Reevaluating Uncertainties in $\bar{B} \to X_s\gamma$ Decay

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Based on
A.G and Gil Paz "Reevaluating Uncertainties in $\bar{B} \to X_s\gamma$"
JHEP 11 (2019) 141

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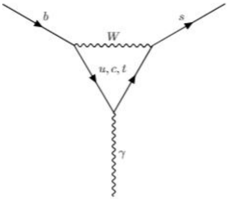
  - Can receive contributions from SM extensions.



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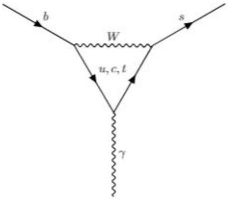

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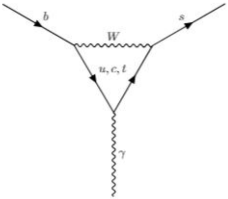

Figure: $b \to s\gamma$ flavor changing neural current (FCNC) in SM

  - SM extensions modify the $C_{7\gamma}$ Wilson coefficient
  - CP violation in $\bar{B} \to X_s\gamma$ can be enhanced by new physics

Photon production

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- Photon can be produced directly:

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- Also, gluon or quark pair can convert to photon

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- At leading power: Only $Q_{7\gamma} - Q_{7\gamma}$ contributes to decay rate
- At $1/m_b$: Γ get $Q_1 - Q_{7\gamma}$, $Q_{8g} - Q_{8g}$ and $Q_{7\gamma} - Q_{8g}$ contributions

Decay rate

- World average for experimental value:

$$\mathcal{B}\left(B \to X_s\gamma\right)\left(E_\gamma > 1.6\ \text{GeV}\right) = (3.32 \pm 0.15) \times 10^{-4}$$

[Y. Amhis et. al. EPJC 77, 895 (2017)]

- NNLO prediction

$$\Gamma\left(\overline{B} \to X_q\gamma\right) = \underbrace{\Gamma\left(b \to X_q^p\gamma\right)}_{\text{Perturbatively calculable}} + \underbrace{\delta\Gamma_{\text{nonp}}}_{\mathcal{O}\left(\frac{\Lambda_{QCD}}{m_b}\right)}$$

- SM prediction (2015) [Misiak et. al. PRL 114, 221801 (2015)]

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- $\delta\Gamma_{\rm nonp} \equiv$ Non-perturbative contribution
  - The largest contribution to the error 5% from $\mathcal{O}(\frac{\Lambda_{\rm QCD}}{m_b})$

Order $1/m_b$ power corrections to $\Gamma(\bar{B} \to X_s\gamma)$

- Non-perturbative effects arise from **Resolved Photon Contributions**

$$\Delta\Gamma \sim \underbrace{\bar{J}}_{\text{Perturbatively calculable}} \otimes \underbrace{h}_{\text{Non perturbative}}$$

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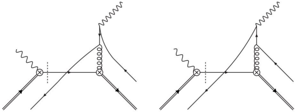

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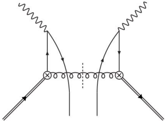
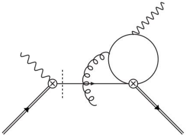


- $Q_1 - Q_{7\gamma}$



Contribution to the non-perturbative error

- 2010 estimates for non-perturbative contribution to error
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  - New Belle result for $Q_{7\gamma} - Q_{8g}$ contribution $\sim 2\%$

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- Now $Q_1^c - Q_{7\gamma}$ is the largest contribution to the error!

  Can we reduce it?

$Q_1^c - Q_{7\gamma}$ contribution

- The contribution to the error from $Q_1^c - Q_{7\gamma}$ is given by

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where

$$\Lambda_{17} = e_c \,\mathrm{Re} \int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1} \left[1 - \underbrace{F\left(\frac{m_c^2 - i\varepsilon}{m_b \omega_1}\right)}_{\text{perturbative}} + \frac{m_b \omega_1}{12 m_c^2} \right] \underbrace{h_{17}\left(\omega_1\right)}_{\text{non-perturbative}}$$

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- Need a new model for h_{17} to **reduce the error**
  - New information on moments of h_{17}: constrain **new model**
  - What can we learn from moments?

Moments of h_{17}

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$$= \int \frac{dr}{2\pi} e^{-i\omega_1 r} \frac{\langle \bar{B}|(\bar{h}S_{\bar{n}})(0)\not{n}(1+\gamma_5)i\gamma^{\perp}\bar{n}_{\beta}(S_{\bar{n}}gG^{\alpha\beta}S_{\bar{n}})(r\bar{n})(S^{\dagger}_{\bar{n}}h)(0)|\bar{B}\rangle}{2M_B}$$

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$$S_n(x) = \mathbf{P} \exp\left(ig \int_{-\infty}^{0} dun \cdot A_s(x + un) \right)$$

  -

$$n^{\mu} \equiv (1,0,0,1) \text{ and } \bar{n}^{\mu} \equiv (1,0,0,-1)$$

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- New result Moments over ω_1

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Moments of the g_{17}

- Procedure to obtain these HQET matrix elements derived in
  [A. Gunawardana and G. Paz, JHEP 07(2017)137 [arXiv:1702.08904]]

$$\langle h_{17} \rangle = 2\lambda_2 = 2\mu_G^2/3$$

$$\left\langle \omega_1^2 h_{17} \right\rangle = \frac{2}{15}\left(5m_5 + 3m_6 - 2m_9\right) \text{ New result}$$

- m_i were extracted from data for the first time in 2016
  [P. Gambino, K. J Healey, S. Turczyk PLB 763, 60 (2016)]

$$\mu_G^2 = 0.355 \pm 0.060 \text{ GeV}^2 \qquad m_5 = 0.072 \pm 0.045 \text{ GeV}^4$$

$$m_6 = 0.060 \pm 0.164 \text{ GeV}^4 \qquad m_9 = -0.280 \pm 0.352 \text{ GeV}^4$$

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- Expect in future
  - Further improvements on HQET matrix elements
  - Belle II or LQCD data $\Rightarrow$ Better constrains on moments

Applications

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- Our model: $\boxed{h_{17}(\omega_1) = \sum_n a_{2n} H_{2n}\left(\frac{\omega_1}{\sqrt{2}\sigma}\right) e^{\frac{-\omega_1^2}{2\sigma}}}$
  - where

$$a_0 = \frac{\langle \omega_1^0 h_{17} \rangle}{\sqrt{2\pi}|\sigma|}, \quad a_2 = \frac{\langle \omega_1^2 h_{17} \rangle - \sigma^2 \langle \omega_1^0 h_{17} \rangle}{4\sqrt{2\pi}|\sigma|^3}, \quad a_4 = \cdots$$

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- $|h_{17}| < 1$ GeV and no peaks beyond $\omega_1 = 1$ GeV

New model vs 2010 model

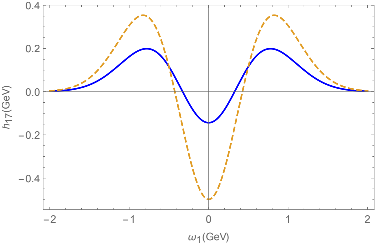
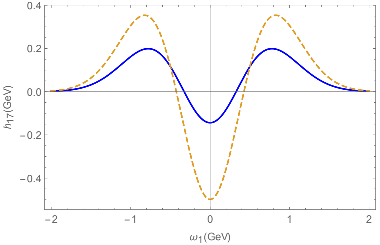


Figure: 2019 model vs 2010 model for h_{17}

New model vs 2010 model



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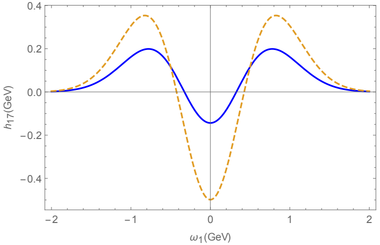

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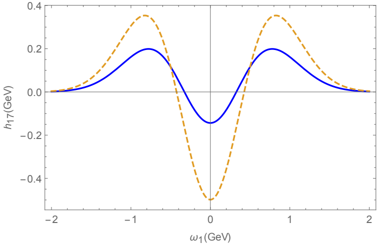

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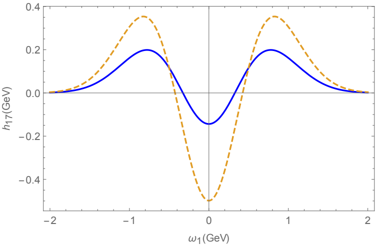

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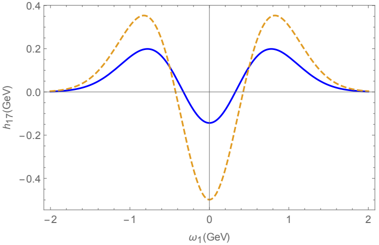

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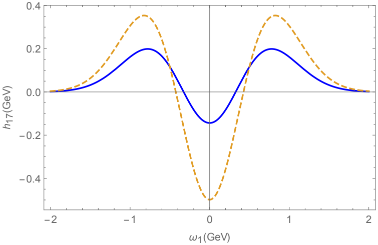

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- Consider also unknown higher moments, up to 6 Hermite polynomials

Phenomenological estimates : nonperturbative uncertainty

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- New results: Our estimates for nonperturbative parameter Λ_{17}

$$\boxed{-24\,\text{MeV} < \Lambda_{17} < +5\,\text{MeV}}$$

  - Compare with the 2010 estimate $-60\,\text{MeV} < \Lambda_{17} < 25\,\text{MeV}$

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- Obtain new estimate for $Q_1 - Q_{7\gamma}$ contribution:

$$\frac{C_1}{C_{7\gamma}}\frac{\Lambda_{17}}{m_b}$$

At $\mu = 1.5$ GeV : $C_1(\mu) = 1.257,\ C_{7\gamma}(\mu) = -0.407$ and $m_b = 4.58$ GeV

- New result: $Q_1 - Q_{7\gamma}$ contribution to nonperturbative uncertainty

$$Q_1 - Q_{7\gamma} \in \boxed{[-0.3, +1.6]\%}$$

  - Compare with 2010 estimate $\in [-1.7, +4.0]\%$

Phenomenological estimates: nonperturbative uncertainty

- New Belle data: $Q_{7\gamma} - Q_{8g}$ nonperturbative uncertainty

$$Q_{7\gamma} - Q_{8g} \in [-1.4, +2]\%$$

[Watanuki et. al. PRD 99, 032012 (2019)]

[Gunawardana and Paz, JHEP 11(2019)141 [arXiv:1908.02812]]

Operator pair	2010 estimates	2019 estimates	
$Q_1 - Q_{7\gamma}$	$[-1.7, +4.0]\%$	$[-0.3, +1.6]\%$	Gunawardana and Paz (20
$Q_{7\gamma} - Q_{8g}$	$[-4.4, +5.6]\%$	$[-1.4, +2]\%$	Watanuki et al. (2019
$Q_{8g} - Q_{8g}$	$[-0.3, +1.9]\%$	$[-0.3, +1.9]\%$	Benzke et al. (2010)
Total uncertainty	$[-6.4, +11.5]\%$	$[-2.0, +5.5]\%$	Gunawardana and Paz (20

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- Total uncertainty is reduced by **half!**
- Total uncertainty obtained by scanning over each uncertainty

CP asymmetry: phenomenological estimates

- Resolved photon contribution to CP asymmetry

$$\mathcal{A}^{\mathrm{SM}}_{X_s\gamma} = \left(1.15 \times \frac{\tilde{\Lambda}^u_{17} - \tilde{\Lambda}^c_{17}}{300\mathrm{MeV}} + 0.71\right)\%$$

$$\tilde{\Lambda}^u_{17} = \frac{2}{3} h_{17}(0)$$

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- Comparison between 2010 and new values

Operator pair	2010 estimates	2019 estimates
$\tilde{\Lambda}^u_{17}$	$[-330\,\mathrm{MeV}, 525\,\mathrm{MeV}]$	$[-660\,\mathrm{MeV}, 660\,\mathrm{MeV}]$
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Table: Comparison : 2010 vs 2019

- Direct CP Asymmetry experimental estimate:

$$A_{CP} = (1.5 \pm 2.0)\,\%$$

[Amhis et. al. EPJC 77, 895 (2017)]

Conclusion

- $\bar{B} \to X_s\gamma$ is a important New Physics probe
- Non perturbative error of the decay rate is 5%
- $Q_1^c - Q_{7\gamma}$ is the largest contribution to the error
- Better estimates for $Q_1^c - Q_{7\gamma}$ obtained from moments of h_{17}
- New estimates for CP asymmetry

Work in progress

- Applications to Inclusive $b \to u$ transitions
  - Systematic parameterization of shape functions using new moment information (work in progress)
  - Update to the BLNP method