Simple Rules for Evanescent Operators in One-Loop Basis Transformations

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FPCP May 2022
Mississippi

arXiv: 2202.01225
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Unterstützt von / Supported by
Alexander von Humboldt
Stiftung / Foundation

Technische Universität München

Standard Model Effective Field Theory

No discovery of New Particles at Large Hadron Collider -> Scale Gap!

- Field content same as that of Standard Model.
- Electroweak symmetry is broken by one Higgs-doublet.
- Full SM gauge symmetry is respected.

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{\mathrm{SM}} + \mathscr{L}_{eff}$$

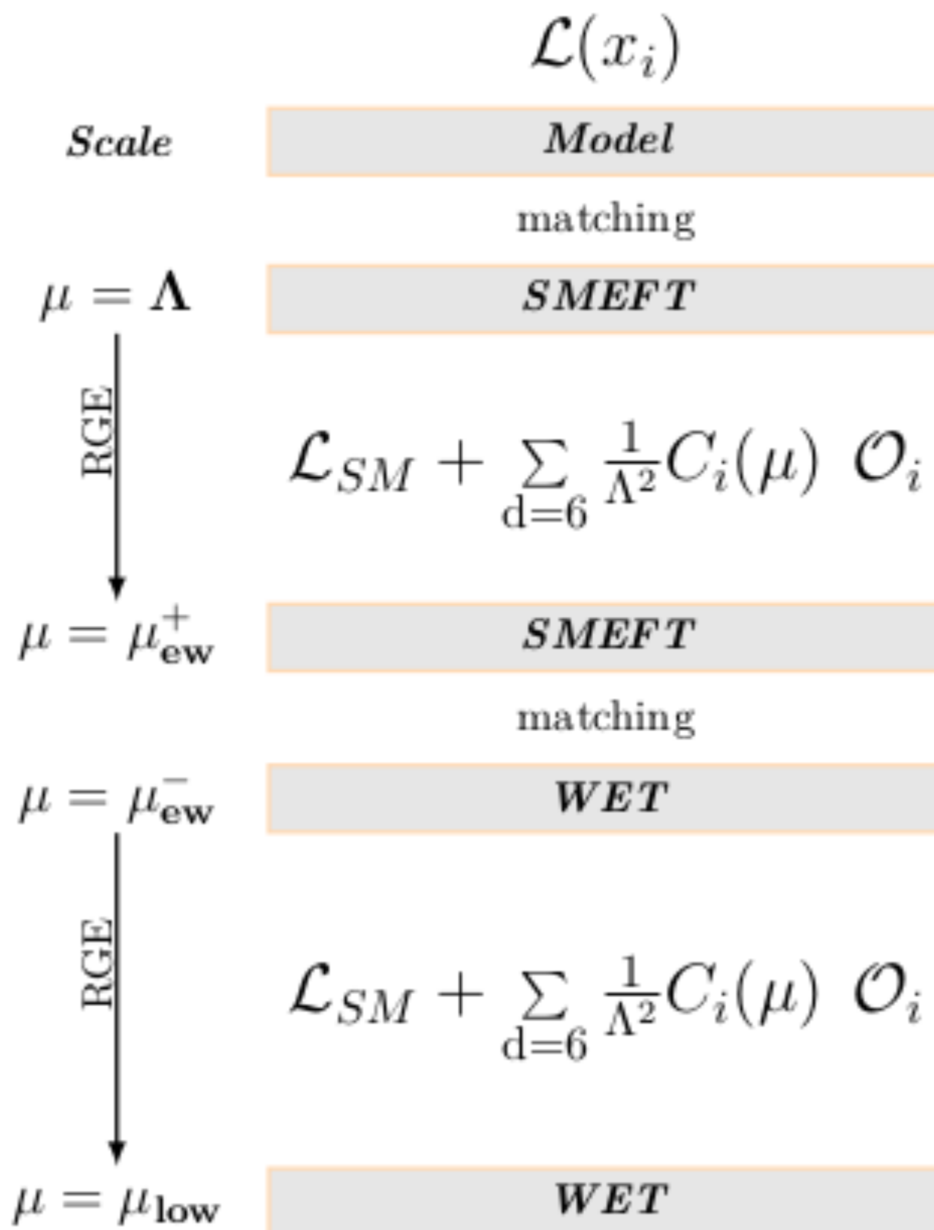
Buchmuller and Wyler 1986

$$\mathscr{L}_{\mathrm{eff}} = \sum_{\mathrm{dim}=5,6,..} C_j O_j$$

15 (Bosonic) +
19 (Single-fermion current) +
25 (Four-fermion) = 59 Operators

Warsaw Basis

General Strategy



1-loop
2-loop (partial)

Warsaw Basis

Tree-level
1-loop

1-loop
2-loop

JMS, BMU, Bern, Flavio bases

SM Effective Field Theory : SMEFT

Weak Effective Theory : WET

The method

Tree-level bases transformation

$$Q_{\rm BMU} = \{Q_1, Q_2, \ldots, Q_N\}$$

$$O_{\rm JMS} = \{O_1, O_2, \ldots, O_N\}$$

JMS= Jenkins, Manohar, Stoffer

BMU = Buras, Misiak, Urban

$$O_{\rm JMS} = \hat{R}^{(0)} Q_{\rm BMU}$$

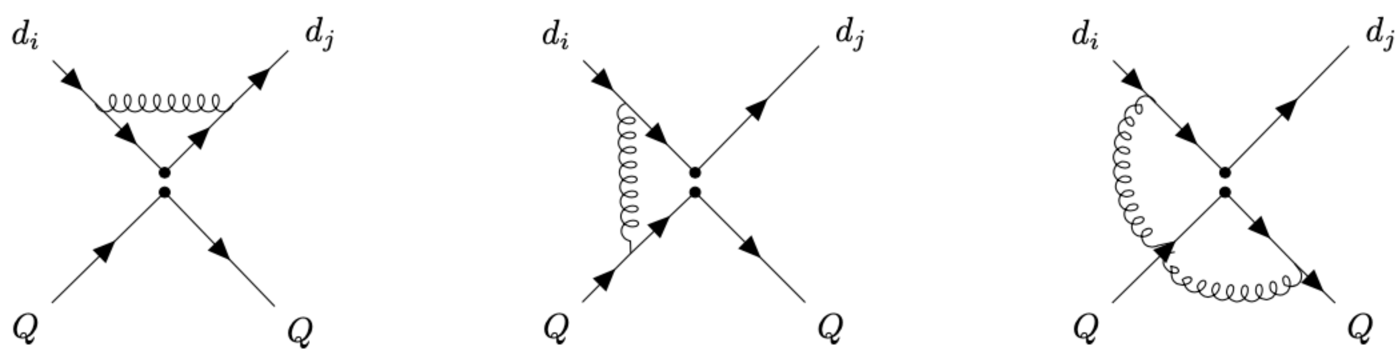
$\hat{R}^{(0)}$ is linear transformation between the two bases

Often such a transformation involves the use of Fierz relations valid only in D=4

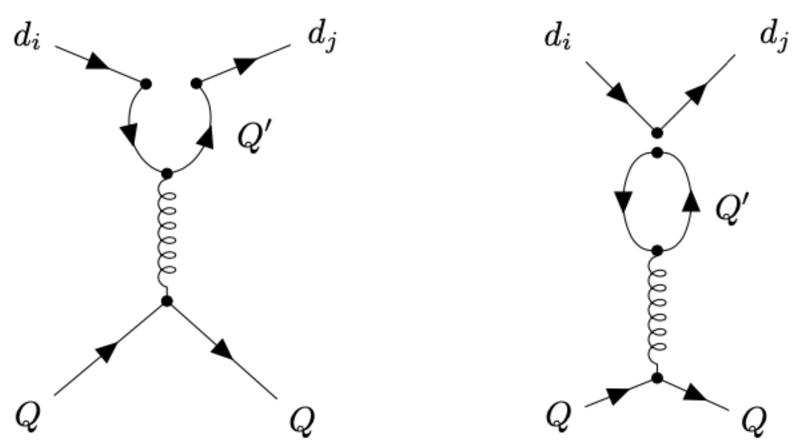
1-loop bases transformation

Step 1: Perform Fierz transformation on every BMU operator

$$\tilde{Q} = \sum_k \omega_k Q_k$$



Current-current diagrams



Penguin diagrams

Step 2: Insert Q_i and $\tilde{Q}_i$ into current-current and QCD penguin diagrams

$$Q_i - \tilde{Q}_i = \mathrm{EV_i}$$

$$\mathrm{EV_i} = \frac{\alpha_s}{4\pi} \sum_r \tilde{\omega}_r Q_r$$

Step 3: Plugin the shifts in the tree-level transformation

$$O_{\mathrm{JMS}} = \hat{R}\ Q_{\mathrm{BMU}}$$

$$\hat{R} = \hat{R}^{(0)} + \frac{\alpha_s}{4\pi}\ \hat{R}^{(1)}$$

Summary of the Rules

1. If no Fierz transformations are required then the corresponding blocks in $\hat{R}^{(0)}$ and $\hat{R}^{(1)}$ are simply equal.

2. If at tree-level Fierz transformations are required but the corresponding shifts vanish, then the corresponding blocks in $\hat{R}^{(0)}$ and $\hat{R}^{(1)}$ are still the same.

3. In certain blocks the necessity of performing Fierz transformation would introduce the shifts which will introduce EV operators and hence contribute to the $\hat{R}^{(1)}$.

Example:
BMJ to JMS conversion at 1-loop level

$\Delta F = 1$ operators in BMU basis

Current-current

$$Q_1 = Q_1^{\mathrm{VLL},u} = (\bar{d}_j^\alpha \gamma_\mu P_L u^\beta)(\bar{u}^\beta \gamma^\mu P_L d_i^\alpha),$$

$$Q_2 = Q_2^{\mathrm{VLL},u} = (\bar{d}_j^\alpha \gamma_\mu P_L u^\alpha)(\bar{u}^\beta \gamma^\mu P_L d_i^\beta),$$

QCD penguin

$$Q_3 = (\bar{d}_j^\alpha \gamma_\mu P_L d_i^\alpha) \sum_q (\bar{q}^\beta \gamma^\mu P_L\, q^\beta), \qquad Q_4 = (\bar{d}_j^\alpha \gamma_\mu P_L d_i^\beta) \sum_q (\bar{q}^\beta \gamma^\mu P_L\, q^\alpha),$$

$$Q_5 = (\bar{d}_j^\alpha \gamma_\mu P_L d_i^\alpha) \sum_q (\bar{q}^\beta \gamma^\mu P_R\, q^\beta), \qquad Q_6 = (\bar{d}_j^\alpha \gamma_\mu P_L d_i^\beta) \sum_q (\bar{q}^\beta \gamma^\mu P_R\, q^\alpha),$$

EW penguin

$$Q_7 = \frac{3}{2} (\bar{d}_j^\alpha \gamma_\mu P_L d_i^\alpha) \sum_q Q_q\, (\bar{q}^\beta \gamma^\mu P_R\, q^\beta), \qquad Q_8 = \frac{3}{2} (\bar{d}_j^\alpha \gamma_\mu P_L d_i^\beta) \sum_q Q_q\, (\bar{q}^\beta \gamma^\mu P_R\, q^\alpha),$$

$$Q_9 = \frac{3}{2} (\bar{d}_j^\alpha \gamma_\mu P_L d_i^\alpha) \sum_q Q_q\, (\bar{q}^\beta \gamma^\mu P_L\, q^\beta), \qquad Q_{10} = \frac{3}{2} (\bar{d}_j^\alpha \gamma_\mu P_L d_i^\beta) \sum_q Q_q\, (\bar{q}^\beta \gamma^\mu P_L\, q^\alpha).$$

Total 40 $\Delta F = 1$ independent operartors

A. J. Buras, M. Misiak, and J. Urban,
Two loop QCD anomalous dimensions of flavor changing four quark operators within and beyond the standard model, Nucl. Phys. B586 (2000) 397–426, [hep-ph/0005183].

$\Delta F = 1$ operators in JMS basis

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$	
$[O_{dd}^{V,LL}]_{prst}$	$(\bar{d}_L^p \gamma_\mu d_L^r)(\bar{d}_L^s \gamma^\mu d_L^t)$	$[O_{dd}^{V,RR}]_{prst}$	$(\bar{d}_R^p \gamma_\mu d_R^r)(\bar{d}_R^s \gamma^\mu d_R^t)$
$[O_{ud}^{V1,LL}]_{prst}$	$(\bar{u}_L^p \gamma_\mu u_L^r)(\bar{d}_L^s \gamma^\mu d_L^t)$	$[O_{ud}^{V1,RR}]_{prst}$	$(\bar{u}_R^p \gamma_\mu u_R^r)(\bar{d}_R^s \gamma^\mu d_R^t)$
$[O_{ud}^{V8,LL}]_{prst}$	$(\bar{u}_L^p \gamma_\mu T^A u_L^r)(\bar{d}_L^s \gamma^\mu T^A d_L^t)$	$[O_{ud}^{V8,RR}]_{prst}$	$(\bar{u}_R^p \gamma_\mu T^A u_R^r)(\bar{d}_R^s \gamma^\mu T^A d_R^t)$

$(\bar{L}L)(\bar{R}R)$		$(\bar{L}R)(\bar{L}R)$+ h.c.	
$[O_{dd}^{V1,LR}]_{prst}$	$(\bar{d}_L^p \gamma_\mu d_L^r)(\bar{d}_R^s \gamma^\mu d_R^t)$	$[O_{dd}^{S1,RR}]_{prst}$	$(\bar{d}_L^p d_R^r)(\bar{d}_L^s d_R^t)$
$[O_{dd}^{V8,LR}]_{prst}$	$(\bar{d}_L^p \gamma_\mu T^A d_L^r)(\bar{d}_R^s \gamma^\mu T^A d_R^t)$	$[O_{dd}^{S8,RR}]_{prst}$	$(\bar{d}_L^p T^A d_R^r)(\bar{d}_L^s T^A d_R^t)$
$[O_{ud}^{V1,LR}]_{prst}$	$(\bar{u}_L^p \gamma_\mu u_L^r)(\bar{d}_R^s \gamma^\mu d_R^t)$	$[O_{ud}^{S1,RR}]_{prst}$	$(\bar{u}_L^p u_R^r)(\bar{d}_L^s d_R^t)$
$[O_{ud}^{V8,LR}]_{prst}$	$(\bar{u}_L^p \gamma_\mu T^A u_L^r)(\bar{d}_R^s \gamma^\mu T^A d_R^t)$	$[O_{ud}^{S8,RR}]_{prst}$	$(\bar{u}_L^p T^A u_R^r)(\bar{d}_L^s T^A d_R^t)$
$[O_{du}^{V1,LR}]_{prst}$	$(\bar{d}_L^p \gamma_\mu d_L^r)(\bar{u}_R^s \gamma^\mu u_R^t)$	$[O_{uddu}^{S1,RR}]_{prst}$	$(\bar{u}_L^p d_R^r)(\bar{d}_L^s u_R^t)$
$[O_{du}^{V8,LR}]_{prst}$	$(\bar{d}_L^p \gamma_\mu T^A d_L^r)(\bar{u}_R^s \gamma^\mu T^A u_R^t)$	$[O_{uddu}^{S8,RR}]_{prst}$	$(\bar{u}_L^p T^A d_R^r)(\bar{d}_L^s T^A u_R^t)$
$[O_{uddu}^{V1,LR}]_{prst}$	$(\bar{u}_L^p \gamma_\mu d_L^r)(\bar{d}_R^s \gamma^\mu u_R^t)$ + h.c.		
$[O_{uddu}^{V8,LR}]_{prst}$	$(\bar{u}_L^p \gamma_\mu T^A d_L^r)(\bar{d}_R^s \gamma^\mu T^A u_R^t)$ + h.c.		

Table 1: Non-leptonic $\Delta F = 1$ operators (baryon and lepton number conserving) in the JMS basis [2]. Note that $Q_{uddu}^{V1,LR}$ and $Q_{uddu}^{V8,LR}$ have Hermitian conjugates. The same holds for the operators $(\overline{L}R)(\overline{L}R)$. This choice of basis eliminates all operators with Dirac structures $\sigma^{\mu\nu}$. The class of operators $(\overline{L}R)(\overline{R}L)$ + h.c. does not contain non-leptonic operators, but only semi-leptonic ones.

- E. E. Jenkins, A. V. Manohar, and P. Stoffer, Low-Energy Effective Field Theory below the Electroweak Scale: Operators and Matching, JHEP 03 (2018) 016, [arXiv:1709.04486]

Shifts for the BMU operators

Shifts due to Penguin insertions

$$Q_1 = \widetilde{Q}_1, \qquad Q_2 = \widetilde{Q}_2 + \frac{1}{3}\frac{\alpha_s}{4\pi}P,$$

$$P = Q_4 + Q_6 - \frac{1}{3}(Q_3 + Q_5).$$

$$Q_{11} = \widetilde{Q}_{11} + \frac{2}{3}\frac{\alpha_s}{4\pi}P, \qquad Q_k = \widetilde{Q}_k, \quad k = 12 - 18\,.$$

$$Q_3 = \widetilde{Q}_3 + \frac{2}{3}\frac{\alpha_s}{4\pi}P, \qquad Q_4 = \widetilde{Q}_4 - \frac{N_f}{3}\frac{\alpha_s}{4\pi}P, \qquad Q_5 = \widetilde{Q}_5, \qquad Q_6 = \widetilde{Q}_6\,,$$

$$Q_7 = \widetilde{Q}_7, \qquad Q_8 = \widetilde{Q}_8, \qquad Q_9 = \widetilde{Q}_9 - \frac{1}{3}\frac{\alpha_s}{4\pi}P, \qquad Q_{10} = \widetilde{Q}_{10} - \frac{1}{3}(N_u - \frac{N_d}{2})\frac{\alpha_s}{4\pi}P.$$

Shifts for the BMU operators

Shifts due to current-current insertions

$$Q_1^{\mathrm{SRR},Q} = \widetilde{Q}_1^{\mathrm{SRR,Q}} + \frac{\alpha_s}{4\pi} \sum_{k=1,2,3,4} a_k Q_k^{\mathrm{SRR,Q}}$$

$$Q_2^{\mathrm{SRR},Q} = \widetilde{Q}_2^{\mathrm{SRR,Q}} + \frac{\alpha_s}{4\pi} \sum_{k=1,2,3,4} b_k Q_k^{\mathrm{SRR,Q}}$$

$$Q_3^{\mathrm{SRR},Q} = \widetilde{Q}_3^{\mathrm{SRR,Q}} + \frac{\alpha_s}{4\pi} \sum_{k=1,2,3,4} c_k Q_k^{\mathrm{SRR,Q}}$$

$$Q_4^{\mathrm{SRR},Q} = \widetilde{Q}_4^{\mathrm{SRR,Q}} + \frac{\alpha_s}{4\pi} \sum_{k=1,2,3,4} d_k Q_k^{\mathrm{SRR,Q}}$$

$$Q_1^{\mathrm{SRR},D} = \widetilde{Q}_1^{\mathrm{SRR,D}} + \frac{\alpha_s}{4\pi} \sum_{k=2,4} A_k Q_k^{\mathrm{SRR,D}},$$

$$Q_2^{\mathrm{SRR},D} = \widetilde{Q}_2^{\mathrm{SRR,D}} + \frac{\alpha_s}{4\pi} \sum_{k=2,4} B_k Q_k^{\mathrm{SRR,D}},$$

$$Q_3^{\mathrm{SRR},D} = \widetilde{Q}_3^{\mathrm{SRR,D}} + \frac{\alpha_s}{4\pi} \sum_{k=2,4} C_k Q_k^{\mathrm{SRR,D}},$$

$$Q_4^{\mathrm{SRR},D} = \widetilde{Q}_4^{\mathrm{SRR,D}} + \frac{\alpha_s}{4\pi} \sum_{k=2,4} D_k Q_k^{\mathrm{SRR,D}}$$

EV contributions for the VLL sector

Tree-level basis change relations

$$[O_{ud}^{V1,LL}]_{11ji} \stackrel{\mathcal{F}}{=} Q_1 + E_1^{\text{VLL}} \,,$$

$$[O_{ud}^{V8,LL}]_{11ji} \stackrel{\mathcal{F}}{=} -\frac{1}{6}Q_1 + \frac{1}{2}Q_2 + E_2^{\text{VLL}} \,,$$

$$[O_{ud}^{V1,LL}]_{22ji} \stackrel{\mathcal{F}}{=} -Q_1 + \frac{1}{3}Q_3 + \frac{2}{3}Q_9 + E_3^{\text{VLL}} \,,$$

$$[O_{ud}^{V8,LL}]_{22ji} \stackrel{\mathcal{F}}{=} \frac{1}{6}Q_1 - \frac{1}{2}Q_2 - \frac{1}{18}Q_3 + \frac{1}{6}Q_4 - \frac{1}{9}Q_9 + \frac{1}{3}Q_{10} + E_4^{\text{VLL}} \,,$$

$$[O_{dd}^{V,LL}]_{jikk} = \frac{2}{3}Q_3 - \frac{2}{3}Q_9 - Q_{11} \,,$$

$$[O_{dd}^{V,LL}]_{jkki} \stackrel{\mathcal{F}}{=} \frac{2}{3}Q_4 - \frac{2}{3}Q_{10} - Q_{11} + E_5^{\text{VLL}} \,,$$

$$[O_{dd}^{V,LL}]_{jiii} = \frac{1}{2}Q_{11} + \frac{1}{2}Q_{14} \,,$$

$$[O_{dd}^{V,LL}]_{jijj} = \frac{1}{2}Q_{11} - \frac{1}{2}Q_{14} \,.$$

$$E_1^{\rm VLL} = [O_{ud}^{V1,LL}]_{11ji} - Q_1 = \widetilde{Q}_1 - Q_1 = 0\,,$$

$$E_3^{\rm VLL} = [O_{ud}^{V1,LL}]_{22ji} - \left(-Q_1 + \frac{1}{3}Q_3 + \frac{2}{3}Q_9\right) = \widetilde{Q}_1 - Q_1 = 0.$$

Fierz: Yes
Evanascent: No

$$\begin{aligned}
E_2^{\rm VLL} &= [O_{ud}^{V8,LL}]_{11ji} - \left(-\frac{1}{6}Q_1 + \frac{1}{2}Q_2\right) \\
&= \frac{1}{6}(Q_1 - \widetilde{Q}_1) - \frac{1}{2}(Q_2 - \widetilde{Q}_2) = -\frac{1}{6}\frac{\alpha_s}{4\pi}P\,, \\
E_4^{\rm VLL} &= = [O_{ud}^{V8,LL}]_{22ji} - \left(\frac{1}{6}Q_1 - \frac{1}{2}Q_2 - \frac{1}{18}Q_3 + \frac{1}{6}Q_4 - \frac{1}{9}Q_9 + \frac{1}{3}Q_{10}\right) \\
&= \frac{1}{6}(\widetilde{Q}_1 - Q_1) + \frac{1}{2}(Q_2 - \widetilde{Q}_2) = \frac{1}{6}\frac{\alpha_s}{4\pi}P\,, \\
E_5^{\rm VLL} &= [O_{dd}^{V,LL}]_{jkki} - \left(\frac{2}{3}Q_4 - \frac{2}{3}Q_{10} - Q_{11}\right) \\
&= \frac{2}{3}(\widetilde{Q}_4 - Q_4) - \frac{2}{3}(\widetilde{Q}_{10} - Q_{10}) \\
&= \frac{2N_f + N_d - 2N_u}{9}\frac{\alpha_s}{4\pi}P.
\end{aligned}$$

Fierz: Yes
Evanascent: Yes

Summary

In an EFT, it might be useful to work simultaneously with more than one operator basis.

JMS basis for WET is useful for the matching to SMEFT, however, for the running below the EW scale at the NLO level, the BMU is more convenient.

We have presented a simple procedure to perform the transformations of the bases at the 1-loop level. Formal techniques are available in the literature :

A. J. Buras and P. H. Weisz, QCD Nonleading Corrections to Weak Decays in Dimensional Regularization and 't Hooft-Veltman Schemes, Nucl. Phys. B333 (1990) 66–99
M. J. Dugan and B. Grinstein, On the vanishing of evanescent operators, Phys. Lett. B256 (1991) 239–244.

See also:

S. Herrlich and U. Nierste, Evanescent operators, scheme dependences and double insertions, Nucl. Phys. B455 (1995) 39–58, [hep-ph/9412375]

A. J. Buras, M. Misiak, and J. Urban, Two loop QCD anomalous dimensions of flavor changing four quark operators within and beyond the standard model, Nucl. Phys. B586 (2000) 397–426, [hep-ph/0005183].

J. Aebischer, C. Bobeth, A. J. Buras, J. Kumar, and M. Misiak, General non-leptonic $\Delta F = 1$ WET at the NLO in QCD, JHEP 11 (2021) 227, [arXiv:2107.10262].

Thanks for your attention!