EDMs: a theoretical review and some recent work

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Flambaum, MP, Ritz, Stadnik, 1912.13129 (PRD2020)
Y. Ema, T. Gao, MP 2108.05398 (PRL2021)
Y. Ema, T. Gao, MP 2202.10524 (PRL, subm.)
Y. Ema, T. Gao, MP 2205.11532 (today!)

Plan

1. Intro: why EDMs
2. Paramagnetic EDMs from Hadronic CP violation
3. Independent constraints on Theta$_{QCD}$, color EDM from semi-leptonic EDM-like operators (C_S).
4. CKM CP-violation → C_S via the "double-penguin" diagram.
5. New indirect constraints on EDMs of muons, c- b- quarks.
6. *Conclusions*

Purcell and Ramsey (1949) ("How do we know that strong interactions conserve parity?" $\longrightarrow |d_n| < 3 \times 10^{-18} e\text{cm}$.)

$$H = -\mu \mathbf{B} \cdot \frac{\mathbf{S}}{S} - d\mathbf{E} \cdot \frac{\mathbf{S}}{S}$$

$d \neq 0$ means that both P and T are broken. If CPT holds then CP is broken as well.

CPT is based on locality, Lorentz invariance and spin-statistics = very safe assumption.

search for EDM = search for CP violation, if CPT holds

Relativistic generalization

$$H_{\mathrm{T,P-odd}} = -d\mathbf{E} \cdot \frac{\mathbf{S}}{S} \rightarrow \mathcal{L}_{\mathrm{CP-odd}} = -d\frac{i}{2}\overline{\psi}\sigma^{\mu\nu}\gamma_5\psi F_{\mu\nu},$$

corresponds to dimension five effective operator and naively suggests $1/M_{\text{new physics}}$ scaling. Due to $SU(2) \times U(1)$ invariance, however, it scales as m_f/M^2.

Current limits translate to multi-TeV sensitivity to M.

Current Experimental Limits

"paramagnetic EDM", Berkeley experiment

$$|d_{\rm Tl}| < 9 \times 10^{-25} e\,{\rm cm}$$ Interpreted $|d_e| < 1.6 \times 10^{-27}$

"diamagnetic EDM", U of Washington experiment

$$|d_{\rm Hg}| < \cancel{2 \times 10^{-28} e\,{\rm cm}}$$

factor of 7 improvement in 2009! And another factor of 4 in 2016

$$|d_{\rm Hg}| < \cancel{3 \times 10^{-29} e\,{\rm cm}}$$ $7.4 \times 10^{-30}\, e$ cm

neutron EDM, ILL experiment

$$|d_n| < \cancel{3 \times 10^{-26} e\,{\rm cm}}$$ $1.8 \times 10^{-26}\, e$ cm

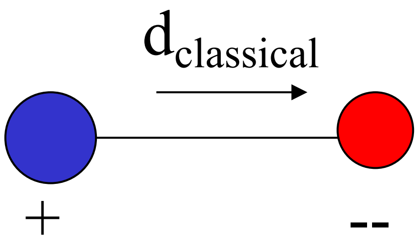
Notice that Thallium EDM is usually quoted as $d_e < 1.6\ 10^{-27}$ e cm bound. It was modestly improved by YbF results. **$|d_e| < 1.1 \times 10^{-29}$**

2013 ThO result by Harvard-Yale collaboration: $\cancel{|d_e| < 8.7 \times 10^{-29}}$

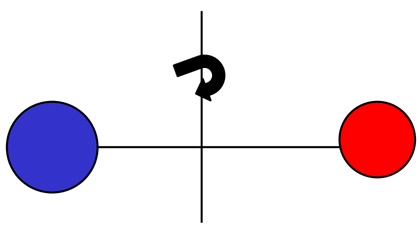
"Confirmed" using different techniques at JILA, $|d_e| < 1.3 \times 10^{-28}$

A small comment on classical EDMs

- Fundamental EDMs are connected to spin, classical EDMs are not.
- A diatomic molecule (like ThO) will have a classical EDM.



- However, in a quantum state with fixed angular momentum classical EDM average to zero, exactly. States with +M and –M projection of angular momentum remain degenerate (at B=0).

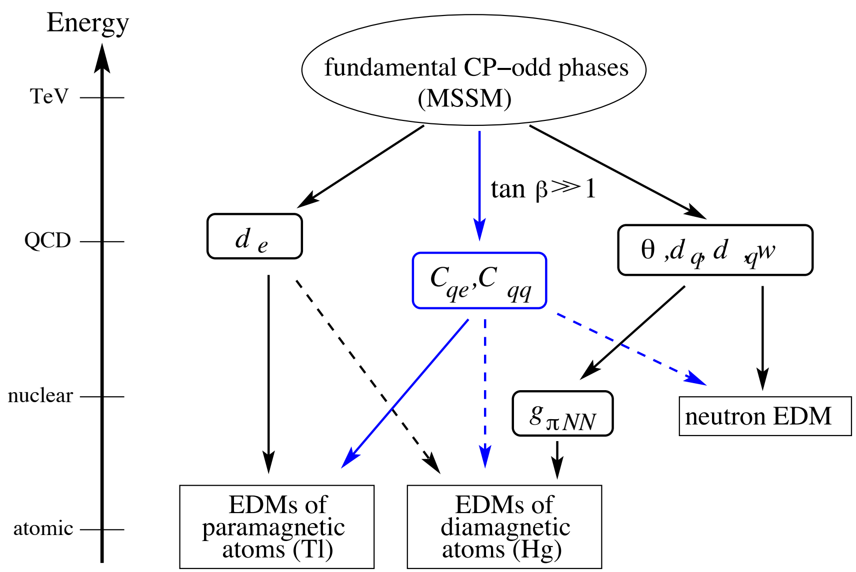


$$< d_{classical} > = 0$$

- If there is fundamental CP-violation, the electric field will induce splitting between +M and –M states, e.g. Zeeman effect but with electric field. EDM experiments are looking for E coupling to spin

BSM physics and EDMs

$$\mathcal{L}_{eff}^{1\mathrm{GeV}} = \frac{g_s^2}{32\pi^2}\,\theta_{QCD} G^a_{\mu\nu}\widetilde{G}^{\mu\nu,a}$$
$$-\frac{i}{2}\sum_{i=e,u,d,s} d_i\,\overline{\psi}_i(F\sigma)\gamma_5\psi_i - \frac{i}{2}\sum_{i=u,d,s}\widetilde{d}_i\,\overline{\psi}_i g_s(G\sigma)\gamma_5\psi_i$$
$$+\frac{1}{3}w\; f^{abc}G^a_{\mu\nu}\widetilde{G}^{\nu\beta,b}G_\beta{}^{\mu,c} + \sum_{i,j=e,d,s,b} C_{ij}\;(\bar{\psi}_i\psi_i)(\bar{\psi}_j i\gamma_5\psi_j) + \cdots$$



- One needs hadronic, nuclear, atomic matrix elements to connect Wilson coefficients to observables

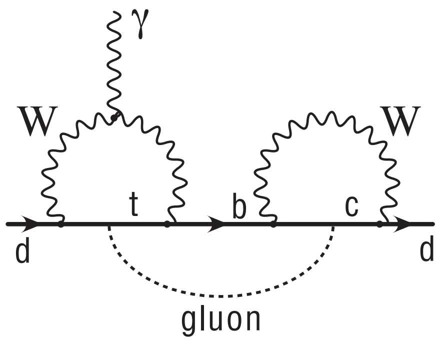
- Extremely high scales [10-100 TeV] can be probed if new physics generating EDMs violates CP maximally.

Two sources of CP-violation in SM

- Theta term of QCD: too large EDMs if theta is arbitrary → new naturalness problem because of EDMs. ($d_n \sim \theta\, m_q/m_n^2$, $\theta < 10^{-10}$)

- Cabibbo-Kobayashi-Maskawa matrix and nearly maximal CP phase → still EDMs are too small to be observable in the next round of EDM experiments.

EDMs from SM sources: CKM



CKM phase generates tiny EDMs:

$$d_d \sim \mathrm{Im}(V_{tb}V_{td}^*V_{cd}V_{cb}^*)\alpha_s m_d G_F^2 m_c^2 \times \text{loop suppression}$$
$$< 10^{-33} e\mathrm{cm}$$

- Quark EDMs identically vanish at 1 and 2 loop levels, $EW^2=0$

(Shabalin, 1981).

- 3-loop EDMs, EW^2QCD^1 are calculated by Khriplovich; Czarnecki, Krause.
- d_e vanishes at EW^3 level (Khriplovich, MP, 1991) < 10^{-38} e cm
- Long distance effects give neutron EDM ~ 10^{-32} e cm; uncertain.

"Paramagnetic" EDMs:

- Paramagnetic EDM (EDM carried by electron spin) can be induced not only by a purely leptonic operator

$$d_e \times \frac{-i}{2}\,\overline{\psi}\sigma_{\mu\nu}\gamma_5 F_{\mu\nu}\psi$$

but by semileptonic operators as well:

$$C_S \times \frac{G_F}{\sqrt{2}}\,\overline{N}N\,\overline{\psi}i\gamma_5\psi$$

- Only a linear combination is limited in any single experiment. ThO 2018 ACME result is:

$|d_e| < 1.1\times10^{-29}$ e cm at $C_S = 0$

$|C_S^{\text{singlet}}| < 7.3\times10^{-10}$ at $d_e = 0$

$$d_e^{\text{equiv}} = d_e + C_S \times 1.5 \times 10^{-20}\,e\,\text{cm}$$

What is sensitivity of paramagnetic EDMs (aka d_e) to hadronic CP violation? Theta term, EDMs of quarks, color EDMs etc? Important given recent progress.

Hadronic CP violation contributing to C_S

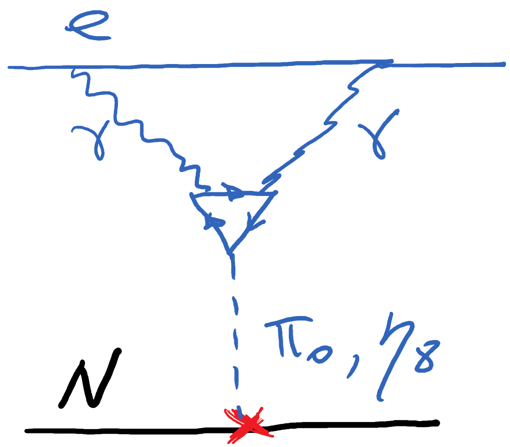
PM

$(\bar{e} i\gamma_5 e)\, \bar{N}N$ operators

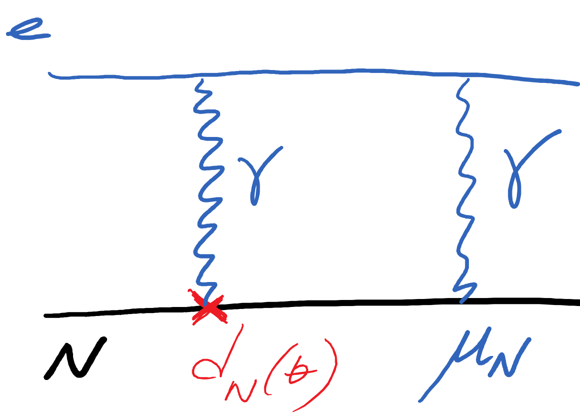
m_q counting:

$\theta\, m_q / m_\pi^{\,2}$

$\sim O(m_q^{\,0})$



Almost complete cancellation of π_0 and η_8 contributions



+ cross diagrams

m_q counting:

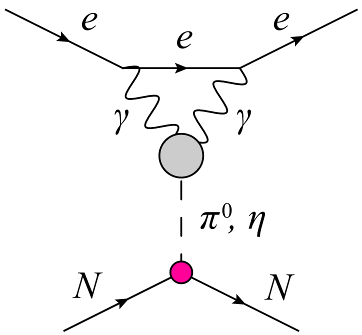
$\sim O(m_q \log{(m_q)})$

Theta terms induces C_S via two-photon exchange

- Th used by ACME collaboration is a spin-less nucleus.
- ThO is mostly sensitive to CP violation in the lepton sector. If CP is broken in the strong interaction sector, *two photon exchange* can communicate it to the electron shells.
- Cutting across the two photons, the intermediate result can be phrased via *CP-odd nuclear polarizability*, $\mathbf{EB}\,\delta(\mathbf{r})$, where E and B are created by an electron.
- Good scale separation is possible, $m_p >> p_F,\ m_\pi >> m_e \sim Z\alpha\, m_e$
- Nuclear uncertainties could be under control if the result is driven by "bulk" [as opposed to valence] nucleons.

LO chiral contribution:

- T-channel pion exchange gives



$$\mathcal{L} = \theta \times \frac{1}{m_\pi^2} \times 0.017 \times 3.5 \times 10^{-7} (\bar{e} i \gamma_5 e)(\bar{n}n - \bar{p}p)$$

$$= (\bar{e} i \gamma_5 e)(\bar{n}n - \bar{p}p) \times \frac{3.2 \times 10^{-13} \theta}{\mathrm{MeV}^2}.$$

implying $|\theta| < 8.4 \times 10^{-8}$ sensitivity. However, adding exchange of η_8,

$$1 \to 1 - \frac{1}{3} \frac{f_\pi^2 m_\pi^2}{f_\eta^2 m_\eta^2} \times \frac{m_d - m_u}{m_d + m_u} \times \frac{A \times \sigma_N}{\frac{m_d - m_u}{2} \langle p | \bar{u}u - \bar{d}d | p \rangle \times (N - Z)}$$

$$1 \to 1 - 0.88 \simeq 0.12.$$

The effect can completely cancel within error bars on nucleon sigma term σ_N.

Photon box diagrams:

- Diagrams are IR divergent but regularized by Fermi momentum in the Fermi gas picture of a nucleus (intermediate N is above Fermi surface).

$$\mathcal{L} = \bar{e} i\gamma_5 e \bar{N} N \times \frac{2m_e \times 4\alpha \times \overline{d\mu} \times 6.2}{\pi p_F} = \bar{e} i\gamma_5 e \bar{N} N \times 2.4 \times 10^{-4} \times \overline{d\mu}$$

$$\overline{d\mu} = \frac{Z}{A}\mu_p d_p + \frac{A-Z}{A}\mu_n d_n = \frac{e}{2m_p} \times (1.08 d_p - 1.16 d_n)$$

- Nucleon EDM (theta) is very much a triplet, $d_p \simeq -d_n \simeq 1.6 \times 10^{-3} e\mathrm{fm}\theta$

Full answer including chiral NLO. (accidental cancellation of π^0 and η)

$$\boxed{C_{SP}(\bar{\theta}) \approx \left[0.1_{\mathrm{LO}} + 1.0_{\mathrm{NLO}} + 1.7_{(\mu d)}\right] \times 10^{-2} \bar{\theta} \approx 0.03\,\bar{\theta}}$$

Limit on theta term from ThO (electron EDM) experiment:

$$\boxed{|\bar{\theta}|_{\mathrm{ThO}} \lesssim 3 \times 10^{-8}}$$

Constraints on other hadronic Wilson coeff.

- Proton EDM, other CP-violating inputs can be limited:

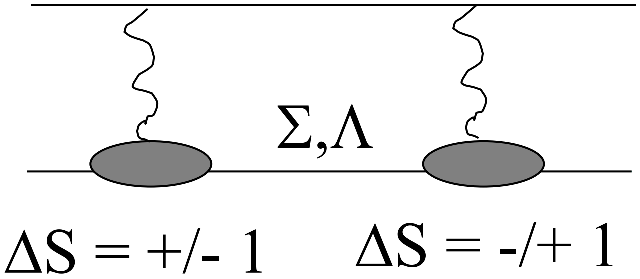
System	$\lvert d_p\rvert$ $(e \cdot \mathrm{cm})$	$\lvert\bar{g}^{(1)}_{\pi NN}\rvert$	$\lvert\tilde{d}_u - \tilde{d}_d\rvert$ (cm)	$\lvert\bar{\theta}\rvert$
ThO	$\mathbf{2 \times 10^{-23}}$	$\mathbf{4 \times 10^{-10}}$	$\mathbf{2 \times 10^{-24}}$	$\mathbf{3 \times 10^{-8}}$
n	—	1.1×10^{-10}	5×10^{-25}	2.0×10^{-10}
Hg	2.0×10^{-25}	1×10^{-12} [a]	5×10^{-27} [a]	1.5×10^{-10}
Xe	3.2×10^{-22}	6.7×10^{-8}	3×10^{-22}	3.2×10^{-6}

- Current constraints on Theta$_{\mathrm{QCD}}$ trail d_n sensitivity by two orders of magnitude

- Given fast progress of recent years with ``paramagnetic'' EDMs, a further increase by ~ 100 will provide comparable sensitivity.

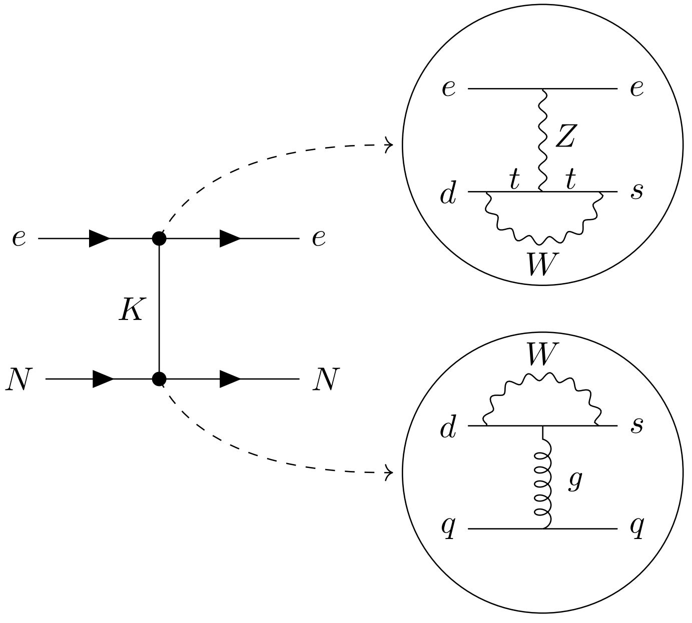
CKM CP-violation and paramagnetic EDMs

- Several groups attempted to calculate d_e (MP, Khriplovich; …)
- The result is small ~ few 10^{-40} e cm. (Yamaguchi, Yamanaka)
- Semileptonic (C_S) operator is more important. MP and Ritz (2012) estimated two-photon mediated EW^2EM^2 effects and found that CS is induced at the level equivalent to ~ 10^{-38} e cm



It turns out that there are much larger contributions at EW^3 order

Union of two-penguins: EW^3 order



- The induced semileptonic operator is calculable in chiral perturbation theory (in m_s expansion)
- The result is large, d_e(equiv) = + 1.0 10^{-35} e cm
- Same EW penguin that is responsible for $B_s \rightarrow \mu\mu$, Re $K_L \rightarrow \mu\mu$

Semileptonic Electroweak Penguin

- The upper part: EW penguin $\mathcal{L}_{\text{EWP}} = \mathcal{P}_{\text{EW}} \times \bar{e}\gamma_\mu\gamma_5 e \times \bar{s}\gamma^\mu(1-\gamma_5)d + (h.c.)$

$$\mathcal{L}_{Uee} = -\frac{if_0^2}{2}\mathcal{P}_{\text{EW}} \times \bar{e}\gamma_\mu\gamma_5 e \times \text{Tr}\left[h^\dagger\left(\partial^\mu U\right)U^\dagger\right] + (h.c.),$$

In the leading order, the dominant diagram is K_S exchange.

$$\mathcal{L}_{Kee} = -2\sqrt{2}f_0 m_e \bar{e}i\gamma_5 e\left(K_S \times \text{Im}\mathcal{P}_{\text{EW}} + K_L \times \text{Re}\mathcal{P}_{\text{EW}}\right)$$

- Lower part: EW[1] B-B-M coupling is related by flavor SU(3) to the s-wave amplitudes of the non-leptonic hyperon decays. Theory fit to decay amplitudes is [surprisingly] good (~5-10%):

$$\mathcal{L}_{\text{SP}} = -a\text{Tr}(\bar{B}\{\xi^\dagger h\xi, B\}) - b\text{Tr}(\bar{B}[\xi^\dagger h\xi, B]) + (h.c.).$$

contains $2^{1/2} f_0^{-1}((b-a)\bar{p}p + 2b\bar{n}n)K_S$

LO kaon exchange result

- Using EW penguin and strong penguin below,

$$\mathcal{L}_{KNN} \simeq -\frac{\sqrt{2}\, G_F \times [m_{\pi^+}]^2 f_\pi}{|V_{ud}V_{us}| f_0} \times 2.84(0.7\bar{p}p + \bar{n}n)$$
$$\times \left(\mathrm{Re}(V_{ud}^* V_{us}) K_S + \mathrm{Im}(V_{ud}^* V_{us}) K_L\right).$$

We calculate C_S

$$C_S \simeq \mathcal{J} \times \frac{N + 0.7Z}{A} \times \frac{13[m_{\pi^+}]^2 f_\pi m_e G_F}{m_K^2} \times \frac{\alpha_{\mathrm{EM}} I(x_t)}{\pi \sin\theta_W^2}$$

$$\mathcal{J} = \mathrm{Im}(V_{ts}^* V_{td} V_{ud}^* V_{us}) \simeq 3.1 \times 10^{-5}$$

That has the following LO scaling

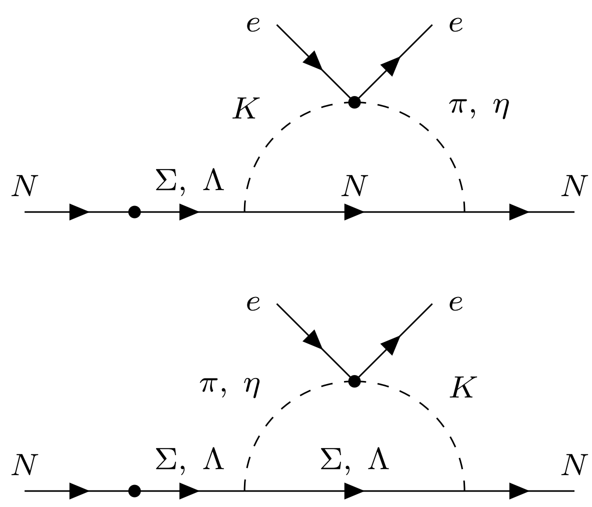
$$G_F C_S \propto \mathcal{J} G_F^3 m_t^2 m_e m_s^{-1} \Lambda_{\mathrm{hadr}}^2$$

Numerically, it is

$$C_S(\mathrm{LO}) \simeq 5 \times 10^{-16}.$$

NLO kaon-pion loop

- We calculate leading order corrections that have $(m_s)^{-1/2}$ scaling



- The loop itself is proportional to $\sim m_K$, but there is a baryonic pole that brings $1/m_s$.

The NLO brings positive contribution of ~ 30%.

$$\frac{C_{S,NLO}(p)}{C_{S,LO}(p)} = \frac{m_K^3(0.77D^2 + 2.7DF - 2.3F^2)}{24\pi f_0^2(m_{\Sigma^+} - m_p)}$$

$$\frac{C_{S,NLO}(n)}{C_{S,LO}(n)} = \frac{m_K^3}{24\pi f_0^2}\Bigg(\frac{(a/b+3)}{2\sqrt{6}(m_\Lambda - m_n)} \times(-0.44D^2 + 3.2DF + 1.3F^2) + \frac{a/b-1}{2\sqrt{2}(m_{\Sigma^0} - m_n)}\left(-0.53D^2 - 1.9DF + 1.6F^2\right)\Bigg)\,.$$

Final result

- Combining $(m_s)^{-1}$ and $(m_s)^{-1/2}$ effects, we get

$$\boxed{\begin{aligned} &C_S(\mathrm{LO}+\mathrm{NLO}) \simeq 6.9\times 10^{-16} \\ &\Longrightarrow\ d_e^{\mathrm{equiv}} \simeq 1.0\times 10^{-35}\, e\,\mathrm{cm}. \end{aligned}}$$

- The result EW^3 much larger than the EW^2EM^2 estimate by ~1000.

- Note that actually establishing the correct sign is tricky.

- The result is under "best possible" theoretical control, and can be improved on the lattice

$$\langle N|i(\bar{s}\gamma_\mu(1-\gamma_5)d-\bar{d}\gamma_\mu(1-\gamma_5)s)|N\rangle_{\mathrm{EW}^1} = \frac{f_S}{m_N}iq_\mu\bar{N}N+\frac{f_T}{m_N}q_\nu\bar{N}\sigma_{\mu\nu}\gamma_5 N.$$

EDMs of heavy flavors

- Among Wilson coefficients of different kind, EDMs of heavy flavours d_i are interesting. i = muon, tau, charm, bottom, top.

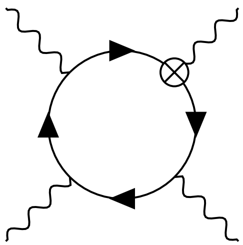
- Muon EDM is limited as a biproduct of BNL g-2 experiment. Can be significantly improved in dedicated beam experiments (PSI, Fermilab)

- There is a creative proposal to measure MDMs and limit EDMs of charmed baryons using thin fixed target and bent crystal technology just before the LHCb experiment (E. Bagli et al, 2017).

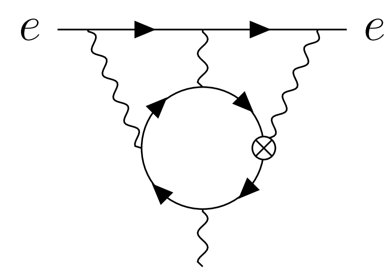
- Heavy flavors contribute to observable EDMs via loops. Top quark EDM is limited indirectly by electron EDM via a two-loop (top-Higgs-gamma) Barr-Zee diagrams. The result is stronger than the direct measurements at LHC.

Muon EDM inside a loop

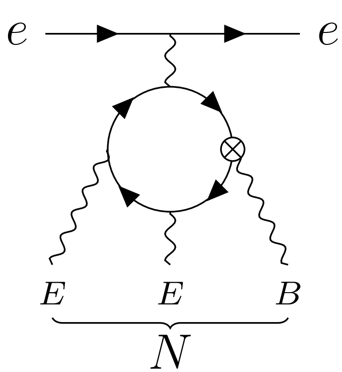
- Muon loop induces E^3B effects, and electron EDM at 3-loops.



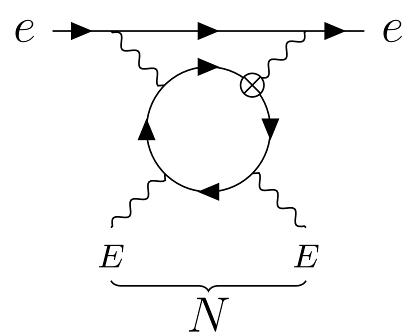
$$\mathcal{L} = -e^4(\tilde{F}_{\alpha\beta}F^{\alpha\beta})(F_{\gamma\delta}F^{\gamma\delta}) \times \frac{d_\mu/e}{96\pi^2 m_\mu^3}$$
$$= -\frac{d_\mu/e}{12\pi^2 m_\mu^3} e^4(\mathbf{E}\cdot\mathbf{B})(\mathbf{E}\cdot\mathbf{E} - \mathbf{B}\cdot\mathbf{B}),$$



Nuclear Schiff moment



Effective C_S operator



d_{Hg}

$d_e^{equiv}(ThO)$

New indirect constraints on muon EDM

- Owing to the fact that the electric field inside a large nucleus is not that small $eE \sim Z\,\alpha\, R_N^{-1} \sim 30$ MeV compared to m_μ, effects formally suppressed by higher power of m_μ win over three-loop electron EDM.

- New results:

Hg EDM experiment: $S_{^{199}\mathrm{Hg}}/e \simeq (d_\mu/e) \times 4.9 \times 10^{-7}\,\mathrm{fm}^2, \quad |d_\mu| < 6.4 \times 10^{-20}\,e\,\mathrm{cm}$

ThO EDM experiment: $d_e^{\mathrm{equiv}} \simeq 5.8 \times 10^{-10}\, d_\mu \implies \boxed{|d_\mu| < 1.9 \times 10^{-20}\,e\,\mathrm{cm}.}$

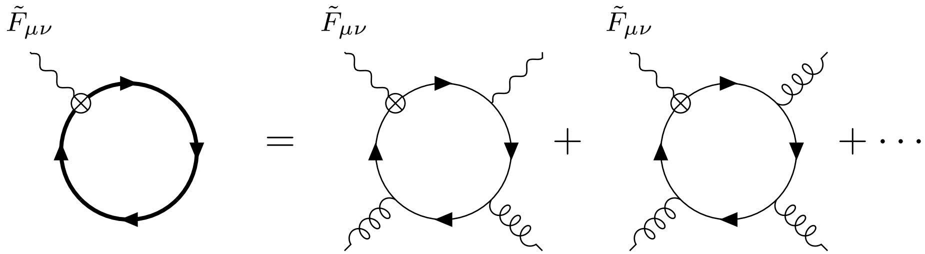
- Factor of 3 and 9 improvement over the BNL constraint, $|d_\mu| < 1.8 \times 10^{-19}$
- New benchmark for the muon beam EDM experiments.

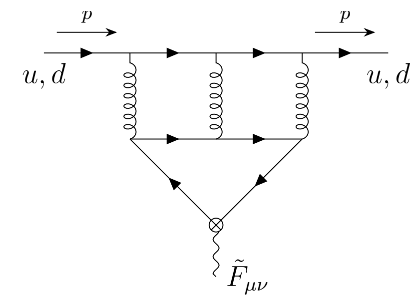
NB: 3-loop contributions calculated by Grozin et al. will be revised

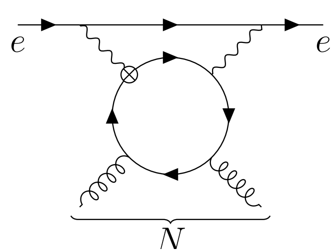
- Tau EDM is constrained by three-loop induced d_e.

Charm and bottom EDMs

Charm loop gives $(\gamma)^2(\text{gluon})^2$ and $(\gamma)^1(\text{gluon})^3$ effective operators

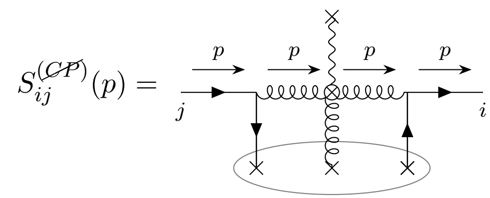






$$\langle N | \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} G^{a\mu\nu} | N \rangle = -\frac{m_N}{9} \bar{N} N,$$

$d_e^{\text{equiv}}(\text{ThO})$



Nonperturbative 3-gluon induced tensor charge

d_n , d_{Hg}

- All EDMs are induced by charm and bottom EDMs.

New indirect constraints on c-, b- quarks EDMs

- New results:

Neutron EDM experiment: $|d_c| < 6 \times 10^{-22}\, e\,\mathrm{cm}, \quad |d_b| < 2 \times 10^{-20}\, e\,\mathrm{cm},$

ThO EDM experiment: $|d_c| < 1.3 \times 10^{-20}\, e\,\mathrm{cm}, \quad |d_b| < 7.6 \times 10^{-19}\, e\,\mathrm{cm},$

- Neutron EDM estimates have uncertainty ~ up to a factor of O(few) due to limitation of QCD sum rule method in this channel. CS derived limits have *minimal* uncertainty, O(10%).
- Independent of (similar order of magnitude) bounds based on RG running of operators, and contribution to the GGGdual Weinberg operator. (Gisbet, Ruiz Vidal; Haisch, Koole)
- The strength of these limits on charm EDMpoints to the conclusion that future charmed baryon EM moment proposal should focus on MDM.

Conclusions

- EDMs are an important tool for searching for flavor-diagonal CP violation. Multi-TeV scales are probed, and can be further improved.
- In *lots of models*, including the SM, the paramagnetic EDMs (*experiments looking for d_e*) are induced by the semi-leptonic operators of (electron pseudoscalar)*(nucleon scalar) type.
- C_S is induced by theta term via a two-photon exchange resulting in sensitivity $|\theta| < 3\times10^{-8}$. Further progress by O(100) for d_e type of experiments will bring the sensitivity to hadronic CP violation on par with current d_n limits.
- CKM induces C_S. The result is large and calculable and is dominated by the EW^3 order. The *equivalent* d_e is found to be $+1.0 \times 10^{-35}$ e cm. This is 1000 times larger than previously believed.
- New indirect limits on muon, charm and bottom provide new target for the EDM beam experiments:

$$|d_c| < 6 \times 10^{-22}\, e\,\mathrm{cm}, \quad |d_b| < 2 \times 10^{-20}\, e\,\mathrm{cm}, \quad |d_\mu| < 1.9 \times 10^{-20}\, e\,\mathrm{cm}.$$