm\pm}(m_N) = \frac{(E_N + m_N \pm p_N)(E_\ell + m_\ell \pm p_\ell)}{\sqrt{(E_\ell + m_\ell)(E_N + m_N)}}$$

For massless neutrinos

$$K_{++}(0) = 2\sqrt{q^2}\beta_\ell, \quad K_{+-}(0) = 2m_\ell\beta_\ell, \quad K_{-+}(0) = K_{--}(0) = 0$$

For massless charged leptons

$$K_{++}(m_N) = 2\sqrt{q^2}\beta_N, \quad K_{-+}(m_N) = 2m_N\beta_N, \quad K_{+-}(m_N) = K_{--}(m_N) = 0$$

$$\beta_\ell \equiv \sqrt{1 - m_\ell^2/q^2} \text{ and } \beta_N \equiv \sqrt{1 - m_N^2/q^2}$$

Leptonic amplitudes

- $L \equiv L\ (q^2, m_\ell, m_N, \theta_\ell, \phi)$

- $\beta = L$: left-handed operators only couple to the SM left-handed neutrinos or SM right-handed antineutrinos. Cannot produce anti-neutrinos with negative helicity

$$L^{V,L}_{\lambda_\ell,-\frac{1}{2},\lambda} = L^{S,L}_{\lambda_\ell,-\frac{1}{2}} = L^{T,L}_{\lambda_\ell,-\frac{1}{2},\lambda\lambda'} = 0$$

$$L^{S,L}_{\frac{1}{2},\frac{1}{2}},\ L^{V,L}_{-\frac{1}{2},\frac{1}{2},\lambda},\ L^{T,L}_{\frac{1}{2},\frac{1}{2},\lambda\lambda'} \propto K_{++}(0) \propto \sqrt{q^2}$$

$$L^{V,L}_{\frac{1}{2},\frac{1}{2},\lambda},\ L^{V,T}_{-\frac{1}{2},\frac{1}{2},\lambda\lambda'} \propto K_{+-}(0) \propto m_\ell$$

Leptonic amplitudes

- $L \equiv L\ (q^2, m_\ell, m_N, \theta_\ell, \phi)$

- $\beta = L$: left-handed operators only couple to the SM left-handed neutrinos or SM right-handed antineutrinos.

- $\beta = R$: right-handed operators only couple to the massive right-handed neutrinos. Its helicity can be both positive and negative.

$$L^{S,R}_{-\frac{1}{2},-\frac{1}{2}},\ L^{V,R}_{\frac{1}{2},-\frac{1}{2},\lambda},\ L^{T,R}_{-\frac{1}{2},-\frac{1}{2},\lambda\lambda'} \propto K_{++}(m_N)$$

$$L^{V,R}_{-\frac{1}{2},-\frac{1}{2},\lambda},\ L^{T,R}_{\frac{1}{2},-\frac{1}{2},\lambda\lambda'} \propto K_{+-}(m_N)$$

$$L^{V,R}_{\frac{1}{2},\frac{1}{2},\lambda},\ L^{T,R}_{-\frac{1}{2},\frac{1}{2},\lambda\lambda'} \propto K_{-+}(m_N)$$

$$L^{S,R}_{\frac{1}{2},\frac{1}{2}},\ L^{V,R}_{-\frac{1}{2},\frac{1}{2},\lambda},\ L^{T,R}_{\frac{1}{2},\frac{1}{2},\lambda\lambda'} \propto K_{--}(m_N)$$

Helicity amplitudes

$\bar{B} \to D$

$$M_D(++) \equiv \mathcal{M}(s,+,+) = \tilde{\mathcal{A}}_1^{++} + \tilde{\mathcal{A}}_2^{++} \cos\theta_\ell \,,$$
$$M_D(-+) \equiv \mathcal{M}(s,-,+) = \tilde{\mathcal{A}}^{-+} \sin\theta_\ell \,,$$
$$M_D(+-) \equiv \mathcal{M}(s,+,-) = \tilde{\mathcal{A}}^{+-} \sin\theta_\ell \,,$$
$$M_D(--) \equiv \mathcal{M}(s,-,-) = \tilde{\mathcal{A}}_1^{--} + \tilde{\mathcal{A}}_2^{--} \cos\theta_\ell \,.$$

- $\beta = L$

$$\tilde{\mathcal{A}}_1^{++} = -(C_{RL}^V + C_{LL}^V) H_{s,s}^V K_{+-}(0) - (C_{LL}^S + C_{RL}^S) H_s^S K_{++}(0) \,,$$
$$\tilde{\mathcal{A}}_2^{++} = -(C_{LL}^V + C_{RL}^V) H_{s,0}^V K_{+-}(0) + 4 C_{LL}^T H_s^T K_{++}(0) \,,$$
$$\tilde{\mathcal{A}}^{-+} = (C_{LL}^V + C_{RL}^V) H_{s,0}^V K_{++}(0) - 4 C_{LL}^T H_s^T K_{+-}(0) \,,$$
$$\tilde{\mathcal{A}}^{+-} = \tilde{\mathcal{A}}_1^{--} = \tilde{\mathcal{A}}_2^{--} = 0 \,,$$

Helicity amplitudes

$\bar{B} \to D$

$$
\begin{aligned}
M_D(++) &\equiv \mathcal{M}(s,+,+) = \tilde{\mathcal{A}}_1^{++} + \tilde{\mathcal{A}}_2^{++} \cos\theta_\ell \,, \\
M_D(-+) &\equiv \mathcal{M}(s,-,+) = \tilde{\mathcal{A}}^{-+} \sin\theta_\ell \,, \\
M_D(+-) &\equiv \mathcal{M}(s,+,-) = \tilde{\mathcal{A}}^{+-} \sin\theta_\ell \,, \\
M_D(--) &\equiv \mathcal{M}(s,-,-) = \tilde{\mathcal{A}}_1^{--} + \tilde{\mathcal{A}}_2^{--} \cos\theta_\ell \,.
\end{aligned}
$$

- $\beta = R$

$$
\begin{aligned}
\tilde{\mathcal{A}}_1^{++} &= (C_{LR}^V + C_{RR}^V) H_{s,s}^V K_{-+}(m_N) + (C_{LR}^S + C_{RR}^S) H_s^S K_{--}(m_N) \,, \\
\tilde{\mathcal{A}}_2^{++} &= -(C_{LR}^V + C_{RR}^V) H_{s,0}^V K_{-+}(m_N) + 4 C_{RR}^T K_{--}(m_N) H_s^T \,, \\
\tilde{\mathcal{A}}^{-+} &= (C_{LR}^V + C_{RR}^V) H_{s,0}^V K_{--}(m_N) - 4 C_{RR}^T H_s^T K_{-+}(m_N) \,, \\
\tilde{\mathcal{A}}^{+-} &= -(C_{LR}^V + C_{RR}^V) H_{s,0}^V K_{++}(m_N) + 4 C_{RR}^T H_s^T K_{+-}(m_N) \,, \\
\tilde{\mathcal{A}}_1^{--} &= -(C_{LR}^V + C_{RR}^V) H_{s,s}^V K_{+-}(m_N) - (C_{LR}^S + C_{RR}^S) H_s^S K_{++}(m_N) \,, \\
\tilde{\mathcal{A}}_2^{--} &= -(C_{LR}^V + C_{RR}^V) H_{s,0}^V K_{+-}(m_N) + 4 C_{RR}^T H_s^T K_{++}(m_N) \,,
\end{aligned}
$$