Beautiful precision

Lattice calculations of form factors for B mesons

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Outline

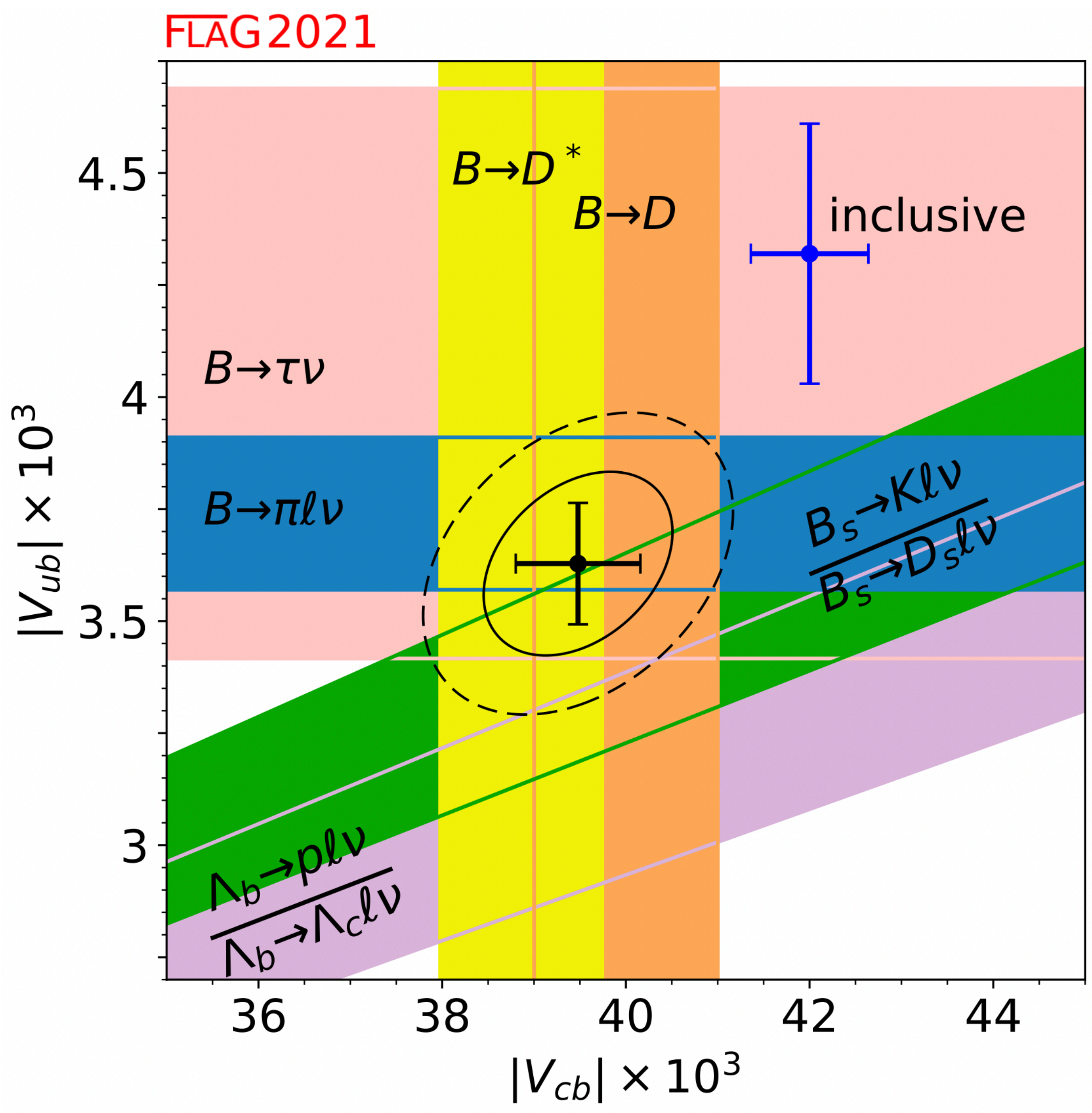
- Motivation, notation, and the nature of the QCD problem
- Lattice QCD and B-mesons
- B-to-vector decays
  - Heavy-to-heavy
- B-to-pseudoscalar decays
  - Heavy-to-heavy
  - Heavy-to-light

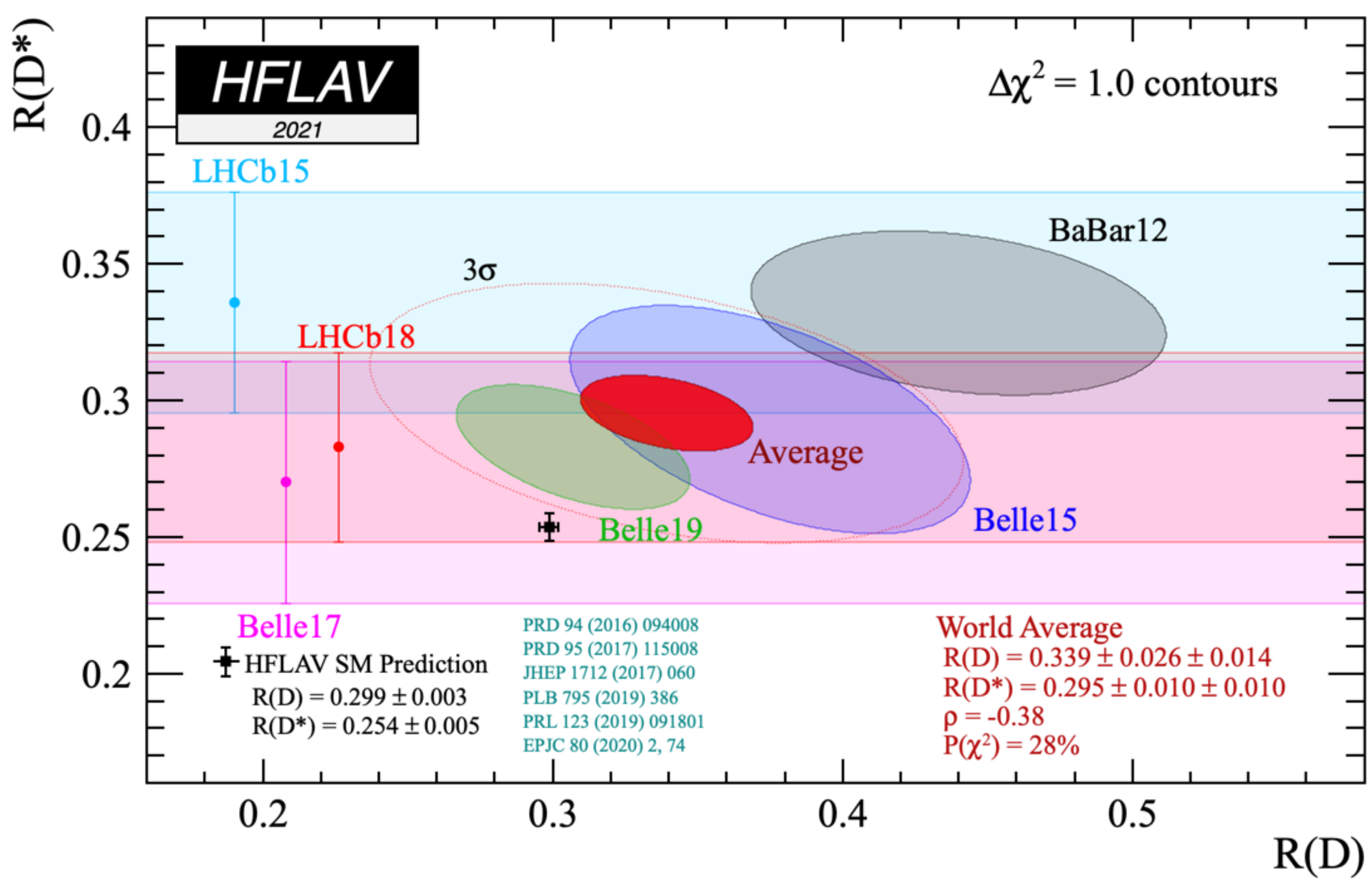
Motivation

Motivation: Big picture

- Scope: Focus on exclusive semileptonic decays of B-mesons
- Extract CKM matrix elements from decays like
  - $|V_{cb}|$: $B_{(s)} \to D_{(s)}^{(*)} \ell\nu$, $B_c \to J/\psi\ \ell\nu$
  - $|V_{ub}|$: $B \to \pi\ell\nu$, $B_s \to K\ell\nu$
- Test loop structure of SM with rare flavor-changing neutral-current (FCNC) decays:
  - $B \to \pi\ell^+\ell^-$, $B \to K\ell^+\ell^-$

Motivation: Tree-level anomalies

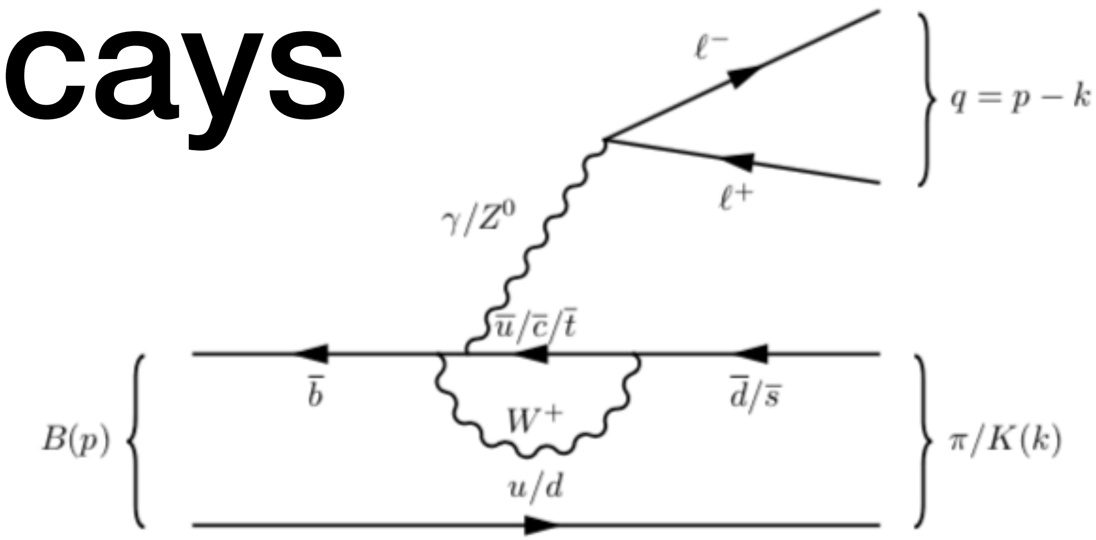




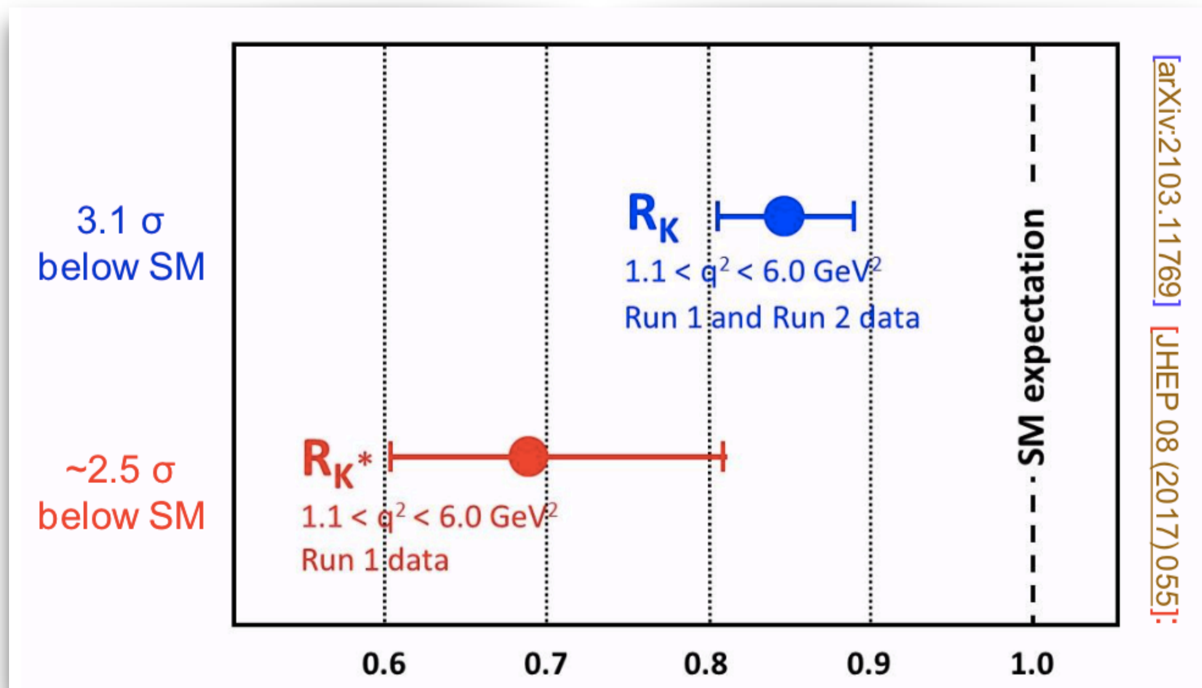
$$R(D) = \frac{\mathcal{B}(B \to D\tau\bar{\nu}_\tau)}{\mathcal{B}(B \to D\mu\bar{\nu}_\mu)}$$

$$R(D) \oplus R(D^\star) \simeq 3\sigma$$

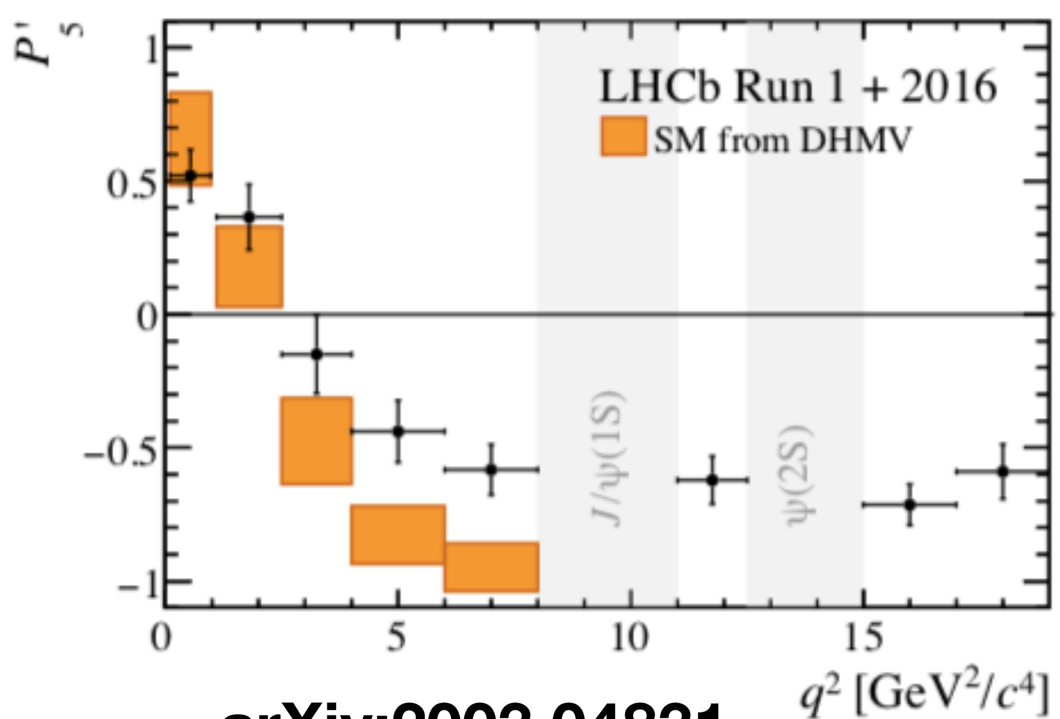
Motivation: Rare Decays



$$B \to K\ell^+\ell^-$$
$$B \to K^*(\to K\pi)\ell^+\ell^-$$



Slide credit: Guy Wilkinson



arXiv:2003.04831
PRL 125, 011802 (2020)

$$R(K) \simeq 3\sigma$$
$$R(K^\star) \simeq 2 - 3\sigma$$

Persistent ~3σ tension with SM in angular distribution P_5'

Experimental Horizons

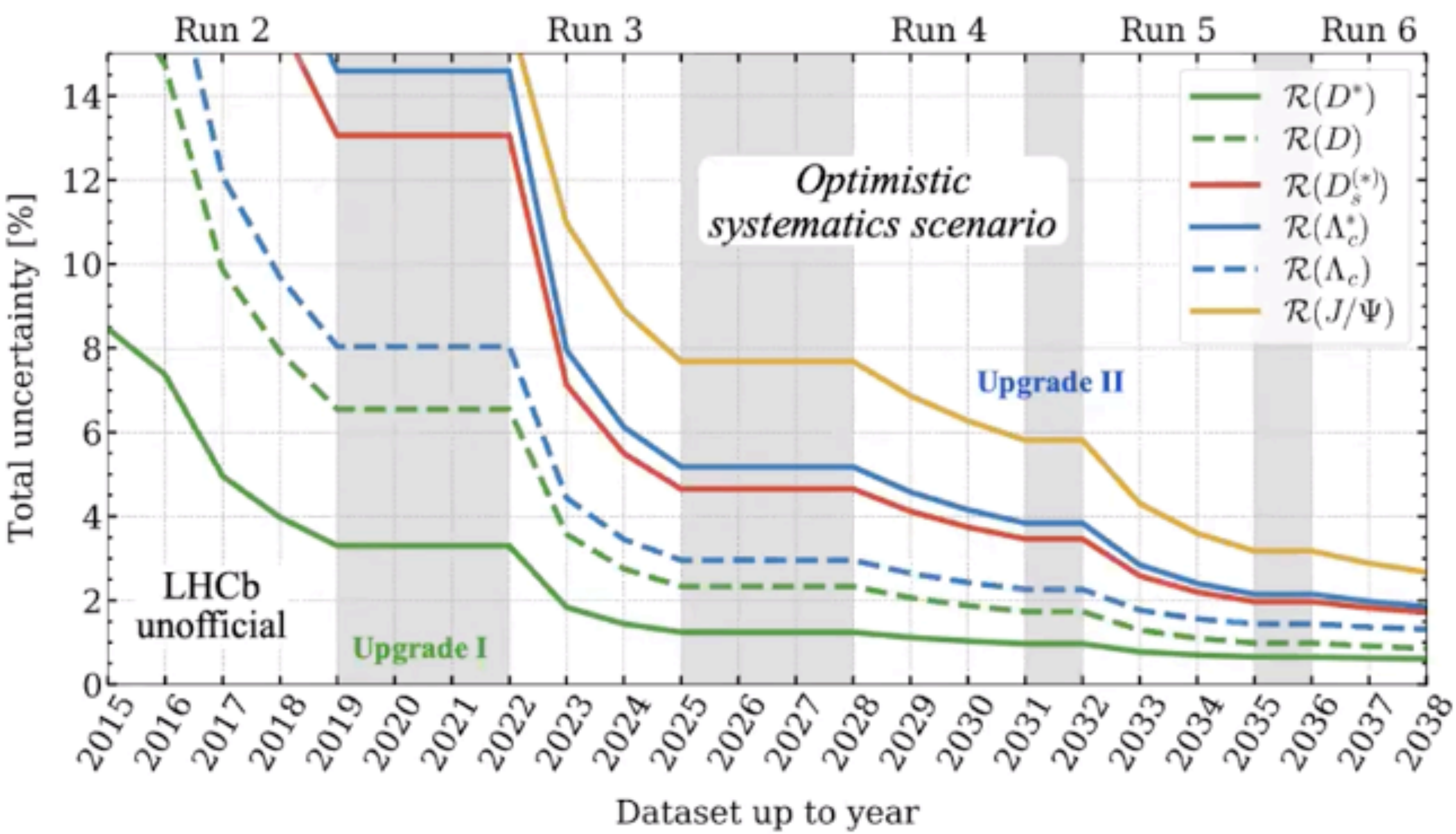
- Tremendous progress in flavor over past 20 years, e.g., with BaBar, Belle, BES III, CDF, D0, ATLAS, CMS
- LHCb: pp at LHC
  - ~10^{12} b-hadrons to date (cf. ~10^7 at LEP)
- Belle II: e^+e^- around $\Upsilon(4s)$ ~ 10.5 GeV
  - Goal: 50 ab^{-1} (50x Belle), roughly 215 fb^{-1} to date
- Exciting measurements are on the horizon

LHCb

Belle II

Improved theory is timely

LHCb projections for Run 3 and Run 4



Extracting CKM matrix elements

Theory + Experiment

- Exclusive B→$\pi\ell\nu$

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2|V_{ub}|^2}{24\pi^3}|p_\pi|^3|f_+(q^2)|^2$$

Measure in experiments

Compute using lattice QCD

$$\langle\pi(p_\pi)|V^\mu|B(p_B)\rangle =$$

$$f_+(q^2)\left[p_B^\mu + p_\pi^\mu - \frac{m_B^2 - m_\pi^2}{q^2}q^\mu\right] + f_0(q^2)\frac{m_B^2 - m_\pi^2}{q^2}q^\mu$$

$$\left|V_{ub}^{\text{excl.}}\right| = (3.70 \pm 0.10 \pm 0.12) \times 10^{-3}$$

Expt.

Theory (LQCD + EW)

- Combined precision ~4%
- Similar errors from theory and experiment
- Belle II, e.g., expects 1-2% errors in near future

A.X. El-Khadra and P. Urquijo
PDG 2021: 76. Semileptonic b-Hadron Decays, Determination of V_{cb}, V_{ub}

Extracting CKM matrix elements

Theory + Experiment

Compute using lattice QCD

- Exclusive B→D$\ell\nu$

Measure in experiments

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{48\pi^3}|V_{cb}|^2(\text{kinematics})\left(\eta_{EW}\mathcal{G}(w)\right)^2$$

$$\mathcal{G}(w) = h_+(w) - \frac{m_B - m_D}{m_B + m_D}h_-(w)$$

"recoil"

$$w \equiv v_B \cdot v_D$$

$$\frac{\langle D(v')|\bar{c}\gamma^\mu b|B(v)\rangle}{\sqrt{m_B m_D}} = h_+(w)\,(v_B + v_D)^\mu + h_-(w)\,(v_B - v_D)^\mu$$

$$\left|V_{cb}^{\text{excl.}}\right| = (39.4 \pm 0.8) \times 10^{-3}$$

- Combined precision for B→D ~ 3%
- Commensurate errors from theory and experiment
- LHCb, e.g., expects 1% errors in near future

A.X. El-Khadra and P. Urquijo
PDG 2021: 76. Semileptonic b-Hadron Decays, Determination of V_{cb}, V_{ub}

Extracting CKM matrix elements

Theory + Experiment

Compute using lattice QCD

- Exclusive B→D*ℓν

Measure in experiments

$$\frac{d\Gamma}{dw} = \frac{G_F^2 m_B^5}{48\pi^3} |V_{cb}|^2 (\text{kinematics}) \left(\eta_{EW} \mathcal{F}(w)\right)^2$$

Tensor structure more elaborate
from vector polarization.
Otherwise same strategy.
Isolate QCD matrix elements.

$$\langle D^*(v', \epsilon) | \bar{c}\gamma^\mu b | B(v) \rangle \supset h_V(w)$$

$$\langle D^*(v', \epsilon) | \bar{c}\gamma^\mu \gamma^5 b | B(v) \rangle$$
$$\supset h_{A_1}(w) + h_{A_2}(w) + h_{A_3}(w)$$

$$\left| V_{cb}^{\text{excl.}} \right| = (39.4 \pm 0.8) \times 10^{-3}$$

- Combined precision for B→D$^{(*)}$ ~ 2%
- Commensurate errors from theory and experiment
- LHCb, e.g., expects 1% errors in near future

A.X. El-Khadra and P. Urquijo
PDG 2021: 76. Semileptonic b-Hadron Decays, Determination of V_{cb}, V_{ub}

LQCD precision achievements over time

CSS2013: Snowmass on the Mississippi
S. Butler et al [arXiv:1311.1076]

		2013		**2013**	**Expected**	**Achieved**
Quantity	CKM element	~~Present~~ expt. error	2007 forecast lattice error	~~Present~~ lattice error	2018 lattice error	2021 FLAG avg
f_K/f_π	$\lvert V_{us}\rvert$	0.2%	0.5%	0.5%	0.15%	0.18%
$f_+^{K\pi}(0)$	$\lvert V_{us}\rvert$	0.2%	–	0.5%	0.2%	0.18%
f_D	$\lvert V_{cd}\rvert$	4.3%	5%	2%	< 1%	0.3%
f_{D_s}	$\lvert V_{cs}\rvert$	2.1%	5%	2%	< 1%	0.2%
$D \to \pi \ell \nu$	$\lvert V_{cd}\rvert$	2.6%	–	4.4%	2%	4.4%
$D \to K \ell \nu$	$\lvert V_{cs}\rvert$	1.1%	–	2.5%	1%	0.6%
$B \to D^* \ell \nu$	$\lvert V_{cb}\rvert$	1.3%	–	1.8%	< 1%	1.7%
$B \to \pi \ell \nu$	$\lvert V_{ub}\rvert$	4.1%	–	8.7%	2%	3%
f_B	$\lvert V_{ub}\rvert$	9%	–	2.5%	< 1%	0.7%
ξ	$\lvert V_{ts}/V_{td}\rvert$	0.4%	2-4%	4%	< 1%	1.3%
ΔM_s	$\lvert V_{ts}V_{tb}\rvert^2$	0.24%	7–12%	11%	5%	4.5%
B_K	$\mathrm{Im}(V_{td}^2)$	0.5%	3.5–6%	1.3%	< 1%	1.3%

Systematic inclusion of QED now becomes necessary

Broad community effort to:
- keep pace with experimental needs
- achieve ~1% precision

- LQCD precision: expected improvements from ~10 years ago have largely been achieved.
- In-progress calculations expect to reach ≲ 1% level for semileptonic B-decays

Lattice QCD and B-mesons

Lattice QCD

- Lattice QCD gives complete non-perturbative definition to the strong interactions
- This framework gives: $$\mathcal{Z} = \int \mathcal{D}[\text{fields}] e^{-S_E[\text{fields}]}$$
- **Fundamental approximations:**
  - UV cutoff: lattice spacing *a* [target: a $\ll$ physical scales]
  - IR cutoff: finite spacetime volume $V = L^3 \times T$ [target: $1 \ll m_\pi L$]

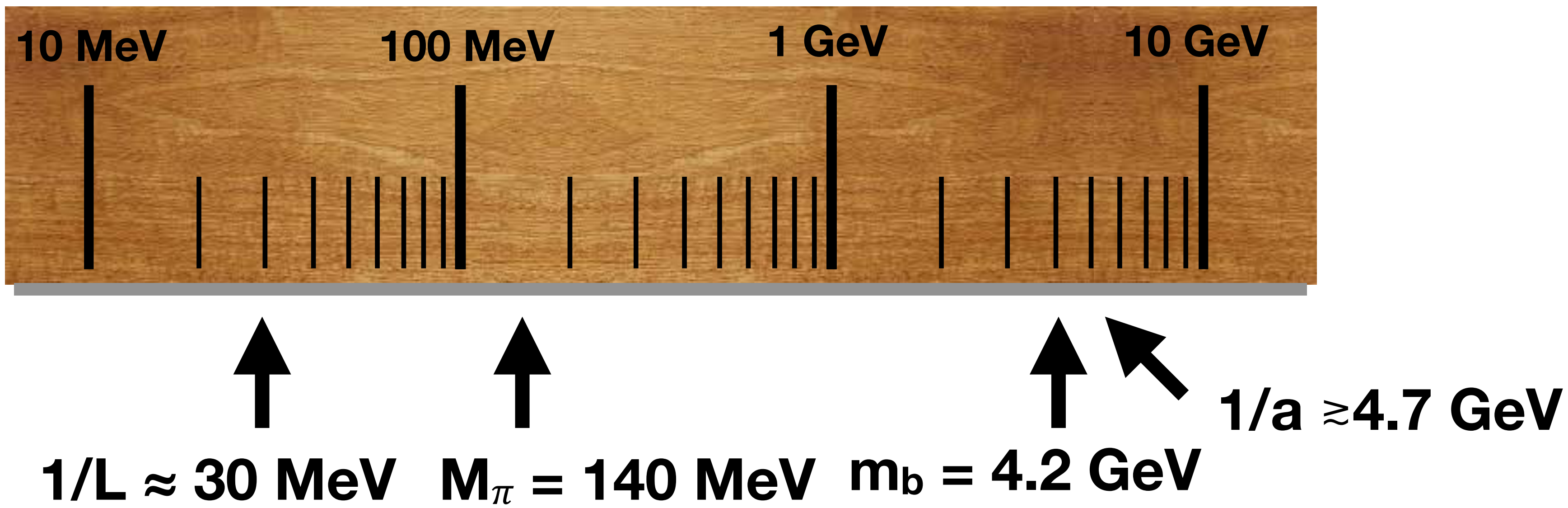
- **Approximations of convenience:**
  - Often: Heavier-than-physical pions: $(m_\pi)^{\text{lattice}} > (m_\pi)^{\text{PDG}}$
  - Often: Isospin limit $m_u = m_d$
  - Often: QCD interactions only, no QED
  - Often: lighter-than-physical or static heavy quarks

Lattice QCD is systematically improvable

- All approximations admit theoretical descriptions via EFT
  - Cutoff dependence ⇔ Symanzik effective theory
  - Finite-volume dependence ⇔ Finite-volume χPT
  - Chiral extrapolation / interpolation ⇔ χPT
  - Heavy quark extrapolation / interpolation ⇔ HQET, NRQCD, etc…
  - QED, isospin breaking ⇔ perturbative expansion of path integral
- Precise treatment of all systematic effects is key to modern high-precision lattice QCD

Chasing beauty

QCD with heavy quarks is a difficult multi-scale problem.



Heavy quarks are hard: lattice artifacts grow like powers $(am_h)^n$ — especially tricky for masses near or above the cutoff

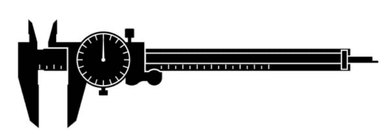
$$\frac{1}{L} \ll M_\pi \ll m_b \ll \frac{1}{a}$$

Chasing beauty

Heavy quarks are hard: lattice artifacts grow like powers $(am_h)^n$ — especially tricky for masses near or above the cutoff

1. Use an effective theory for heavy quarks (b, sometimes c)
   - "FNAL interpretation," NRQCD, RHQ, Oktay-Kronfeld
   - Good: Solves problem with artifacts (am_h)
   - No free lunch: EFTs require matching, which introduces systematic effects
   - (1-3)% total errors

2. Use highly-improved relativistic light-quark action on fine lattices
   - Good: advantageous renormalization, continuum limit
   - No free lunch: simulations still need $am_h < 1$ and often an extrapolation to the physical bottom mass
   - (< 1)% total errors possible



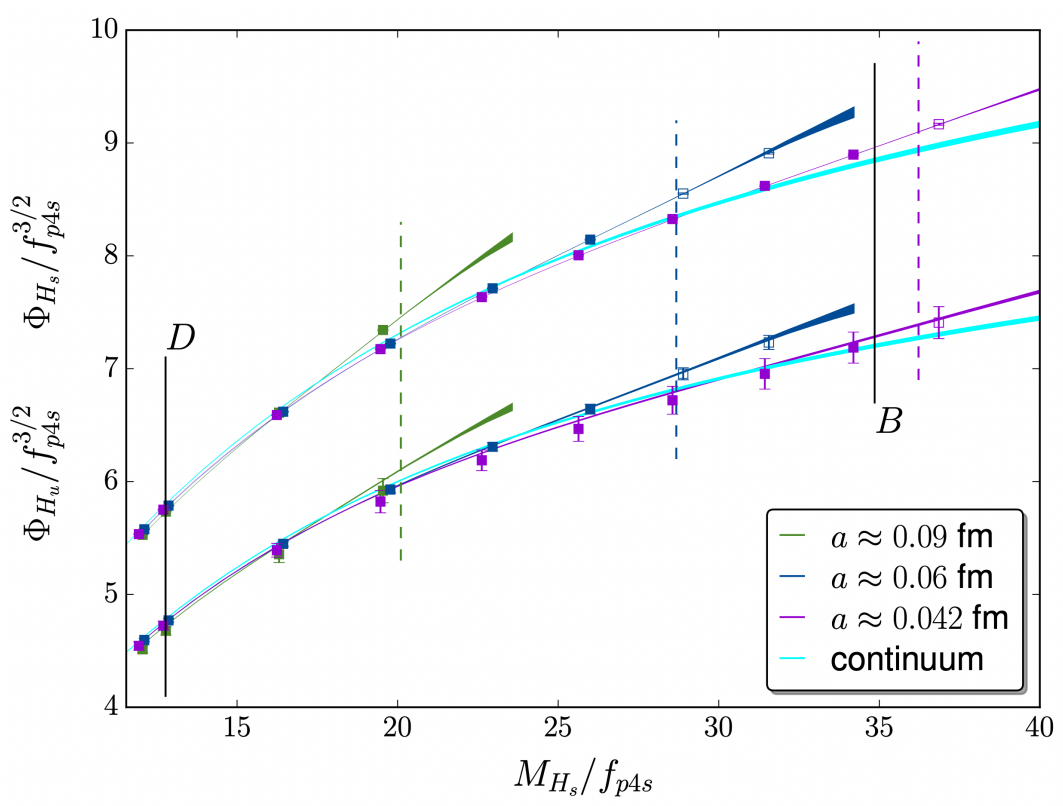
Chasing beauty

- Many different treatments used in the literature:

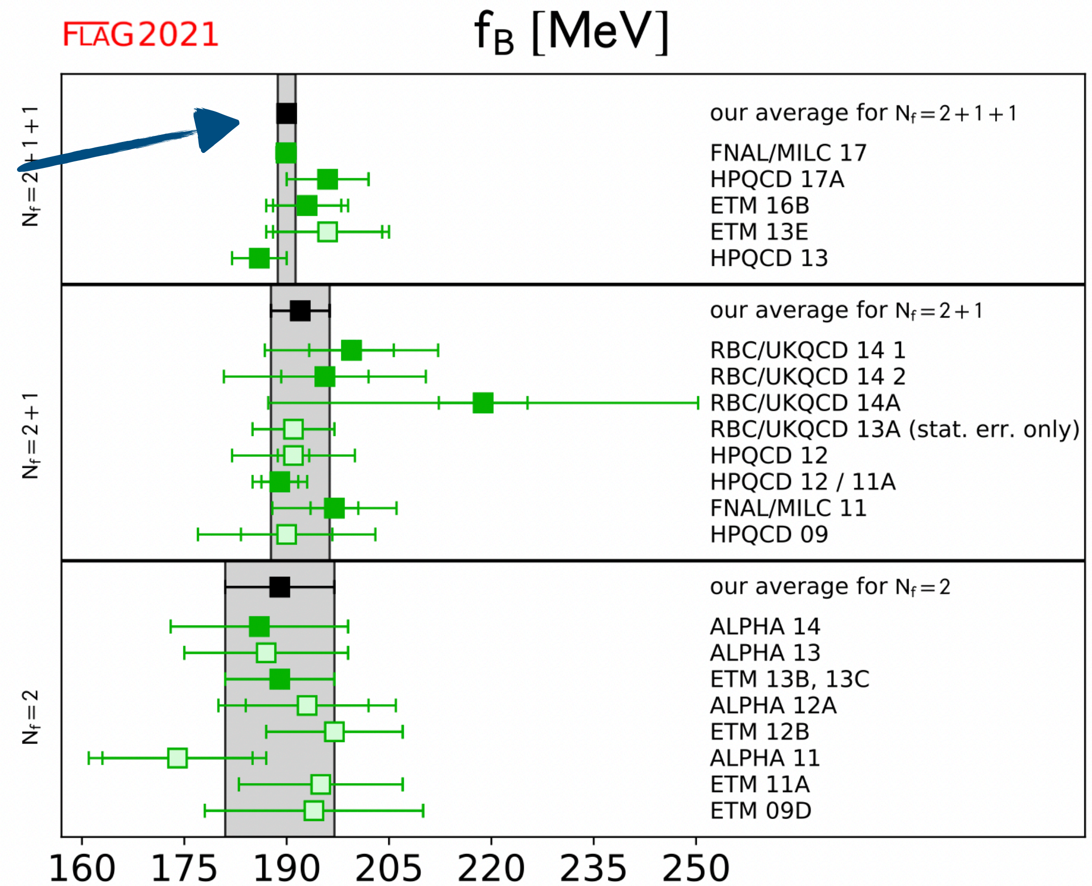
Group	Heavy valence		Sea	"Generation"
HPQCD	NRQCD	on	ASQTAD	I
HPQCD	NRQCD	on	HISQ	II
HPQCD	HISQ	on	HISQ	III
FNAL/MILC	Fermilab	on	ASQTAD	1
FNAL/MILC	Fermilab	on	HISQ	2
FNAL/MILC	HISQ	on	HISQ	3
JLQCD	Möbius DW	on	Möbius DW	
LANL/SWME	Oktay-Kronfeld	on	HISQ	
RBC/UKQCD	RHQ	on	DW	
ETMC	Twisted mass	on	Twisted mass	

Leptonic Decays: A success story

Bazavov et al [FNAL/MILC, arXiv:1712.09262, PRD 2018



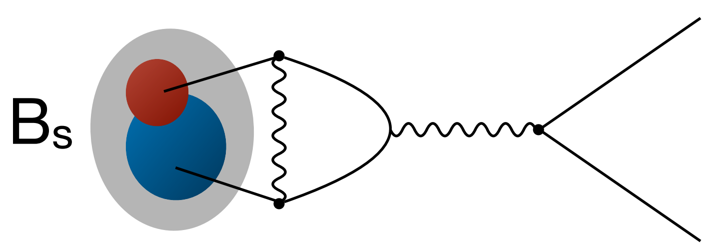
0.6% (!)



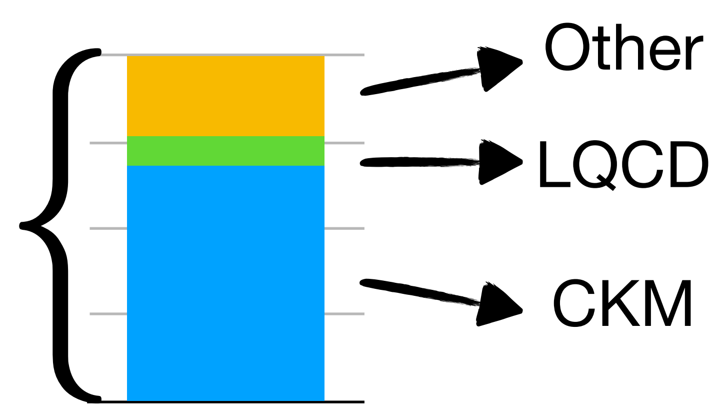
SM prediction for rare leptonic decay rate

Beneke et al, arXiv:1908.07011, JHEP 2019

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-) = 3.660(38) \times 10^{-9}$$



Lattice QCD value for f_{Bs} is now a sub-dominant source of uncertainty



B to V Decays

Heavy-to-heavy: $B \to D^*$, $B_s \to D_s^*$, $B_c \to J/\psi$

$B \to D^*$

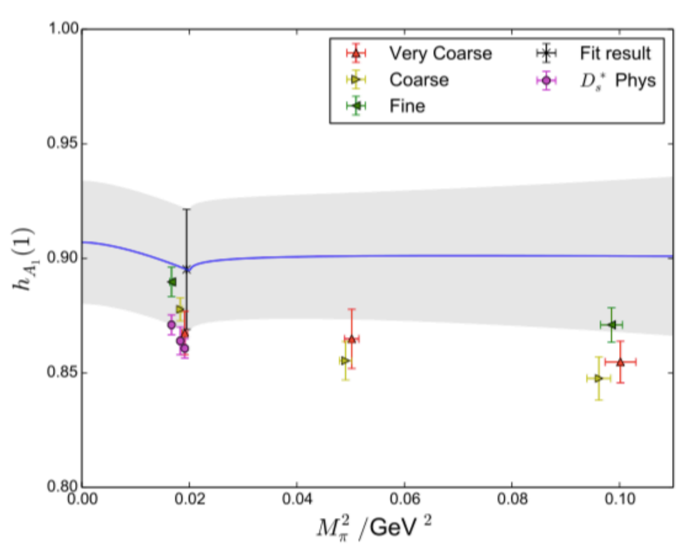
Zero-recoil only (w=1)

HPQCD "Generation II"

arXiv:1711.11013
PRD 97 (2018) 5, 054502

- (N_f=2+1+1) MILC HISQ ensembles
- Lattice spacings [0.09, 0.12, 0.15] fm
- Heavy b: NRQCD
- $h_{A1}(1)$ = 0.895(10)(24), $B \to D^*$
- $h_{A1}(1)$ = 0.883(12)(28), $B_s \to D_s^*$

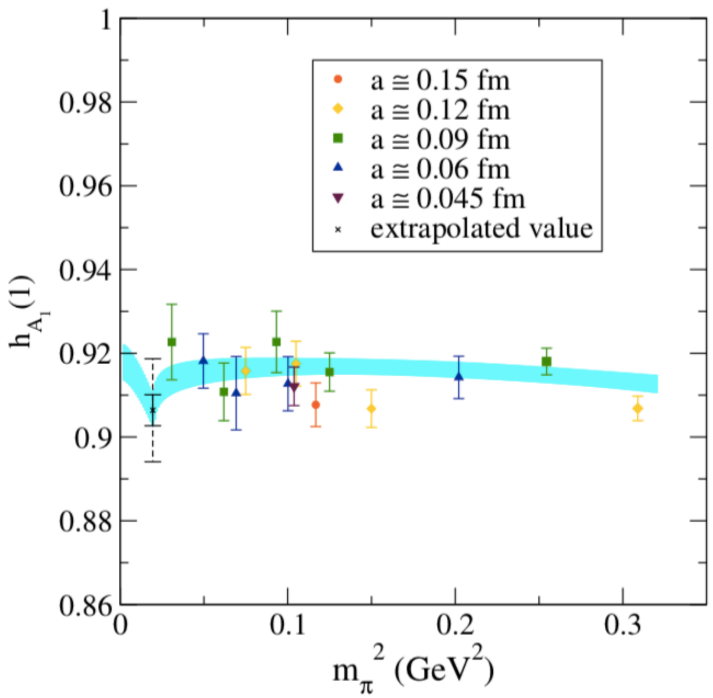
Uncertainty	$h_{A_1}(1)$	$h^s_{A_1}(1)$	$h_{A_1}(1)/h^s_{A_1}(1)$
α_s^2	2.1	2.5	0.4
$\alpha_s\Lambda_{QCD}/m_b$	0.9	0.9	0.0
$(\Lambda_{QCD}/m_b)^2$	0.8	0.8	0.0
a^2	0.7	1.4	1.4
$g_{D^*D\pi}$	0.2	0.03	0.2
Total systematic	2.7	3.2	1.7
Data	1.1	1.4	1.4
Total	2.9	3.5	2.2



FNAL/MILC "Generation 1"

arXiv:1403.0635
PRD 89 (2014) 11, 114504

- (N_f=2+1) MILC asqtad ensembles
- Lattice spacings in [0.045 - 0.15] fm
- Light valence: asqtad staggered
- Heavy b/c: FNAL interpretation
- $h_{A1}(1)$=F(1) = 0.906(4)(13)



Uncertainty	$h_{A_1}(1)$
Statistics	0.4%
Scale (r_1) error	0.1%
χPT fits	0.5%
$g_{D^*D\pi}$	0.3%
Discretization errors	1.0%
Perturbation theory	0.4%
Isospin	0.1%
Total	1.4%

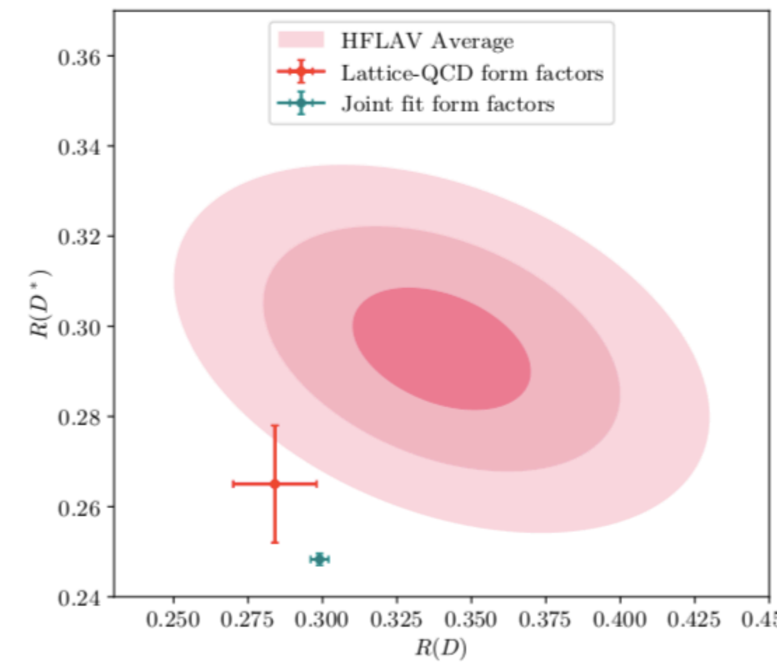
B→D*: FNAL/MILC

arXiv:2105.14019

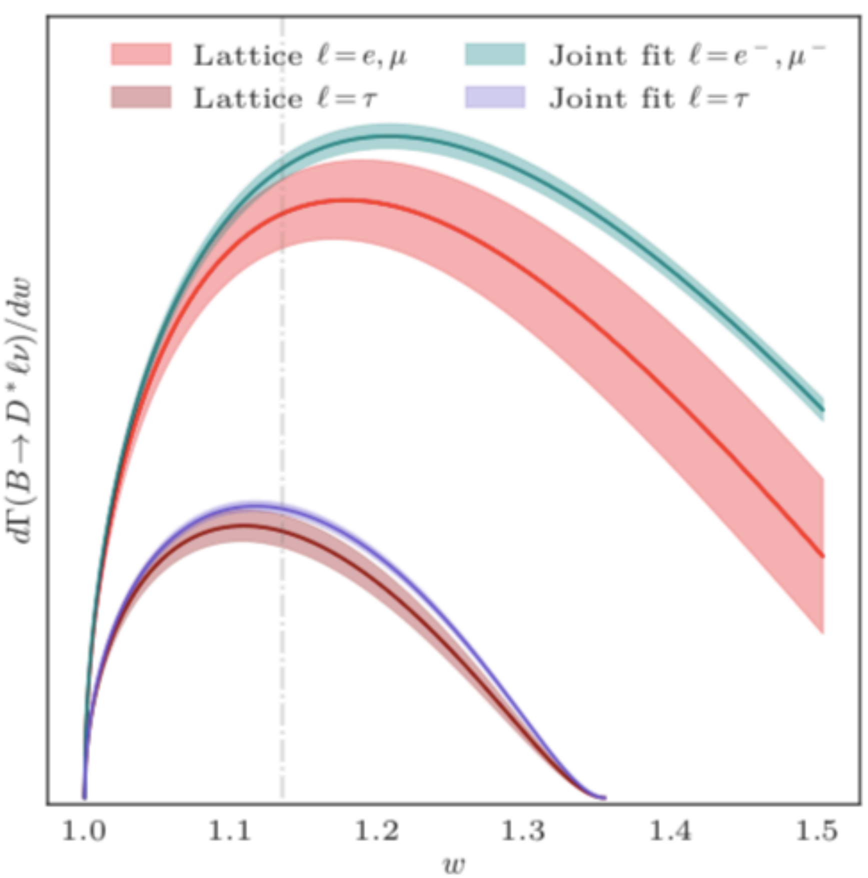
“Generation 1”

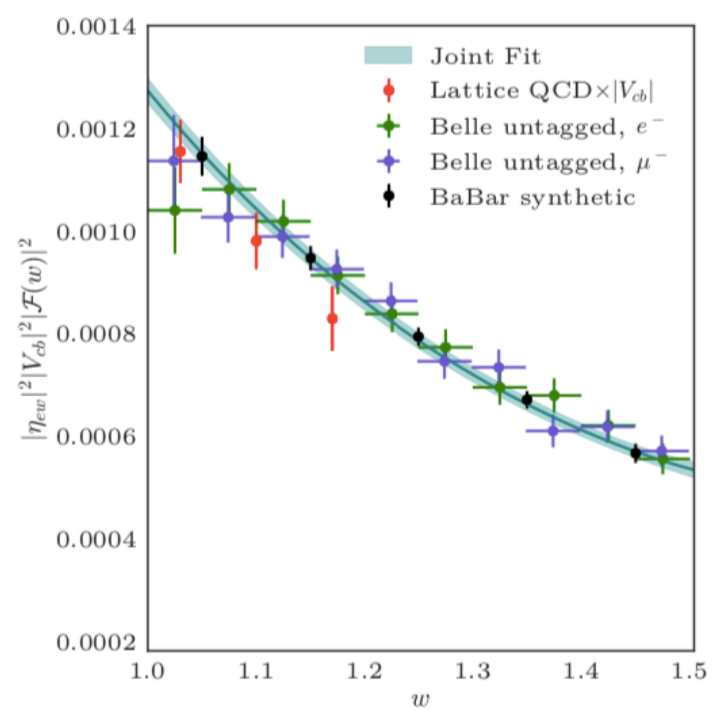
Full kinematic range

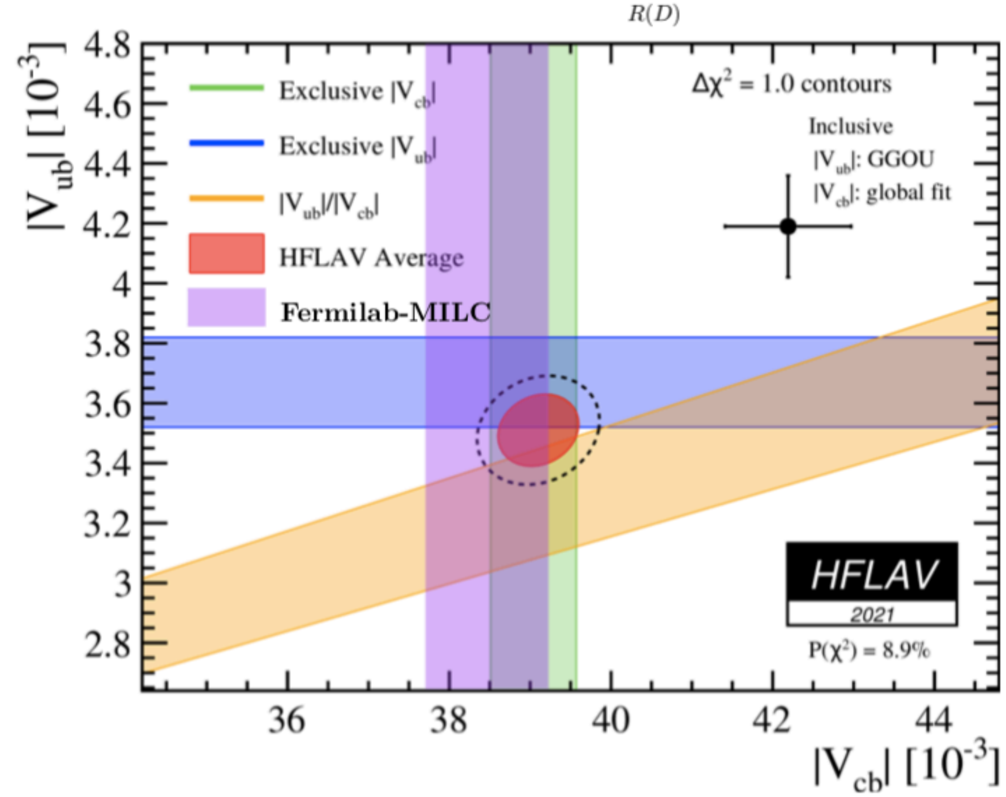
- (N_f=2+1) MILC asqtad ensembles
- Lattice spacings in [0.045 - 0.15] fm
- Valence b/c: FNAL interpretation
- World-first calculation away from $q^2=q^2_{max}$
- $|V_{cb}| = (38.40 \pm 0.66_{th} \pm 0.34_{exp})\times 10^{-3}$
- $R(D^*) = 0.265 \pm 0.013$



Confirms tension for R(D), R(D*)







In progress: B→D*

Talks @ Barolo workshop
19-23 April 2022

JLQCD

arXiv:2112.13775
Lattice21 Proceedings

- T. Kaneko @ Barolo
- (N_f = 2+1) Möbius domain wall
- Lattice spacings [0.44 - 0.8] fm
- $M_\pi \geq$ 230 MeV
- Also computing B→D

HPQCD

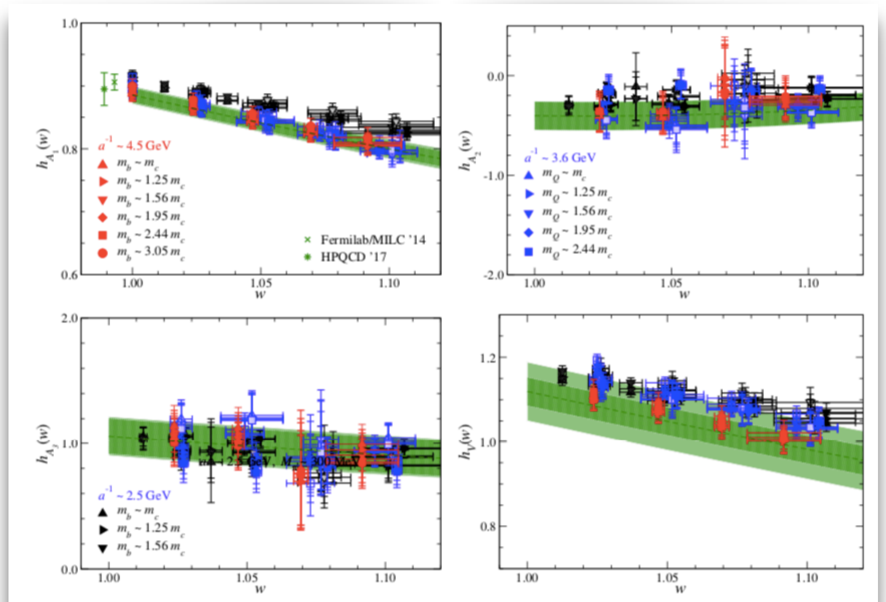
"Generation III"

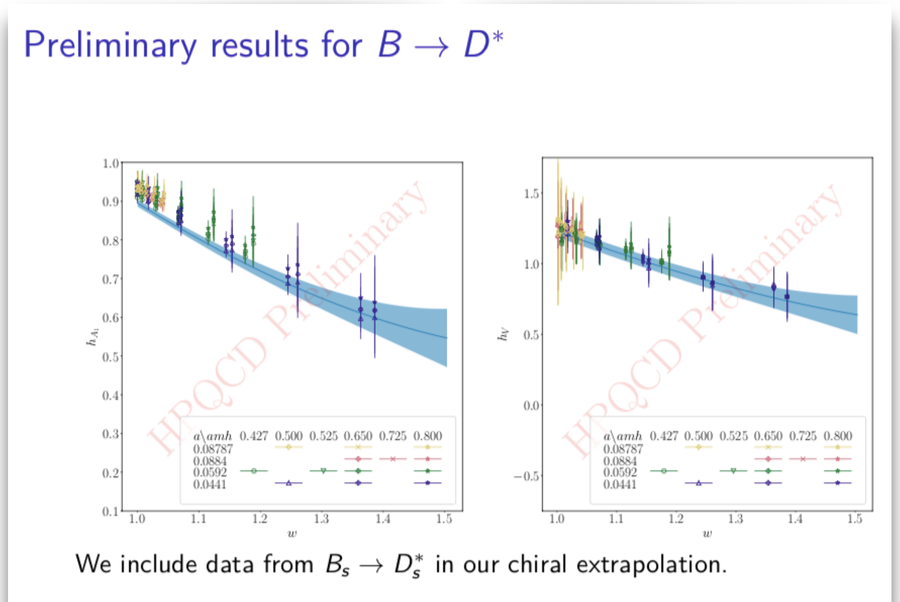
- J. Harrison @ Barolo
- all-HISQ setup
- Also computing $B_s \to D_s^*$

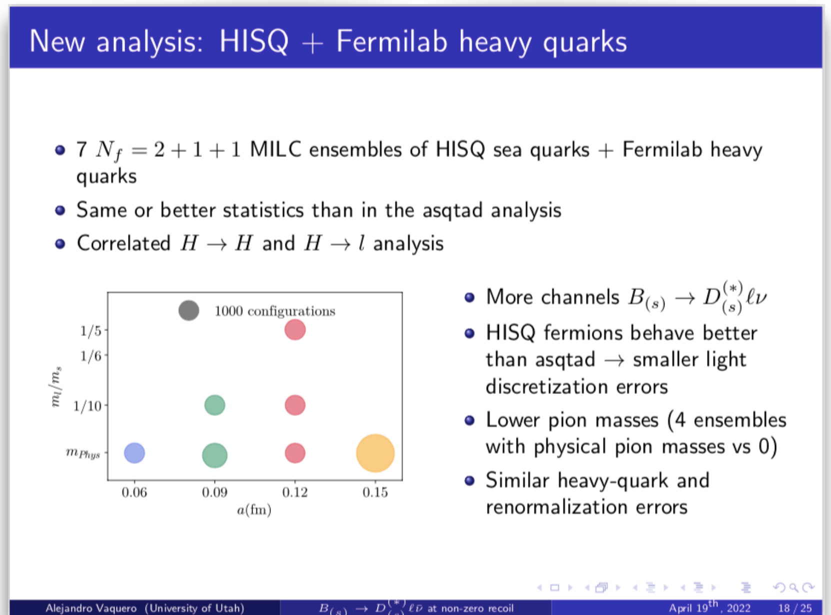
FNAL/MILC

"Generation 2"

- A. Vaquero @ Barolo
- Also computing $B_s \to D_s^*$, $B_{(s)} \to D_{(s)}$







arXiv:2105.11433

$B_s \to D_s^*$: HPQCD

"Generation III"

Full kinematic range

- (N_f=2+1+1) MILC HISQ ensembles
- Lattice spacings: [0.04 - 0.09] fm
- Valence quarks: all HISQ
- $R(D_s^*) = 0.2442(79)_{latt}(35)_{EM}$
- $|V_{cb}| = 43.0(2.1)_{latt}(1.7)_{exp}(0.4)_{EM} \times 10^{-3}$

"… a model-independent determination of $|V_{cb}|$ using $B_s \to D_s^*$ will require a reduction in uncertainty by a factor of ≈ 3 to reach the same precision as that quoted for the exclusive determination using $B \to D^*$ at zero-recoil."

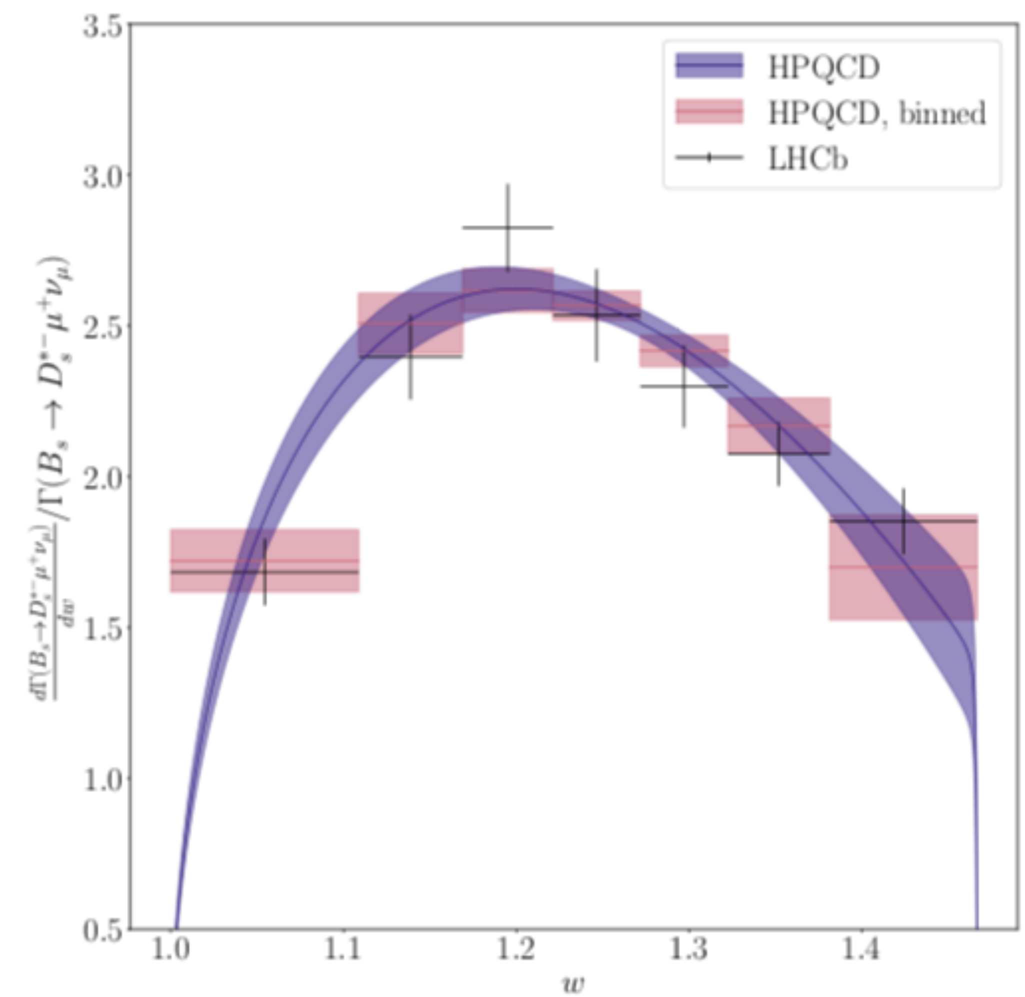
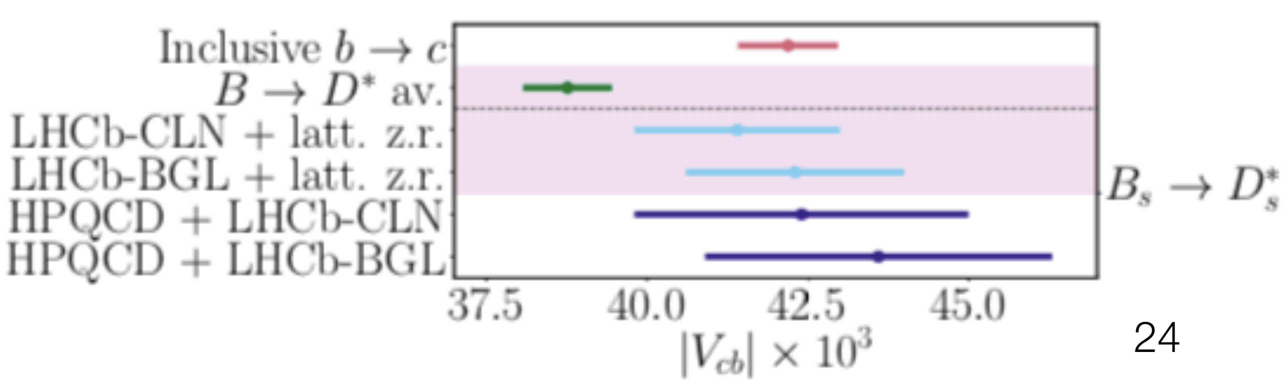


FIG. 11. The differential rate $d\Gamma/dw$ for $B_s^0 \to D_s^{*-}\mu^+\nu_\mu$ as a function of the recoil $w = v_{B_s} \cdot v_{D_s^*}$ and normalised by the total decay rate calculated from our form factors is given by the purple band. We also show our rate integrated across bins and measurements by LHCb [54].

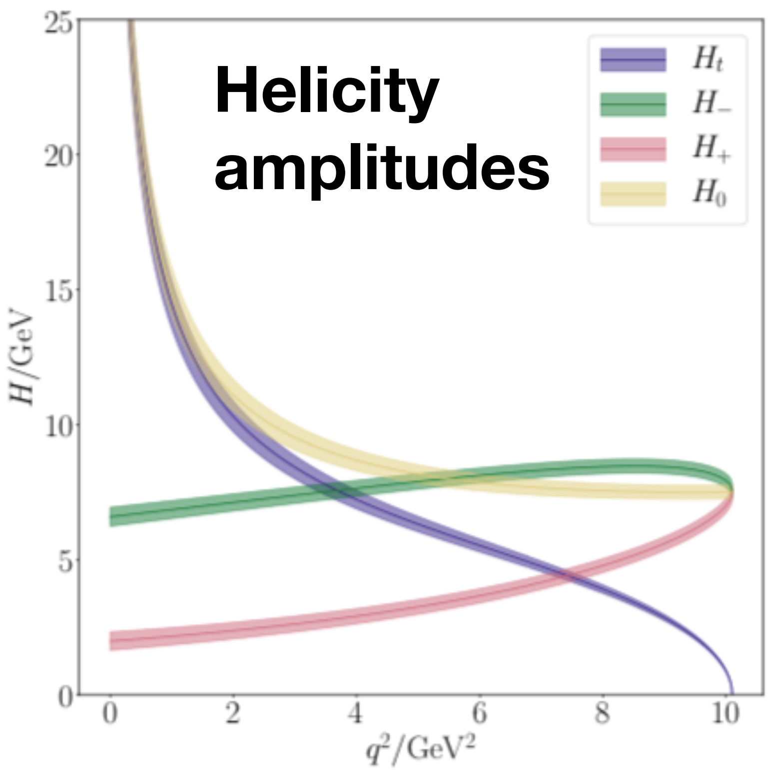


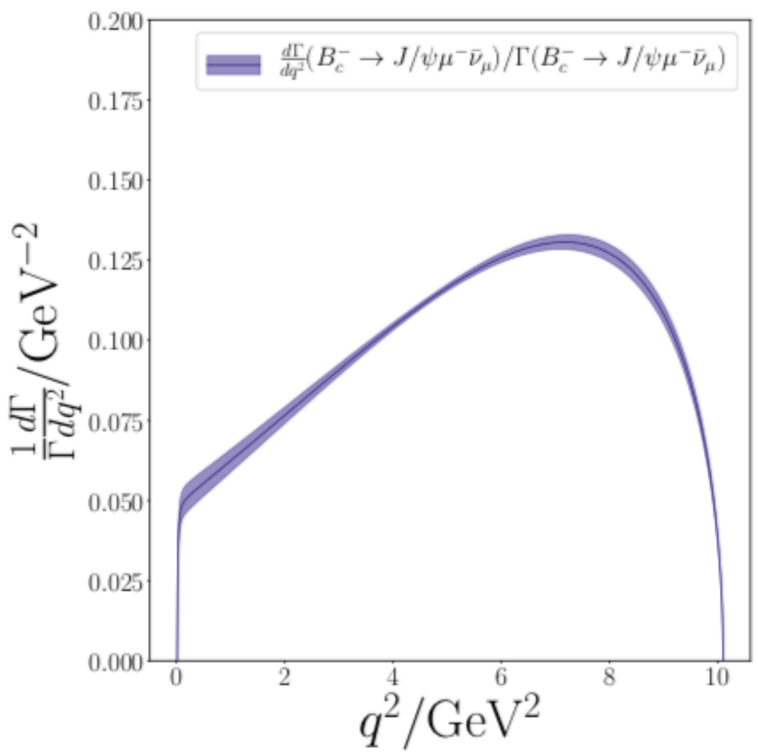
$B_c \rightarrow J/\psi$: HPQCD

"Generation III"

arXiv:2007.06957
PRD 102 (2020) 9, 094518
arXiv:2007.06956
PRL 125 (2020) 22, 222003

- (N_f=2+1+1) MILC HISQ ensembles
- Lattice spacings: [0.04 - 0.09] fm
- Valence quarks: all HISQ
- $\Gamma(B_c \rightarrow J/\psi \mu \bar{\nu}) / |\eta_{EW} V_{cb}|^2 = 1.73(12) \times 10^{13}\ s^{-1}$ [7%]
- $Br(B_c \rightarrow J/\psi \mu \bar{\nu}) = 0.0150(11)_{thy}(10)_{|\eta EW\ Vcb|}(3)_{lifetime}$
- $R(J/\psi)$=0.2582(38) [1.5%]





$f^T_{J/\psi}$: HPQCD

arXiv:2008.02024
PRL 125 (2020) 22, 222003

"Generation III"

- Ensembles: 4x (N_f=2+1+1) MILC HISQ
- Lattice spacings: [0.04 - 0.09] fm
- Valence quarks: all HISQ
- $f^T_{J/\psi}$(2 GeV) = 0.3927(27) GeV in MSbar

VII. CONCLUSIONS

We have shown here that it is possible to renormalise lattice tensor currents to give accurate results for continuum matrix elements in the $\overline{\text{MS}}$ scheme using nonperturbative determination of intermediate renormalisation factors in momentum-subtraction schemes. A key requirement is that the nonperturbative renormalisation factors should be obtained at multiple values of the renormalisation scale, μ, so that μ-dependent nonperturbative (condensate) contamination of Z_T can be fitted and removed. This contamination would otherwise give a systematic error of 1.5% using the RI-SMOM scheme and 3% using the RI′-MOM scheme in our calculation.

Crucial for rare loop decays like $B \to K\ell^+\ell^-$

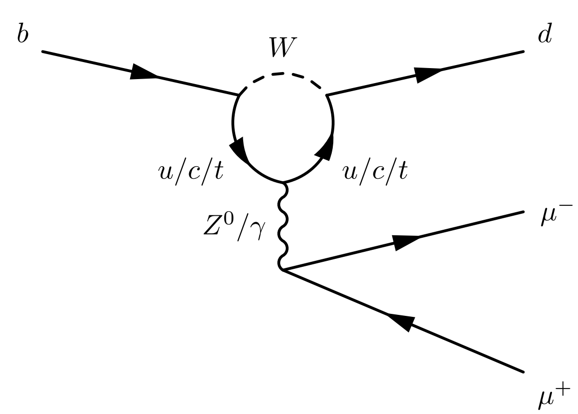


Table VII: Error budget for ratio J/ψ vector and tensor decay constants

	$f^T_{J/\psi}/f^V_{J/\psi}$
$(am_c)^2 \to 0$	0.11
$(\tilde{a}\mu)^2 \to 0$	0.27
Z_T	0.12
Z_V	0.14
Missing α_s^4 term	0.06
Statistics	0.41
Sea mistuning	0.04
Condensates	0.07
Total	0.54

B to P Decays

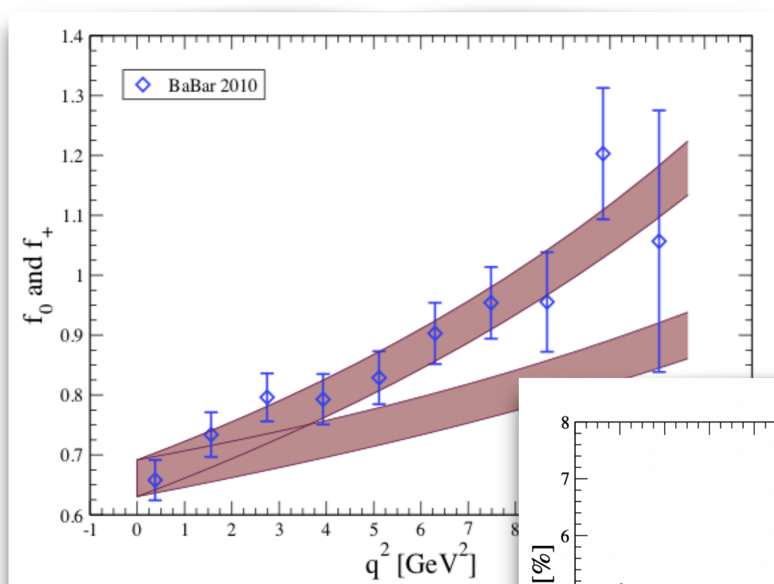
Heavy-to-heavy: $B \to D$, $B_s \to D_s$, $B_c \to B_{s(d)}$

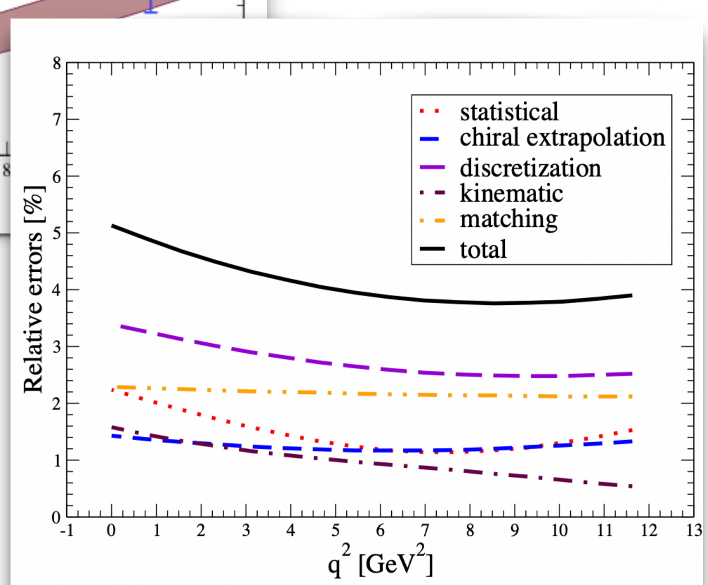
B→D

HPQCD "Generation I"

arXiv:1505.03925
PRD 92 (2015) 5, 054510

- (N_f=2+1) MILC asqtad Ensembles
- Lattice spacings [0.09, 0.12] fm
- Heavy b: NRQCD
- R(D) = 0.300(8), G(1) = 1.035(40)

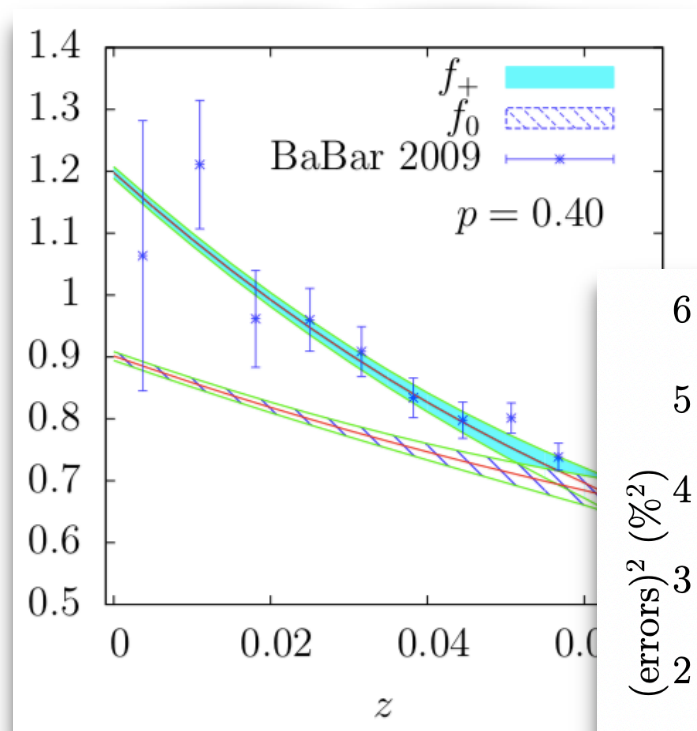


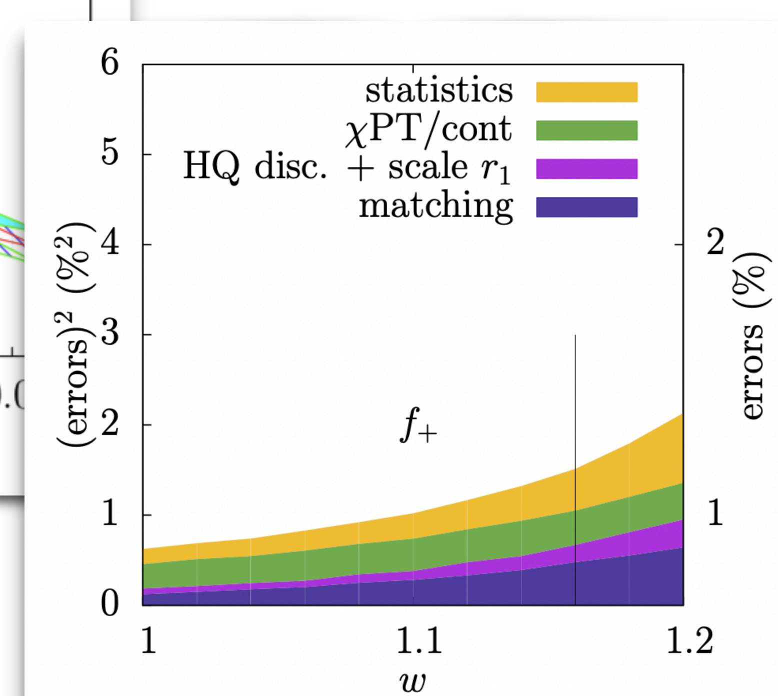


FNAL/MILC "Generation 1"

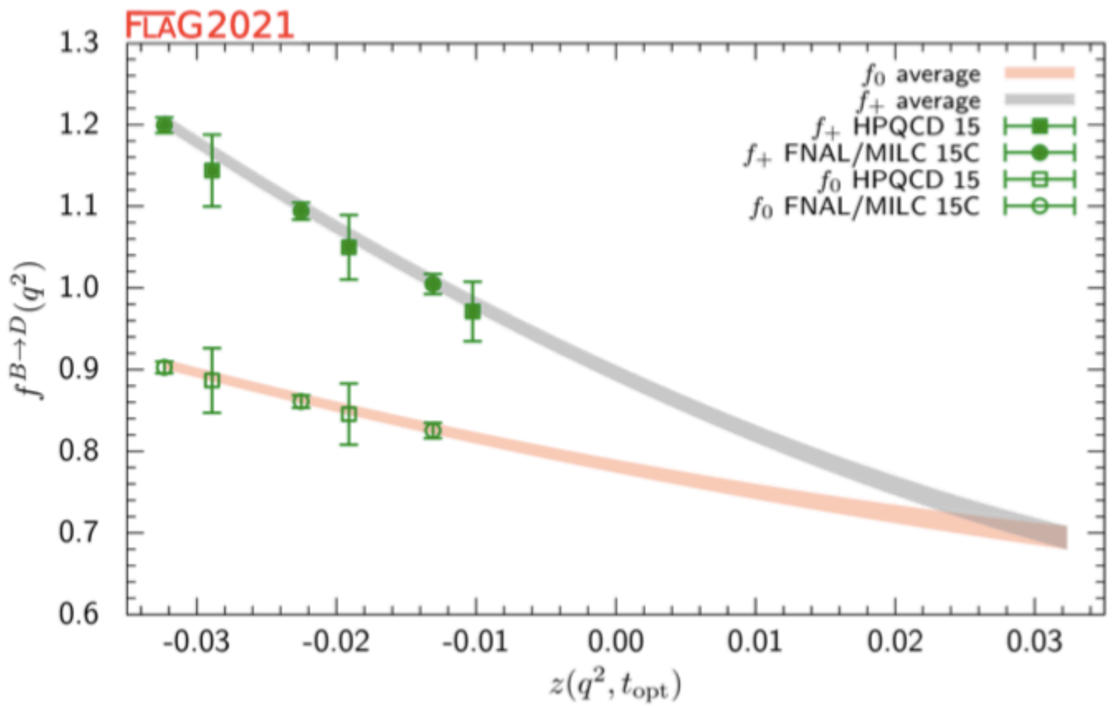
arXiv:1503.07237
PRD 92 (2015) 3, 034506

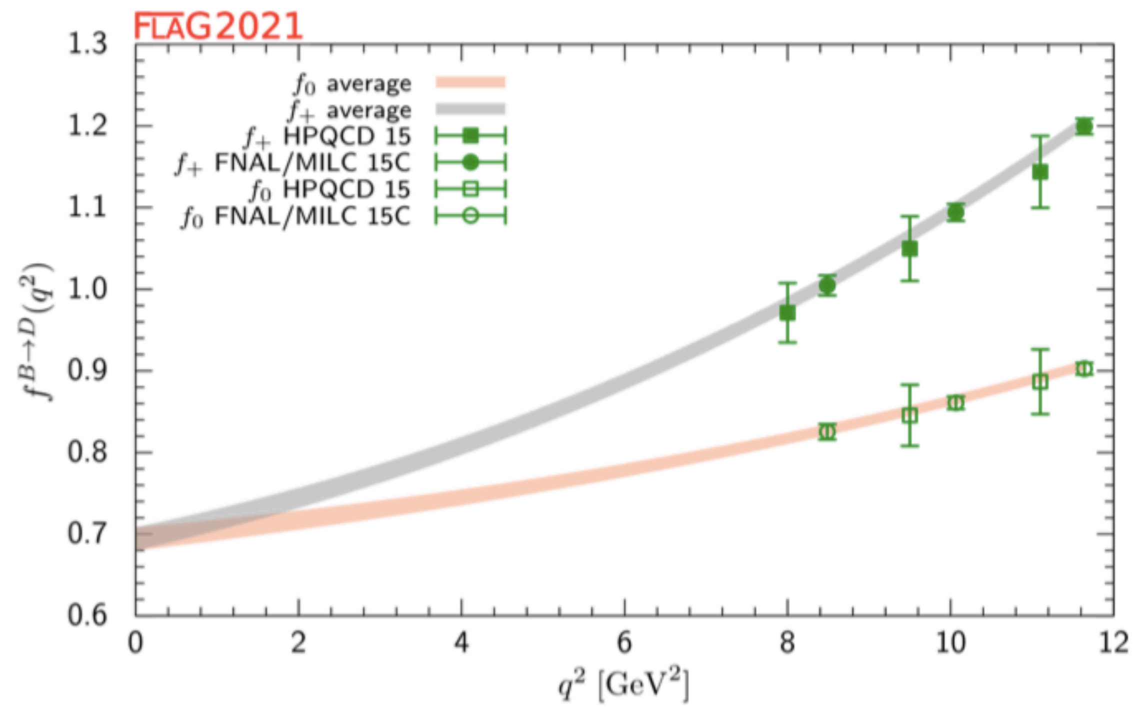
- (N_f=2+1) MILC asqtad ensembles
- Lattice spacings [0.045 - 0.12] fm
- Heavy b/c: FNAL interpretation
- R(D) = 0.299(11), G(1) = 1.054(4)(8)

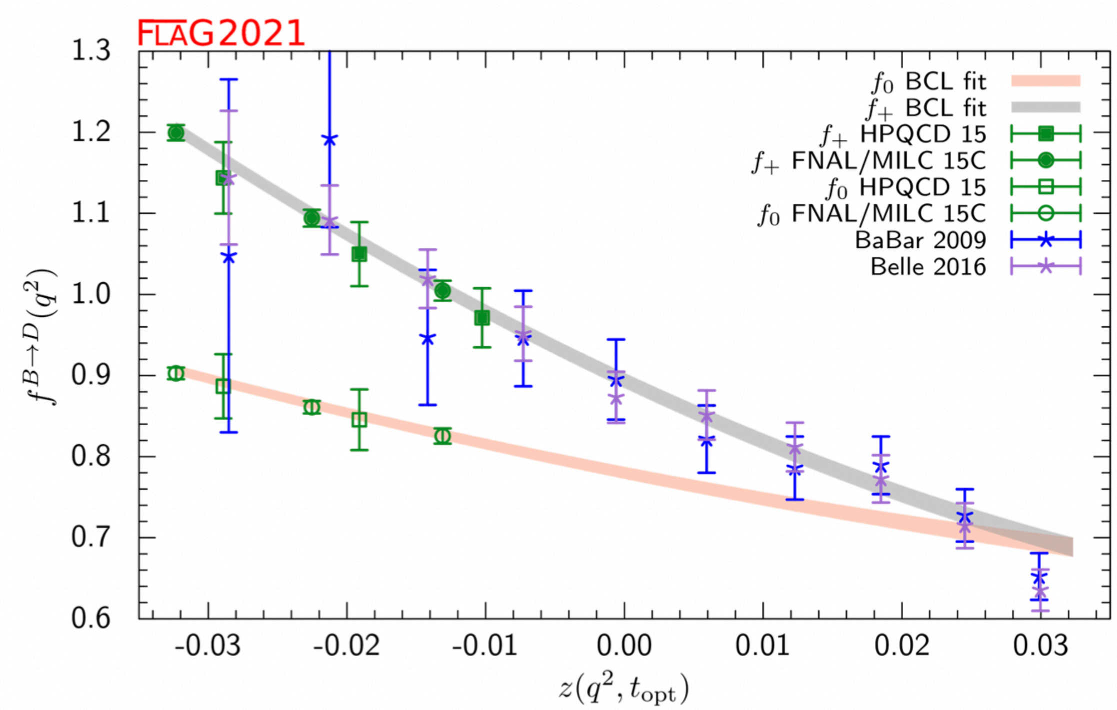


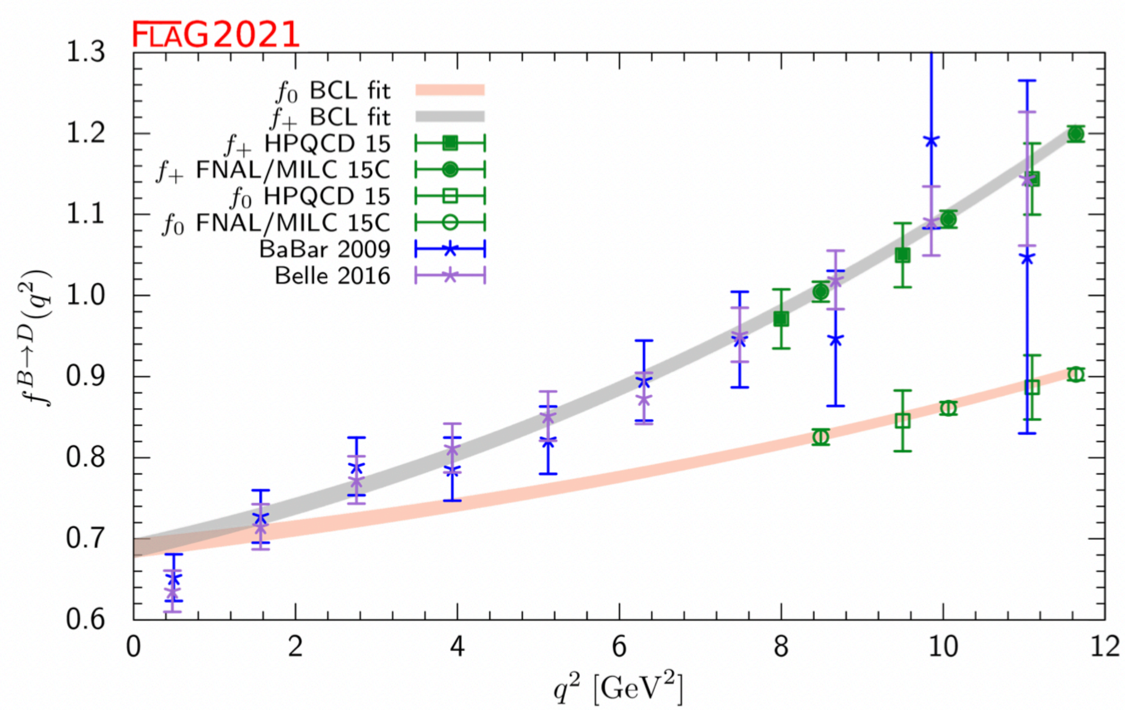


B→D: Summary


FLAG2021
f_0 average
f_+ average
f_+ HPQCD 15
f_+ FNAL/MILC 15C
f_0 HPQCD 15
f_0 FNAL/MILC 15C
$f^{B\to D}(q^2)$
$z(q^2, t_{\rm opt})$


FLAG2021
f_0 average
f_+ average
f_+ HPQCD 15
f_+ FNAL/MILC 15C
f_0 HPQCD 15
f_0 FNAL/MILC 15C
$f^{B\to D}(q^2)$
q^2 [GeV^2]


FLAG2021
f_0 BCL fit
f_+ BCL fit
f_+ HPQCD 15
f_+ FNAL/MILC 15C
f_0 HPQCD 15
f_0 FNAL/MILC 15C
BaBar 2009
Belle 2016
$f^{B\to D}(q^2)$
$z(q^2, t_{\rm opt})$


FLAG2021
f_0 BCL fit
f_+ BCL fit
f_+ HPQCD 15
f_+ FNAL/MILC 15C
f_0 HPQCD 15
f_0 FNAL/MILC 15C
BaBar 2009
Belle 2016
$f^{B\to D}(q^2)$
q^2 [GeV^2]

$B_c \rightarrow B_{s(d)}$

HPQCD "Generation II + III"

arXiv:2003.00914
PRD 102 (2020) 1, 014513

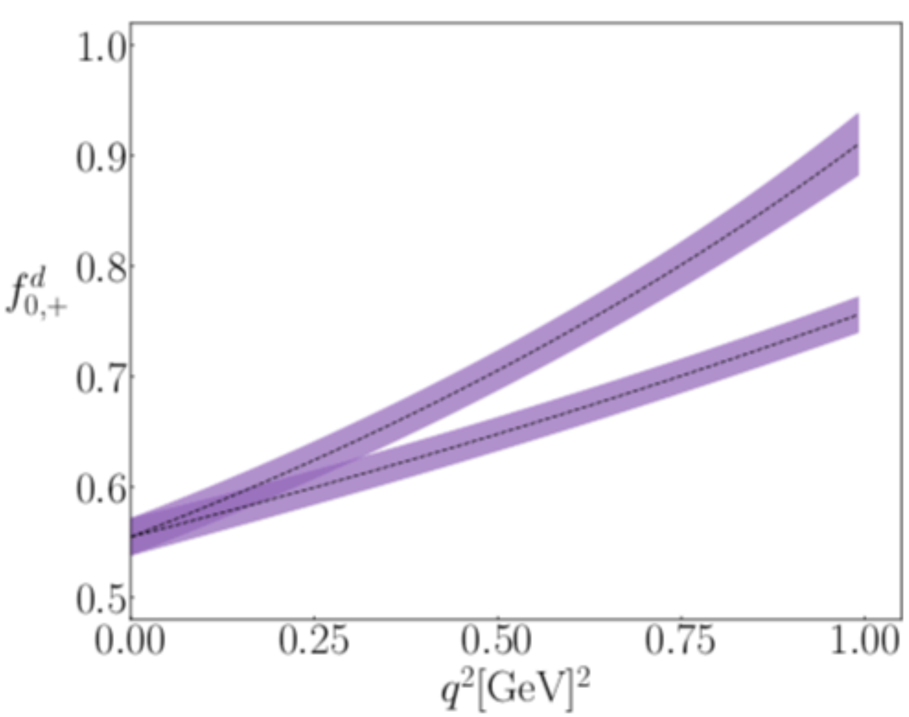
- (N_f=2+1+1) MILC HISQ ensembles
- Lattice spacings: [0.06 - 0.15] fm
- Valence bottom: NRQCD, HISQ
- $\Gamma(B_c^+ \rightarrow B_s^0 \bar{\ell} \nu) = 26.25(90)_{CKM}(83)_{latt} \times 10^9\ s^{-1}$
- $\Gamma(B_c^+ \rightarrow B^0 \bar{\ell} \nu) = 1.650(61)_{CKM}(84)_{latt} \times 10^9\ s^{-1}$

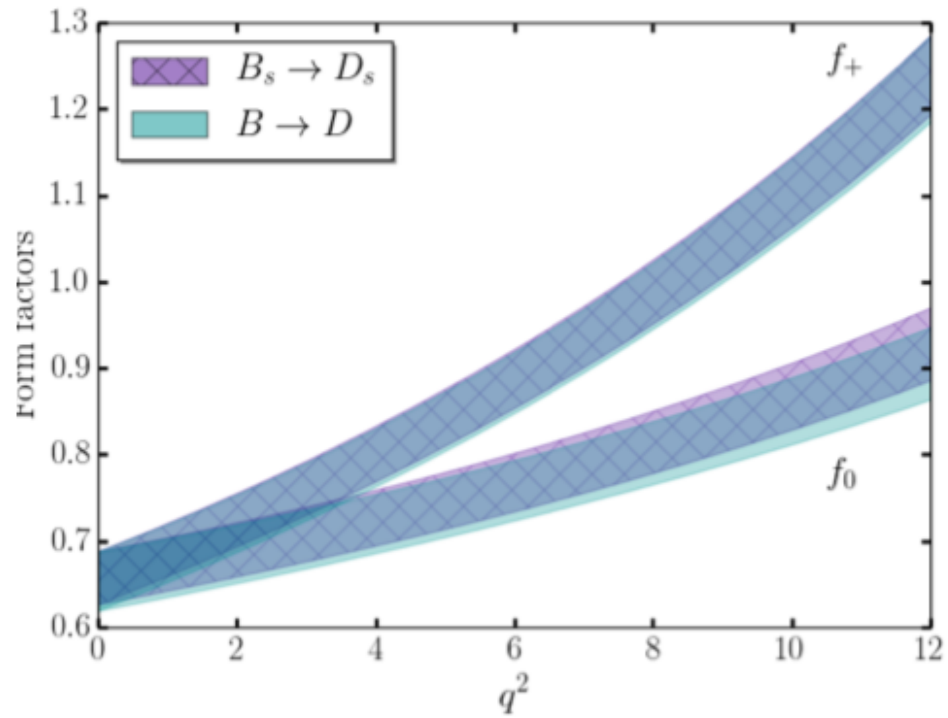
$B_s \rightarrow D_s$

HPQCD "Generation I"

arXiv:1703.09728
PRD 95 (2017) 11, 114506

- (N_f=2+1) MILC asqtad ensembles
- Lattice spacings: [0.09, 0.12] fm
- Light valence and charm: HISQ
- Heavy b: NRQCD
- G(1)=1.068(40)





Error budget, q^2=0

Type	Partial uncertainty (%)
Statistical	1.22
Chiral extrapolation	0.80
Quark mass tuning	0.66
Discretization	2.47
Kinematic	0.71
Matching	2.21
total	3.70

B to P Decays

Heavy-to-light: B→$\pi\ell\nu$, B→K$\ell^+\ell^-$ B→$\pi\ell^+\ell^-$

B→π

Zero-recoil only (q^2_{max})

FNAL/MILC **"Generation 1"**

arXiv:1503.07839
PRD 92 (2015) 1, 014024

- (N_f=2+1) MILC asqtad ensembles
- Lattice spacings: [0.045 - 0.12] fm
- Heavy b: FNAL interpretation
- $M_\pi \geq 180$ MeV
- Full physical q^2
- $|V_{ub}|=3.72(16)\times10^{-3}$

RBC/UKQCD

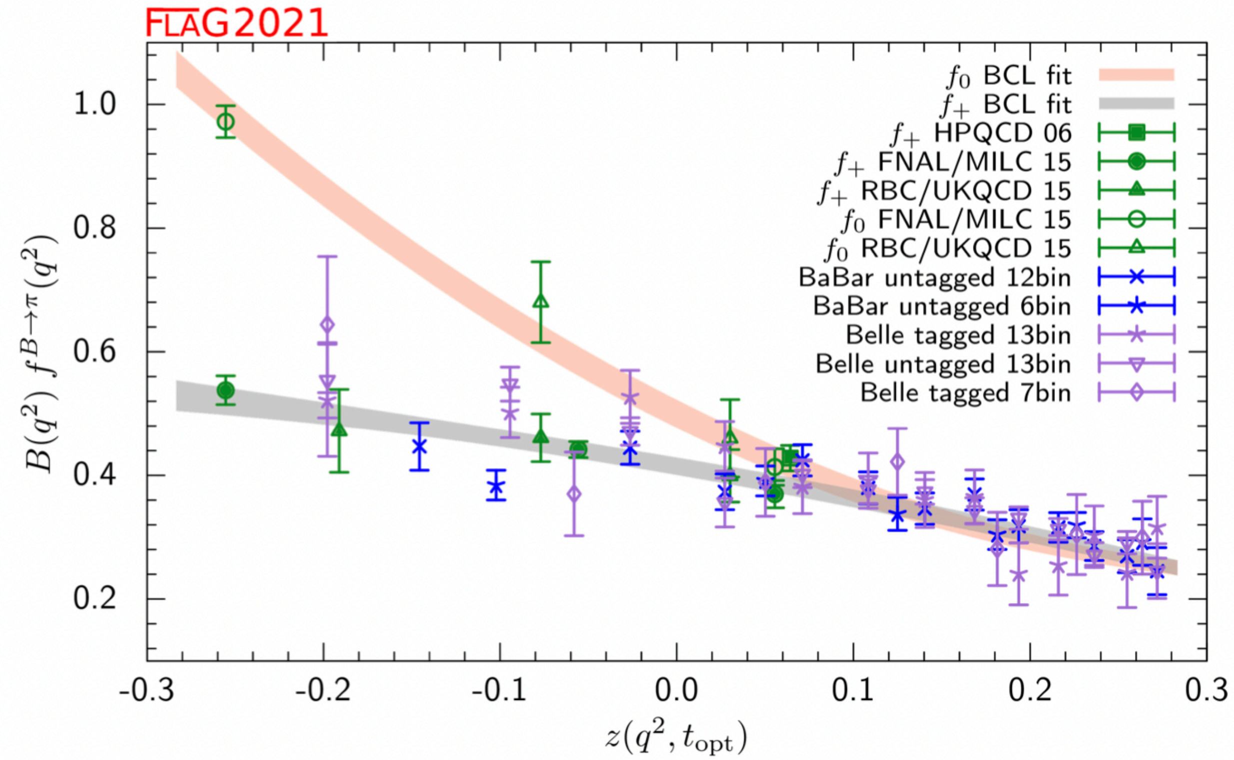
arXiv:1501.05373
PRD 91 (2015) 7, 074510

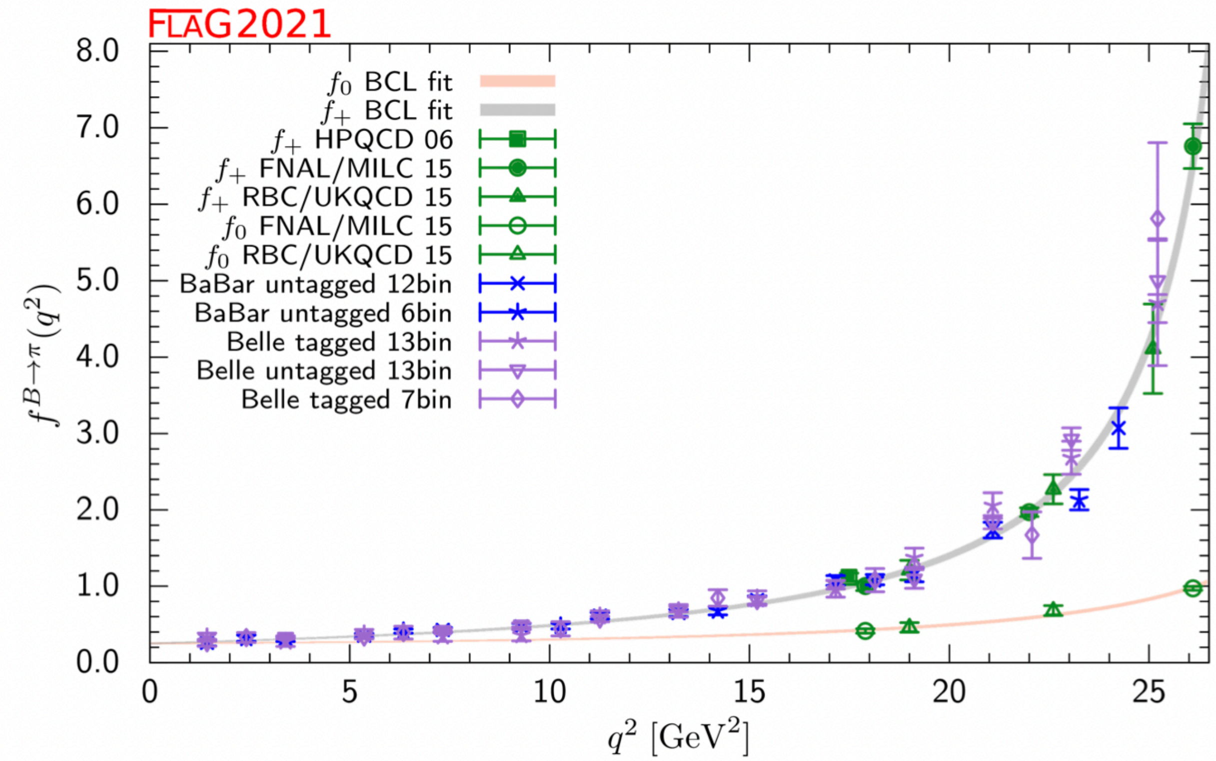
- (N_f=2+1) domain-wall fermions
- Lattice spacings: [0.09, 0.11] fm
- $M_\pi \geq 290$ MeV
- Heavy b: relativistic heavy quark
- Full physical q^2
- $|V_{ub}|=3.61(32)\times10^{-3}$

HPQCD **"Generation II"**

arXiv:1510.07446
PRD 93 (2016) 3, 034502

- (Nf=2+1+1) MILC HISQ ensembles
- Lattice spacings [0.09 - 0.15] fm
- Heavy b: NRQCD





$B \to \pi$

Zero-recoil only (q^2_{max})

FNAL/MILC "Generation 1"

arXiv:1503.07839
PRD 92 (2015) 1, 014024

- (N_f=2+1) MILC asqtad ensembles
- Lattice spacings: [0.045 - 0.12] fm
- Heavy b: FNAL interpretation
- $M_\pi \geq 180$ MeV
- Full physical q^2
- $|V_{ub}|=3.72(16)\times10^{-3}$

RBC/UKQCD

arXiv:1501.05373
PRD 91 (2015) 7, 074510

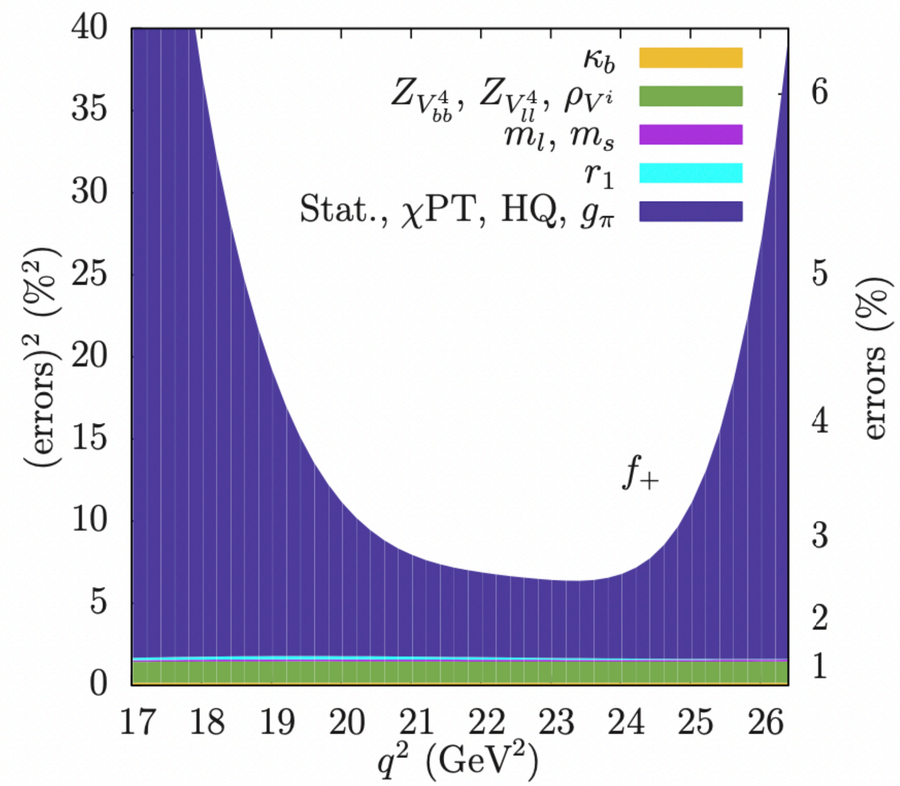
- (N_f=2+1) domain-wall fermions
- Lattice spacings: [0.09, 0.11] fm
- $M_\pi \geq 290$ MeV
- Heavy b: relativistic heavy quark
- Full physical q^2
- $|V_{ub}|=3.61(32)\times10^{-3}$

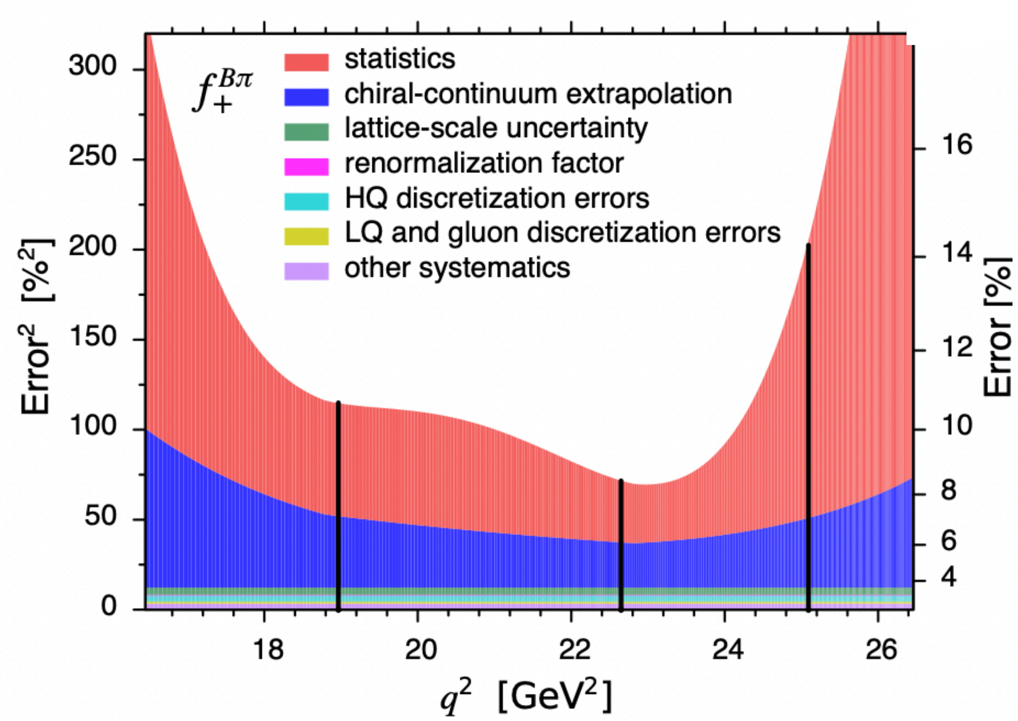
HPQCD "Generation II"

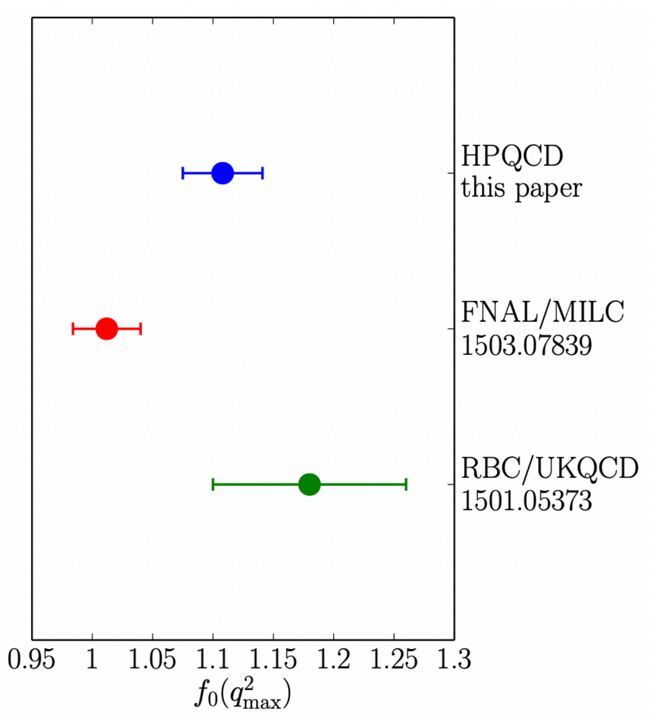
arXiv:1510.07446
PRD 93 (2016) 3, 034502

- (N_f=2+1+1) MILC HISQ ensembles
- Lattice spacings [0.09 - 0.15] fm
- Heavy b: NRQCD

$$\left.\frac{f_0^{B\to\pi}(q^2_{max})}{f_B/f_\pi}\right|_{m_\pi=0} = 0.987(51)$$





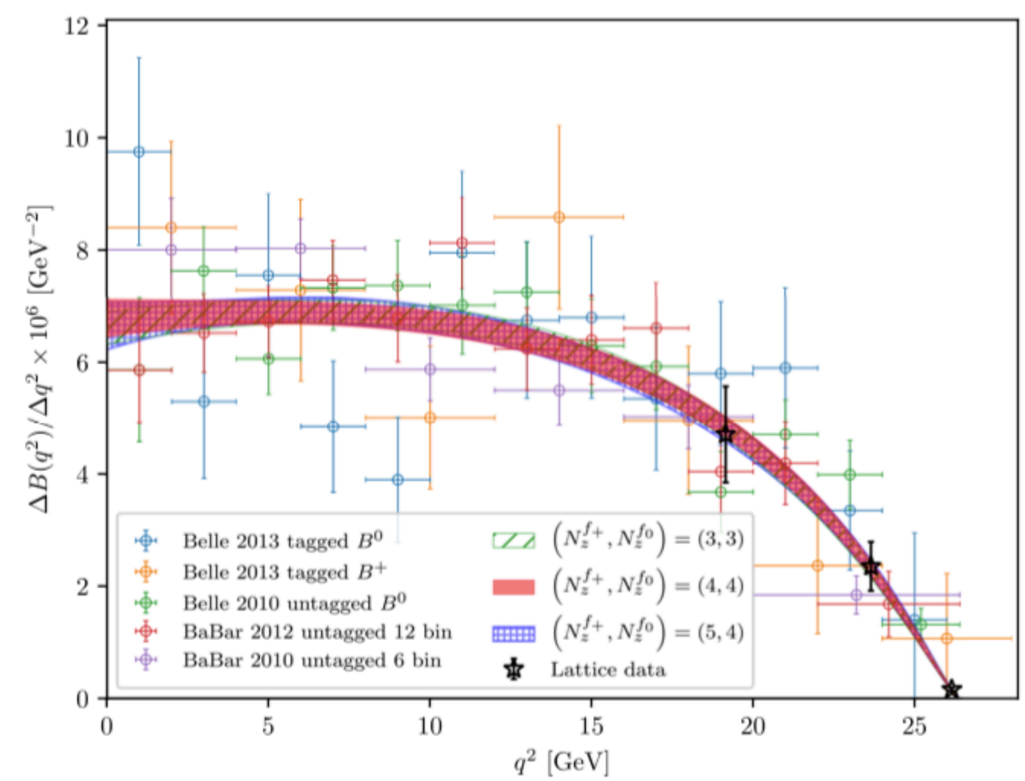


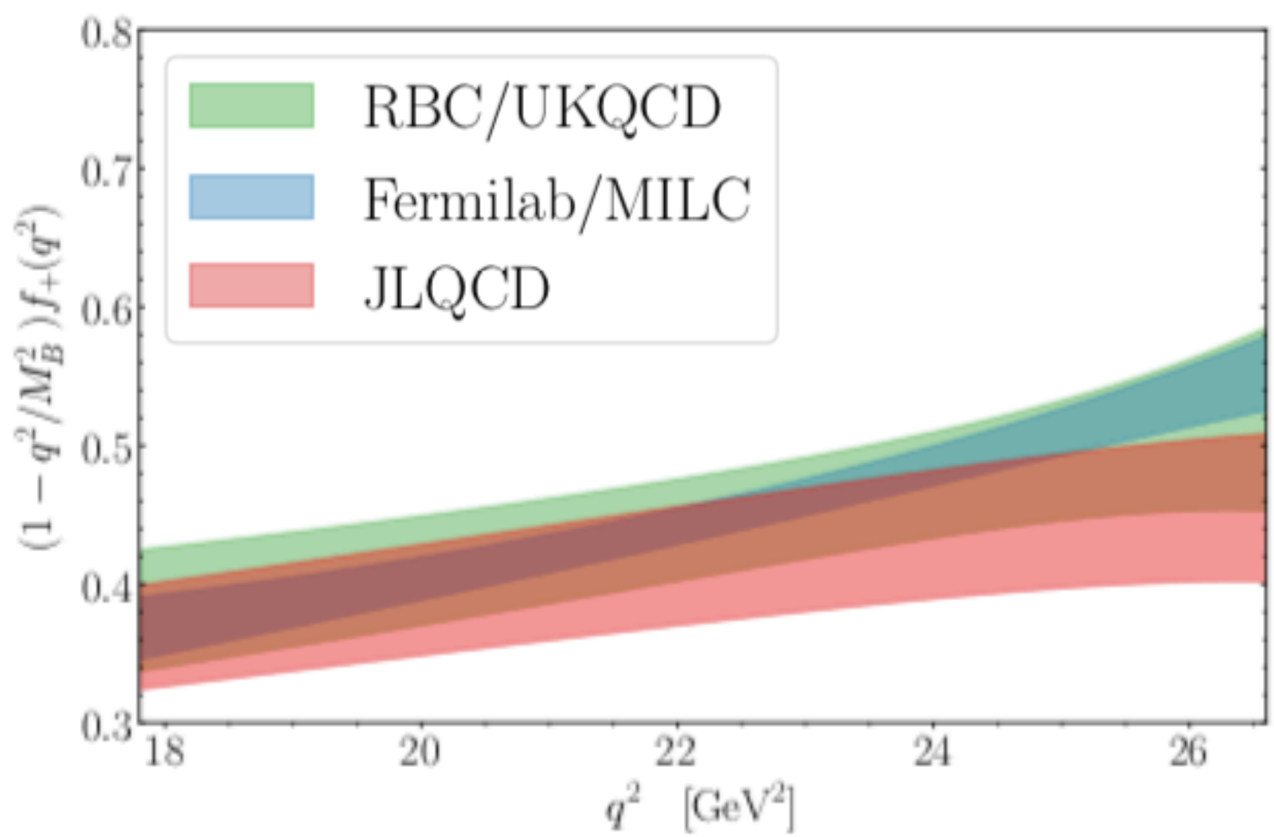
B→π (New!)

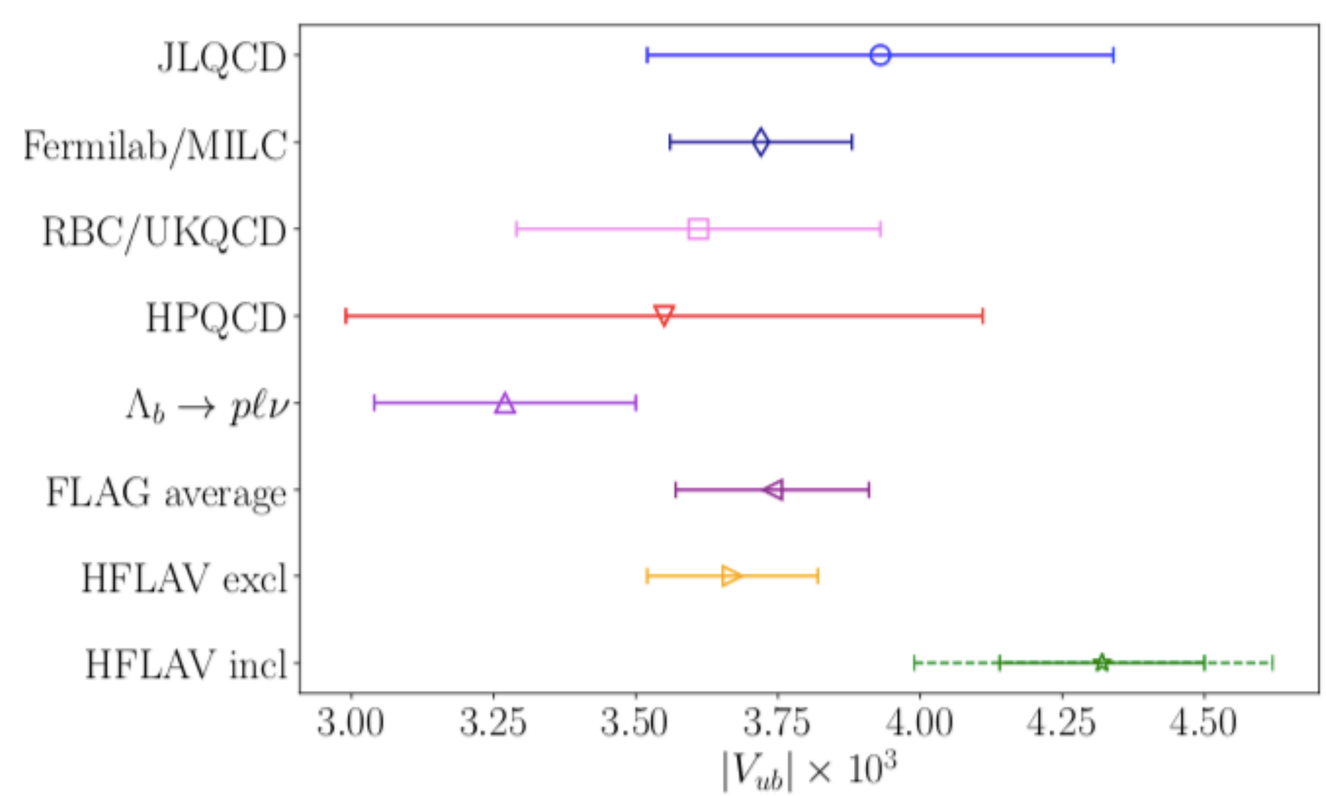
JLQCD

arXiv:2203.04938

- (Nf=2+1) Möbius domain wall fermions
- Lattice spacings [0.044 - 0.08] fm
- $M_\pi \geq 225$ MeV







$B_s \to K$

FNAL/MILC "Generation 1"

arXiv:1901.02561
PRD 100 (2019) 3, 034501

- (N_f=2+1) MILC asqtad ensembles
- Lattice spacings [0.06, 0.09, 0.12] fm
- $M_\pi \geq 180$ MeV
- Heavy b: FNAL interpretation

RBC/UKQCD

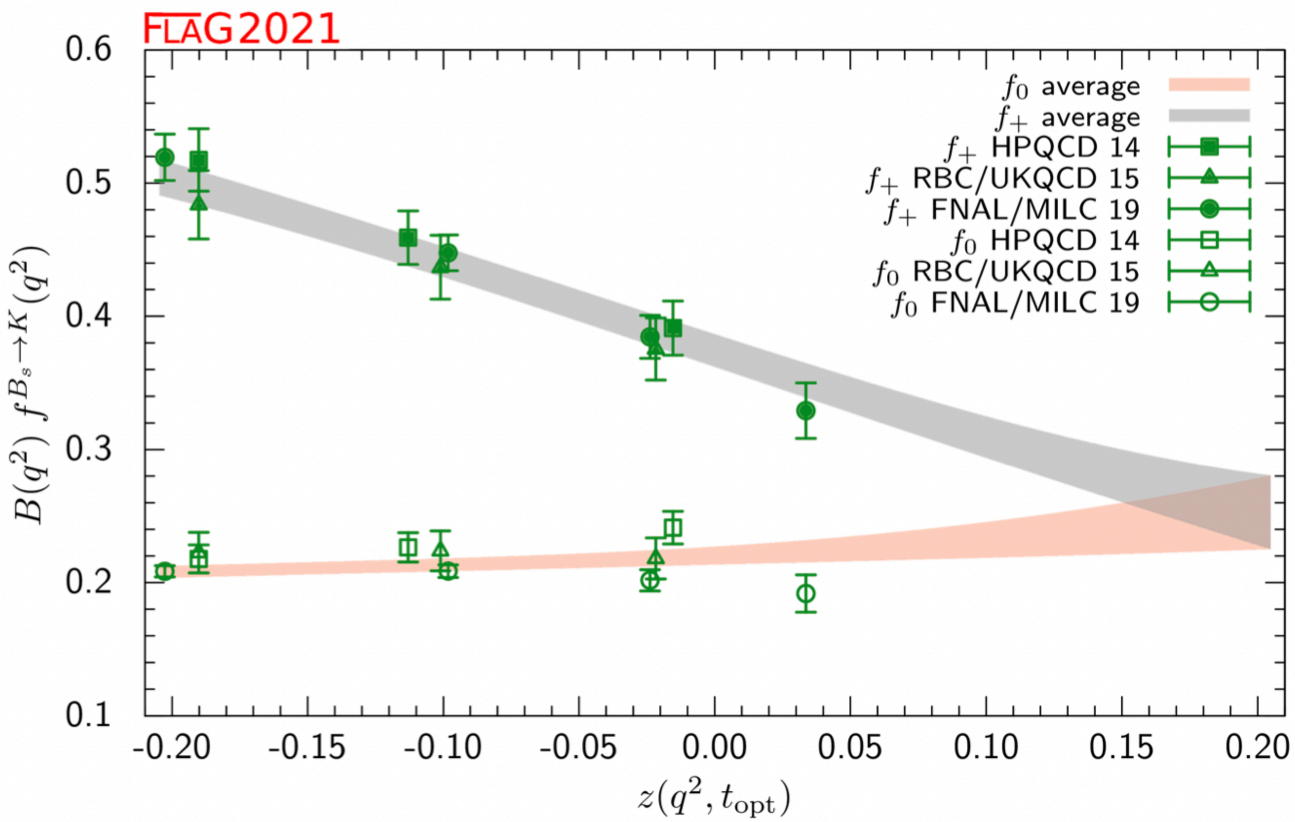
arXiv:1501.05373
PRD 91 (2015) 7, 074510

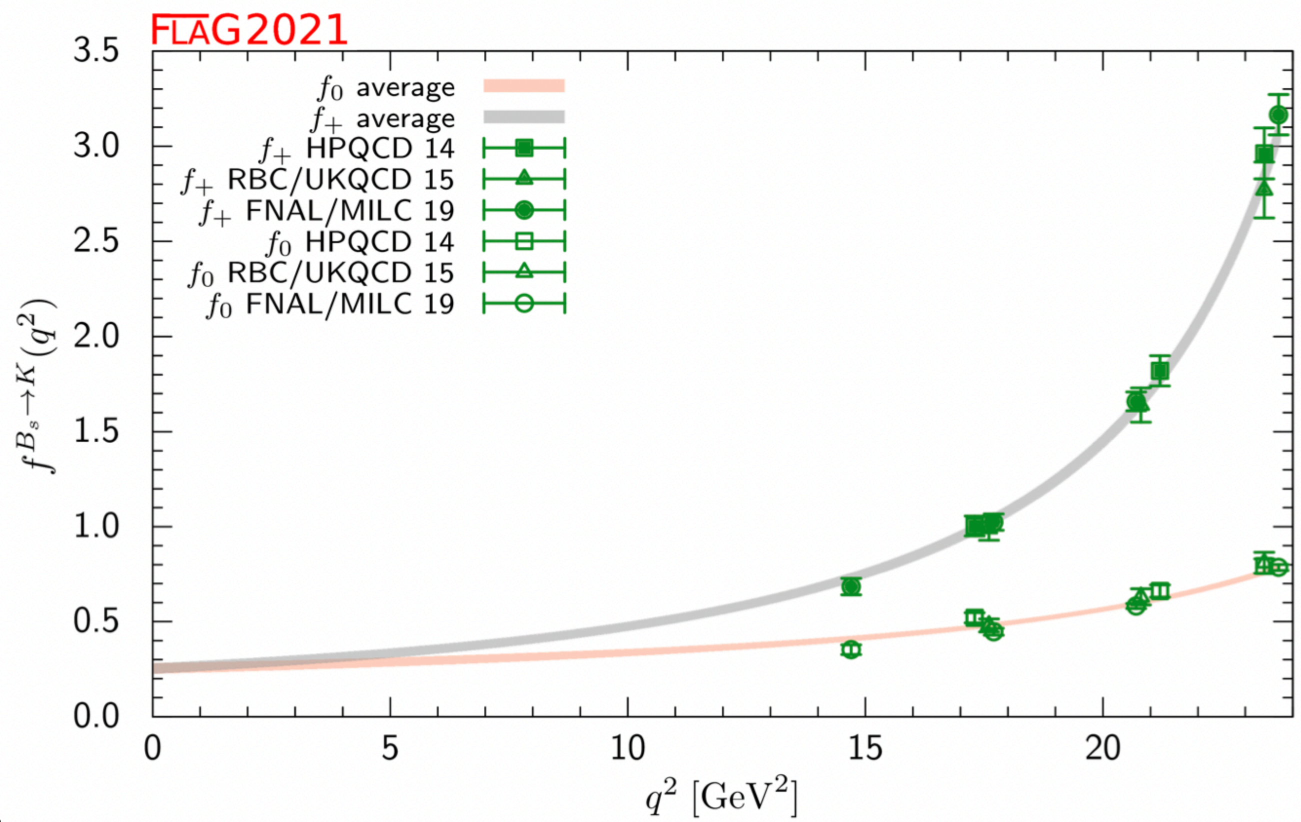
- (N_f=2+1) domain-wall fermions
- Lattice spacings: [0.09, 0.11] fm
- $M_\pi \geq 290$ MeV
- Heavy b: relativistic heavy quark
- Full physical q^2
- $|V_{ub}|=3.61(32)\times 10^{-3}$

HPQCD "Generation I"

arXiv:1406.2279
PRD 90 (2014) 054506

- (N_f=2+1) MILC asqtad ensembles
- Lattice spacings [0.06, 0.09, 0.12] fm
- $M_\pi \geq 260$ MeV
- Heavy b: NRQCD





$B_s \to K$

FNAL/MILC "Generation 1"

arXiv:1901.02561
PRD 100 (2019) 3, 034501

- (Nf=2+1) MILC asqtad ensembles
- Lattice spacings [0.06, 0.09, 0.12] fm
- $M_\pi \geq 180$ MeV
- Heavy b: FNAL interpretation

RBC/UKQCD

arXiv:1501.05373
PRD 91 (2015) 7, 074510

- (N_f=2+1) domain-wall fermions
- Lattice spacings: [0.09, 0.11] fm
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HPQCD "Generation I"

arXiv:1406.2279
PRD 90 (2014) 054506

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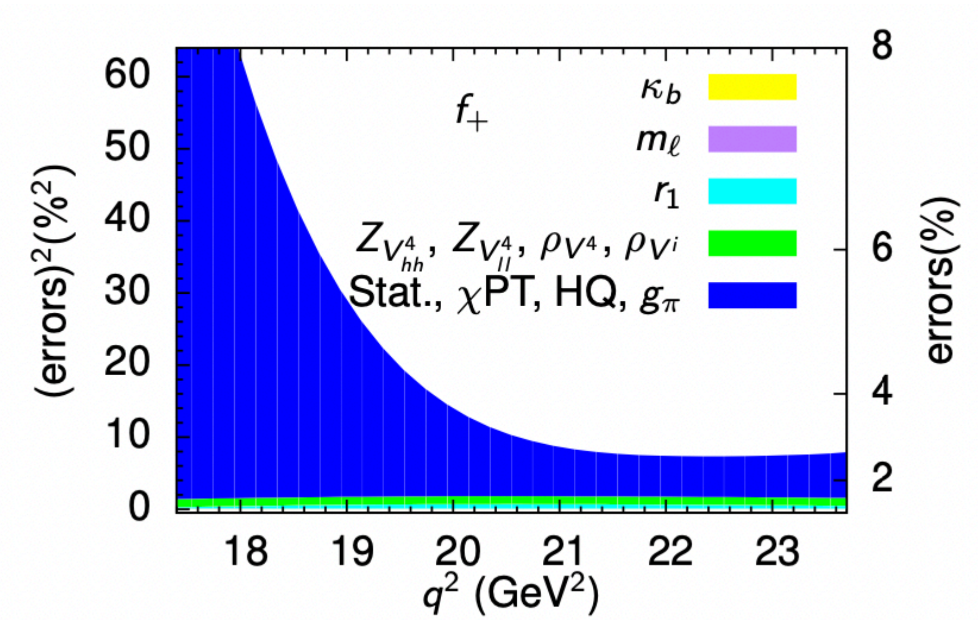
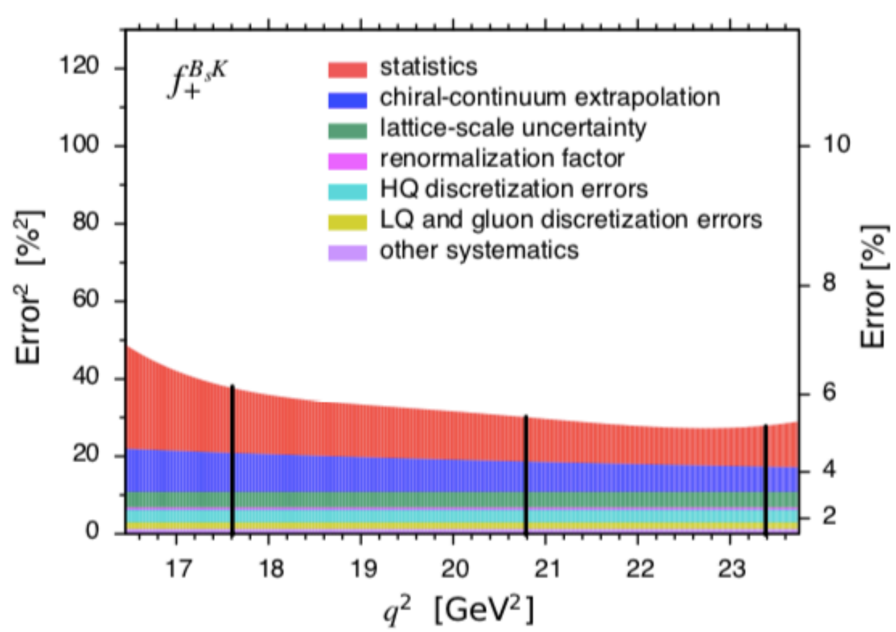
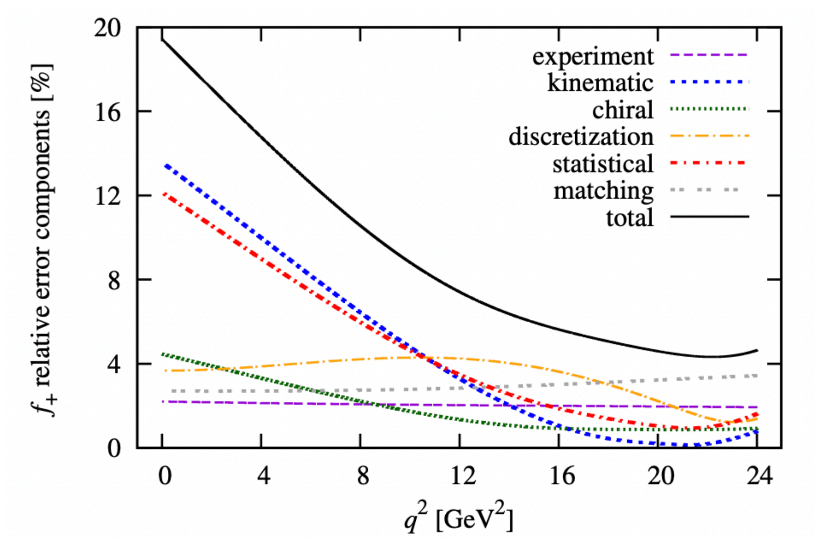






B→Kℓ+ℓ-

FNAL/MILC “Generation 1”

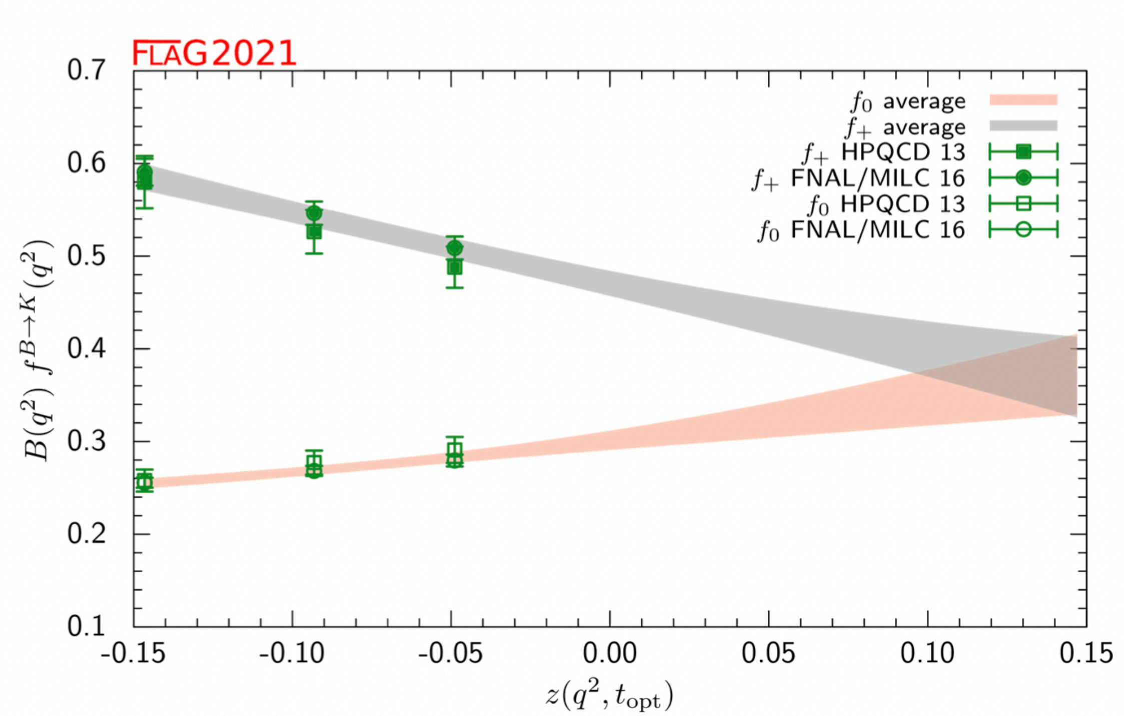
arXiv:1509.06235
PRD 93 (2016) 2, 025026

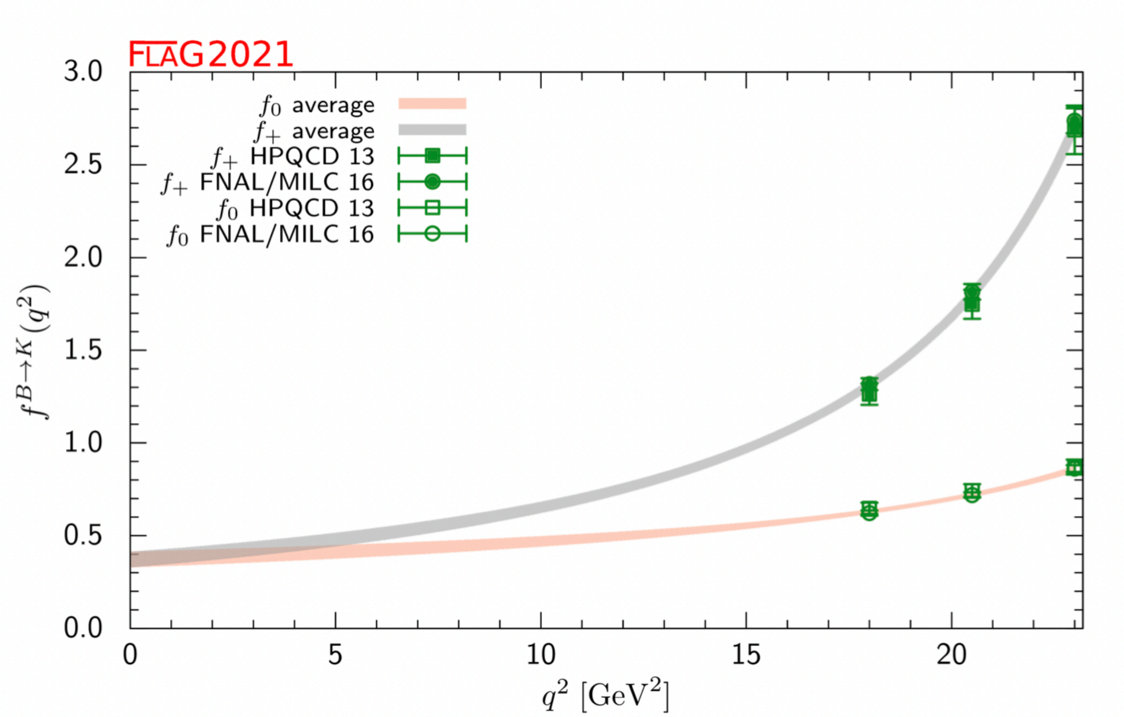
- (N_f=2+1) MILC asqtad ensembles
- Lattice spacings: [0.045 - 0.12] fm
- Heavy b: FNAL interpretation
- $M_\pi \geq$ 280 MeV

HPQCD “Generation I”

arXiv:1306.2384
PRD 88 (2013) 5, 054509

- (Nf=2+1) MILC asqtad ensembles
- Lattice spacings [0.09 - 0.12] fm
- Heavy b: NRQCD





B→K$\ell^+\ell^-$

FNAL/MILC “Generation 1”

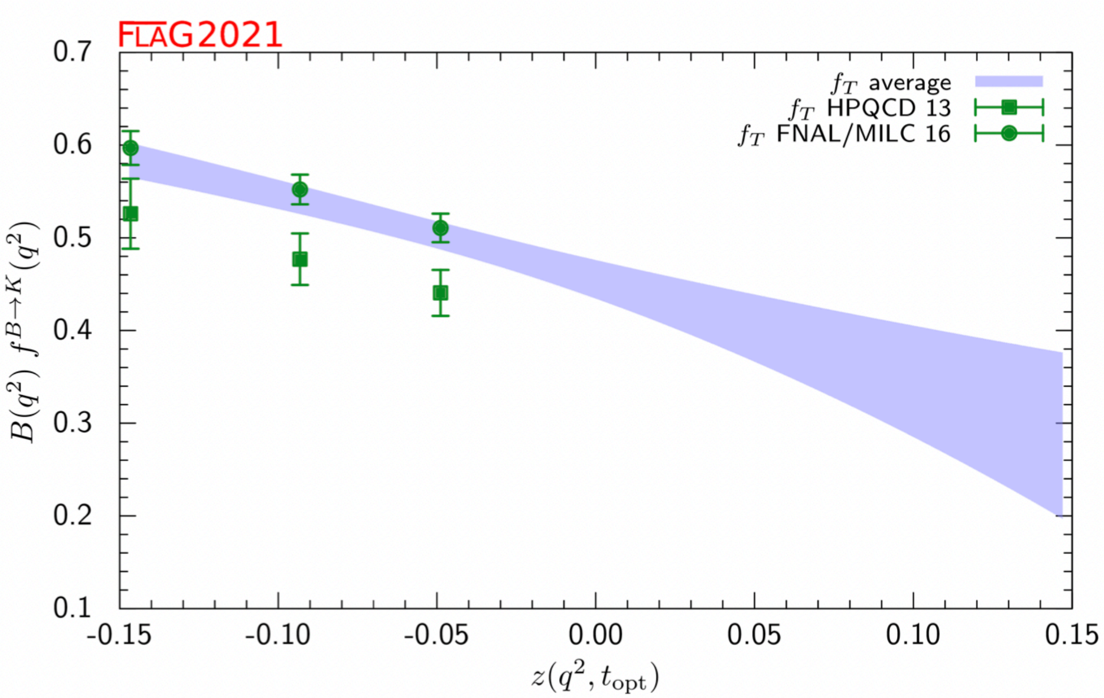
arXiv:1509.06235
PRD 93 (2016) 2, 025026

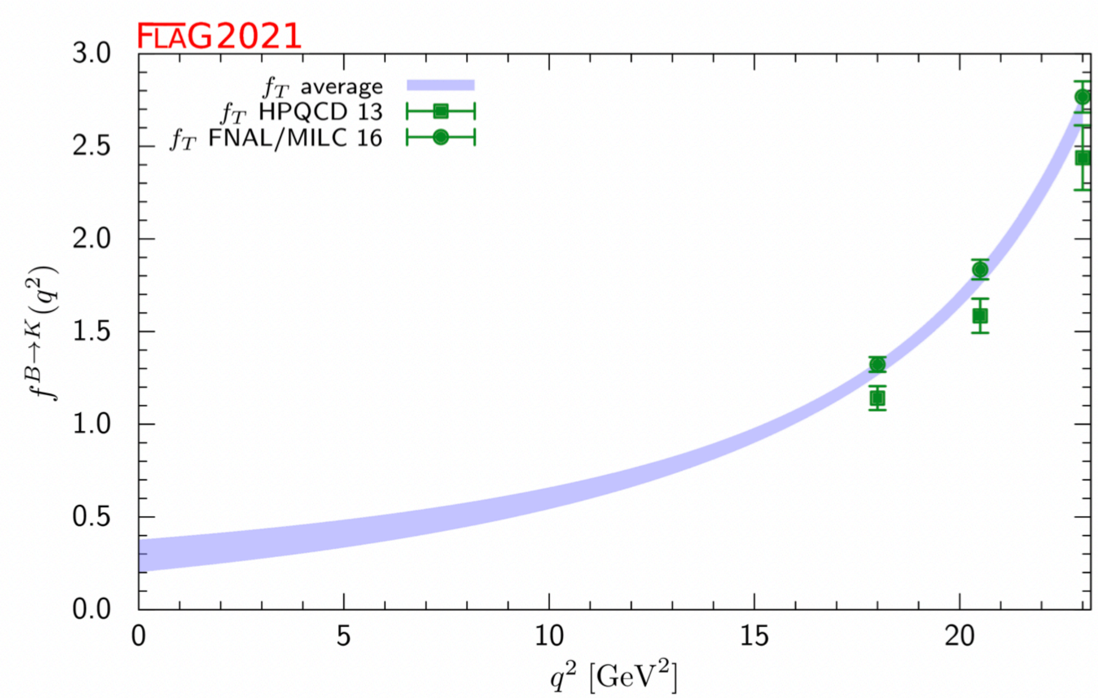
- (N_f=2+1) MILC asqtad ensembles
- Lattice spacings: [0.045 - 0.12] fm
- Heavy b: FNAL interpretation
- $M_\pi \geq$ 280 MeV

HPQCD “Generation I”

arXiv:1306.2384
PRD 88 (2013) 5, 054509

- (Nf=2+1) MILC asqtad ensembles
- Lattice spacings [0.09 - 0.12] fm
- Heavy b: NRQCD



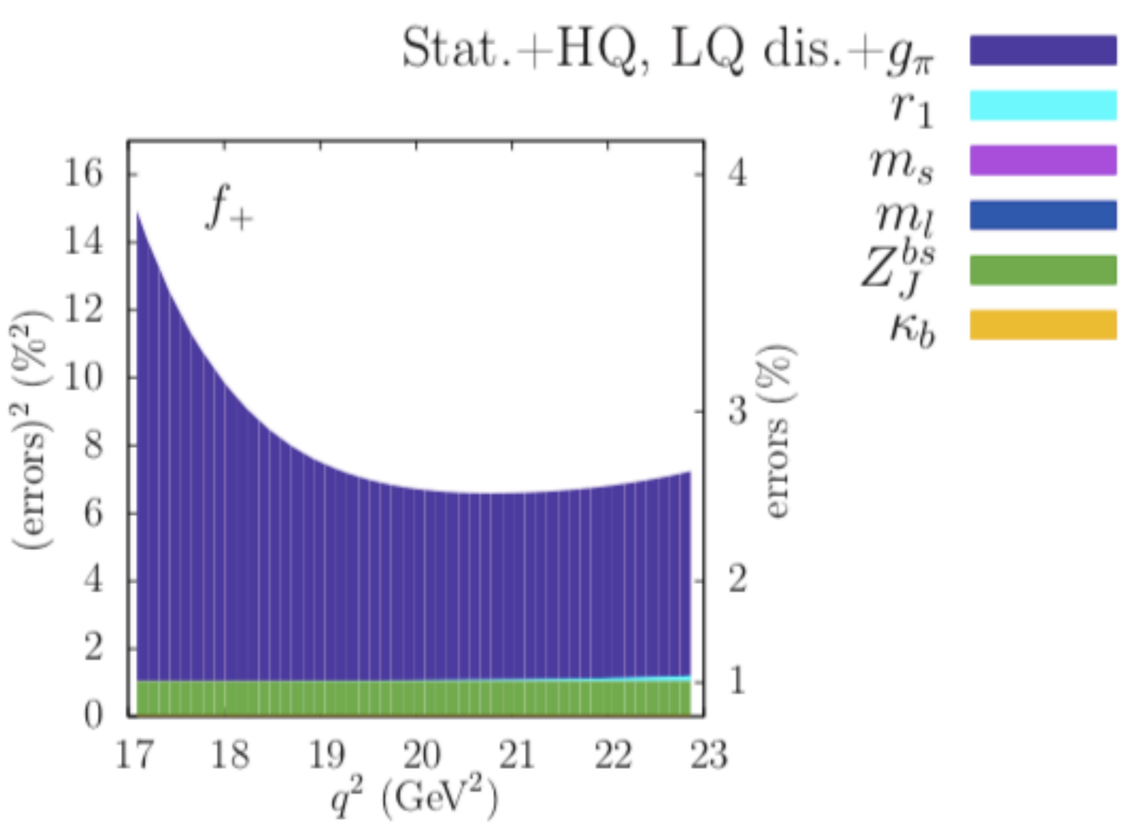


B→K$\ell^+\ell^-$

FNAL/MILC "Generation 1"

arXiv:1509.06235
PRD 93 (2016) 2, 025026

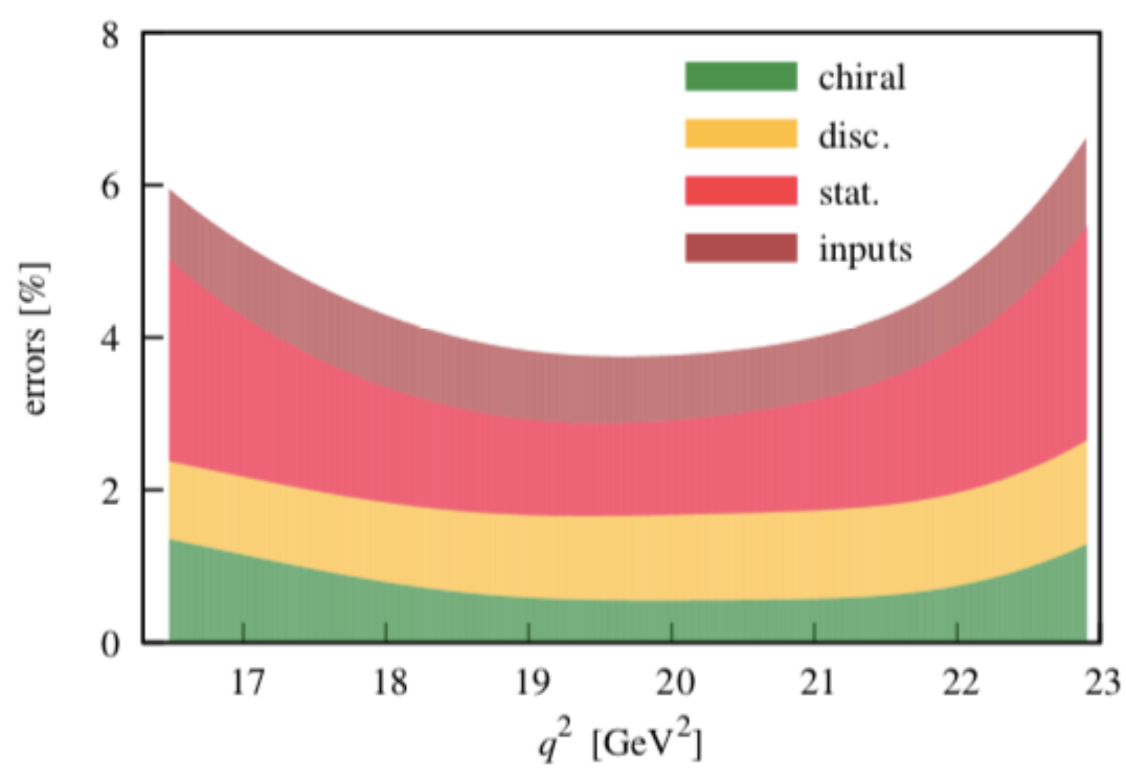
- (N_f=2+1) MILC asqtad ensembles
- Lattice spacings: [0.045 - 0.12] fm
- Heavy b: FNAL interpretation
- $M_\pi \geq$ 280 MeV



HPQCD "Generation I"

arXiv:1306.2384
PRD 88 (2013) 5, 054509

- (Nf=2+1) MILC asqtad ensembles
- Lattice spacings [0.09 - 0.12] fm
- Heavy b: NRQCD

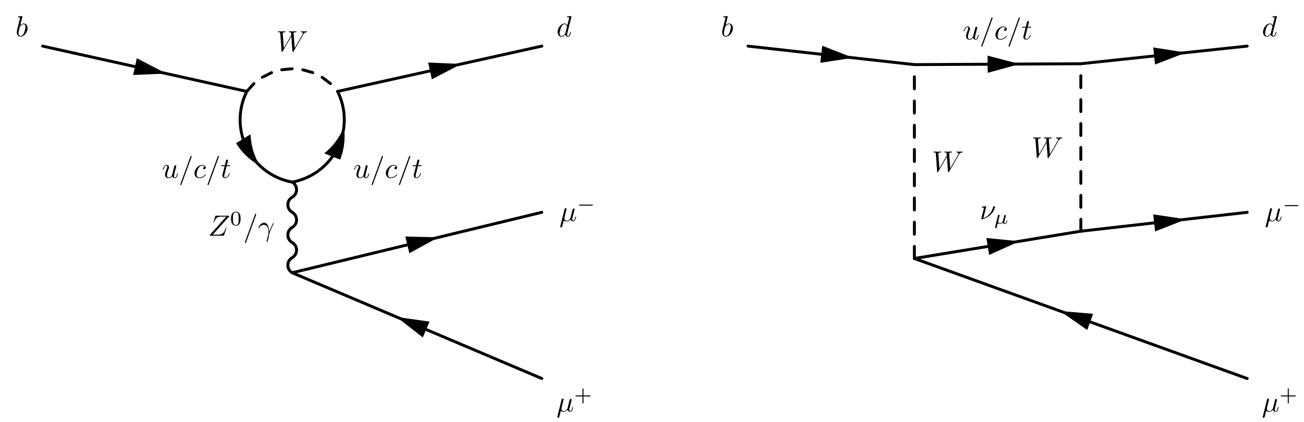


B→$\pi\ell^+\ell^-$

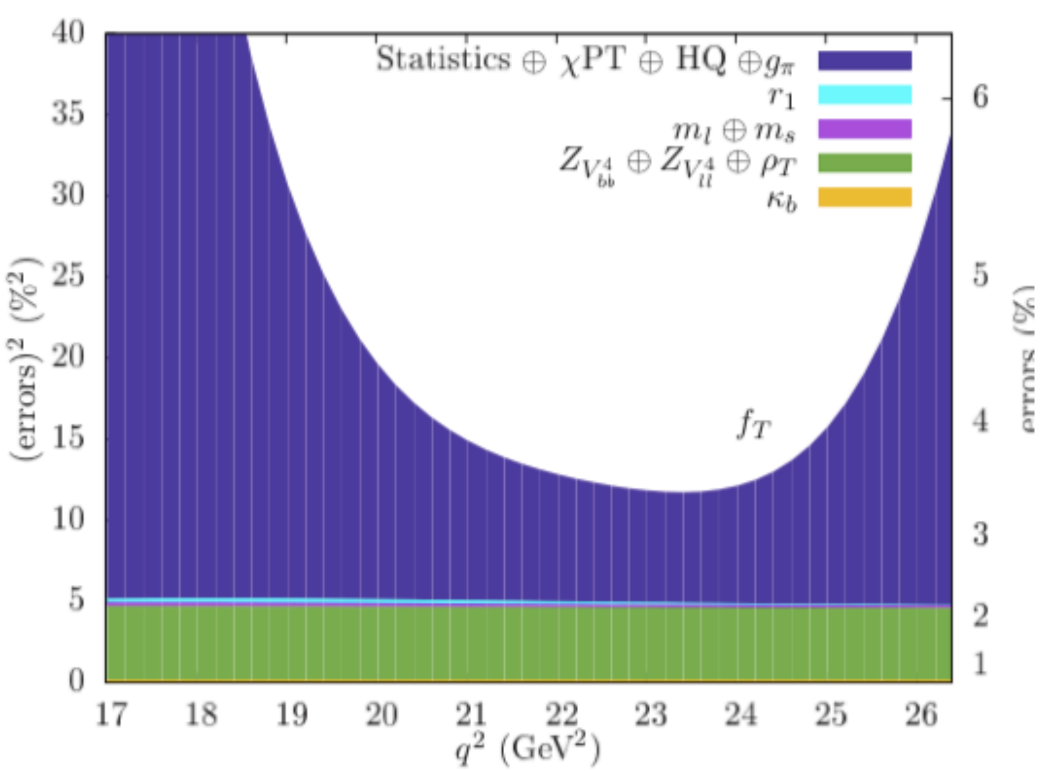
FNAL/MILC "Generation 1"

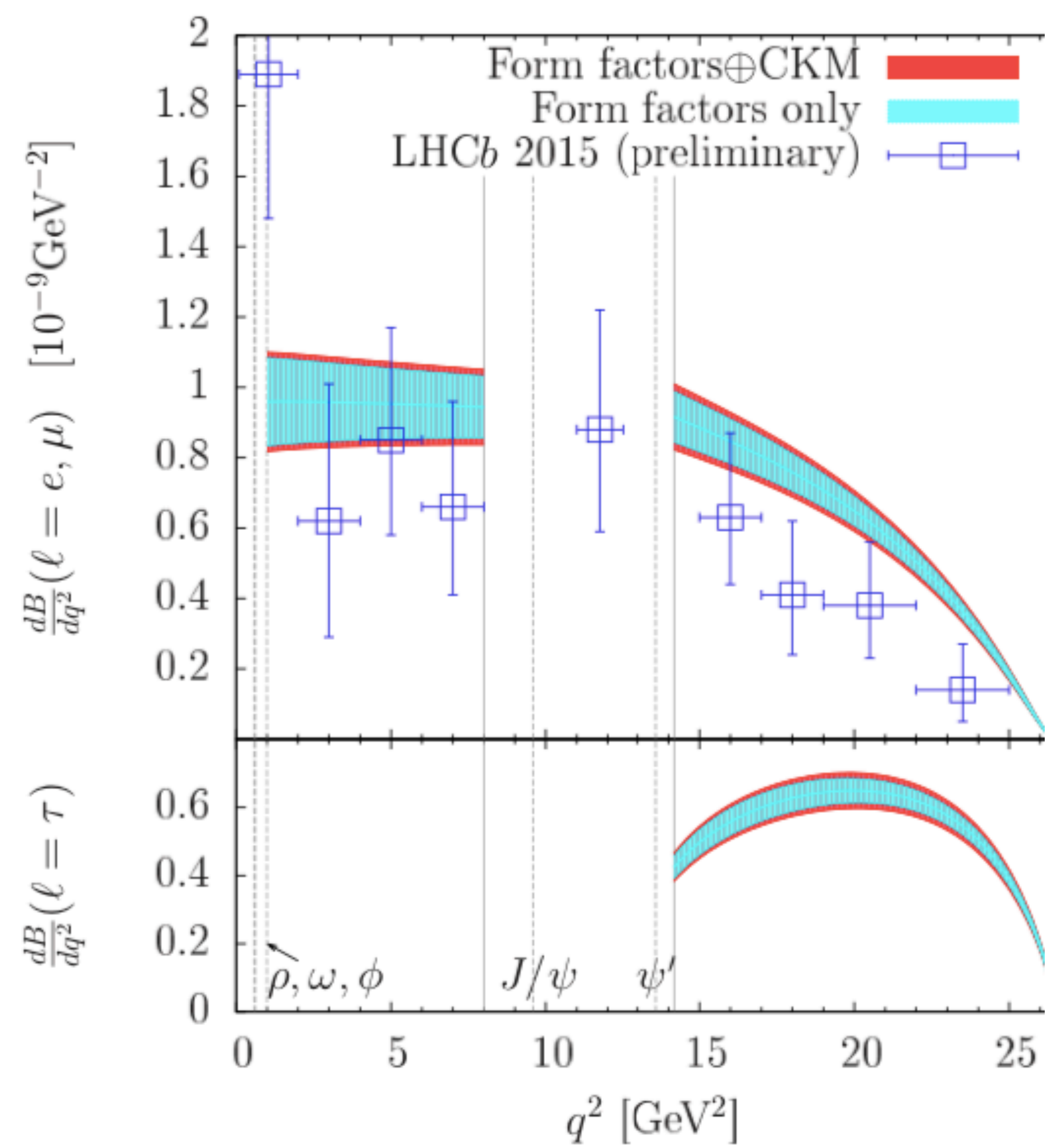
arXiv:1509.06235

PRL 115 (2015) 15, 152002



- (N_f=2+1) MILC asqtad ensembles
- Lattice spacings: [0.045 - 0.12] fm
- Heavy b: FNAL interpretation
- $M_\pi \geq$ 175 MeV





FNAL/MILC all-HISQ campaign

- **Campaign goal**: precise (≲ 1%) form factors for decays of B and D meson to pseudoscalars
  - D mesons: $D_{(s)} \to \pi$, K
  - B mesons: $B_{(s)} \to D_{(s)}$, π, K
  - Full set of scalar, vector, and tensor currents
- Ensembles: $N_f = (2+1+1)$ dynamical sea quarks generated by the MILC collaboration
- Valence quarks:
  - Light and strange quarks match the sea
  - Heavy quarks: range from 0.9 m_c up to cutoff ($ma \sim 1$)
- Eventual target: lattice spacings from 0.15 fm—0.03 fm
- **All 3pt functions are fully blinded**

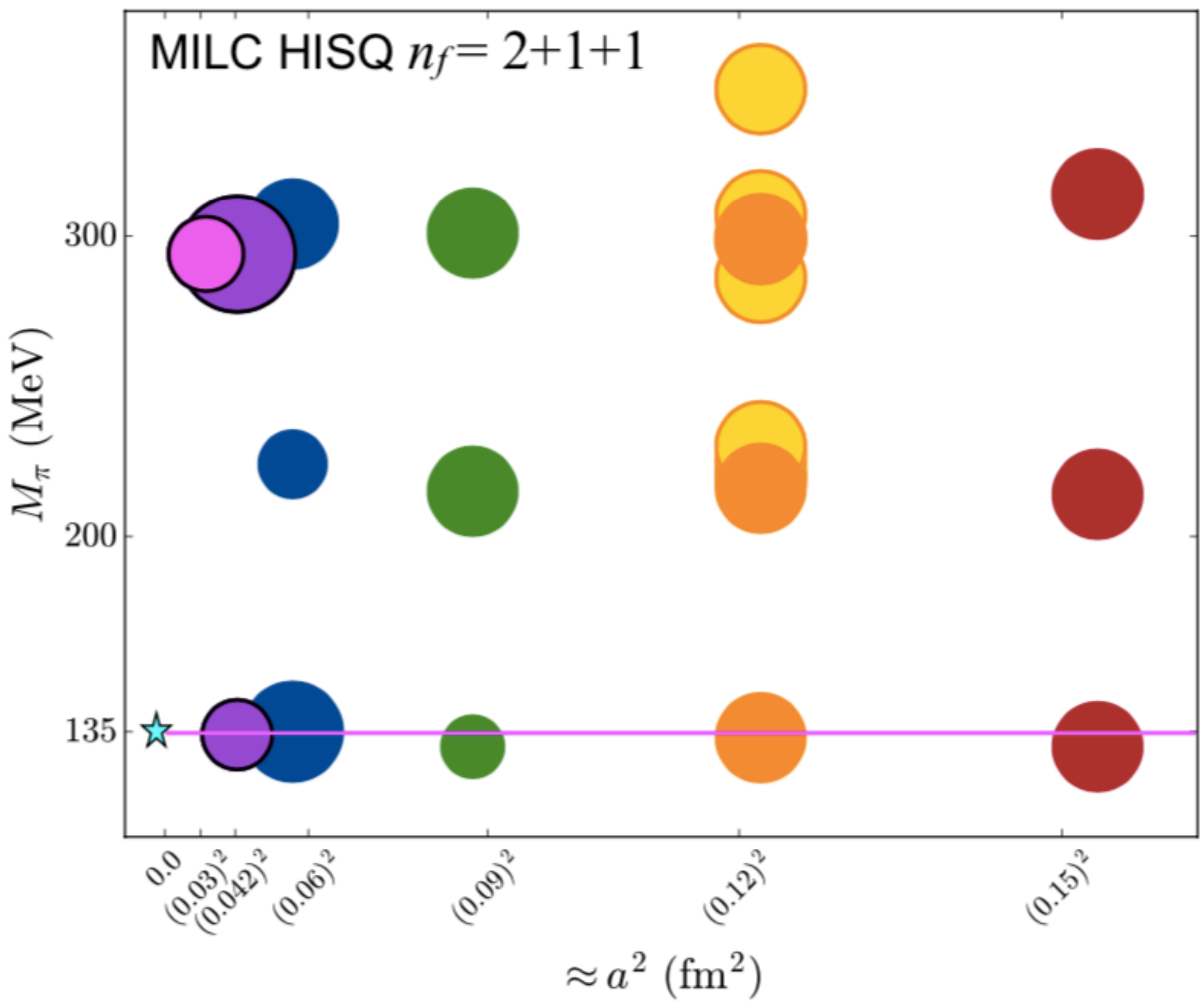
FNAL/MILC all-HISQ campaign

all-HISQ working group:

- Carleton DeTar
- Aida El-Khadra
- Elvira Gamiz
- Steve Gottlieb
- Jim Simone
- WJ
- Andreas Kronfeld
- Andrew Lytle
- Alex Vaquero

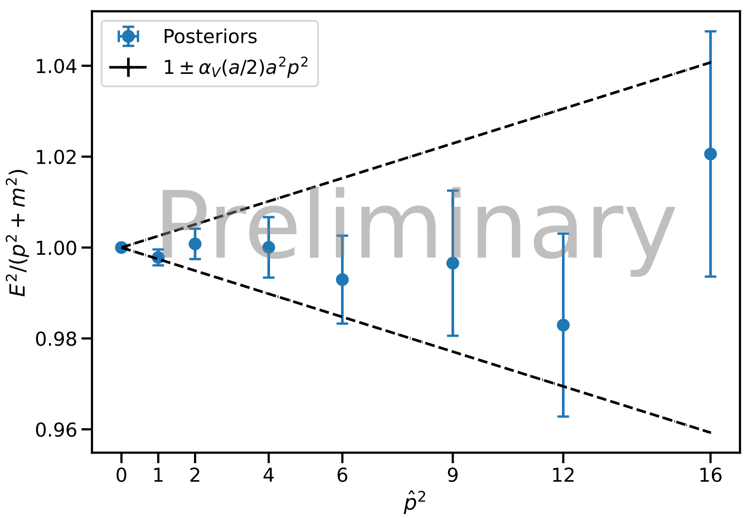
Analysis leads:
D-decays: WJ
B-decays:

Andrew Lytle (UIUC)

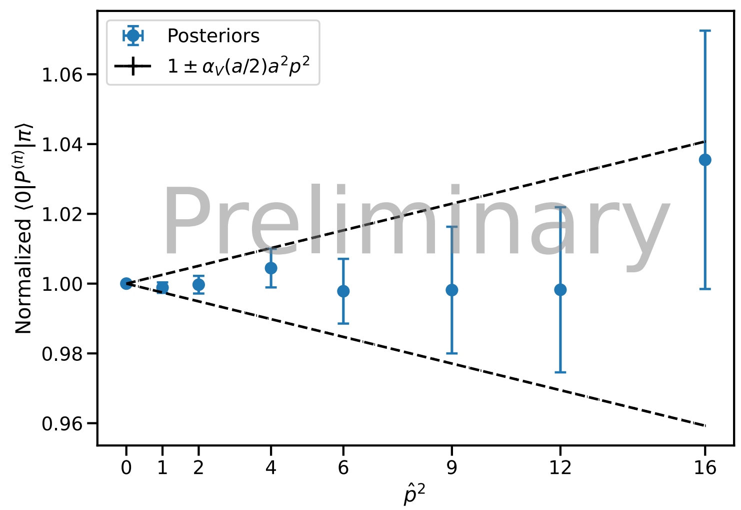


FNAL/MILC all-HISQ campaign

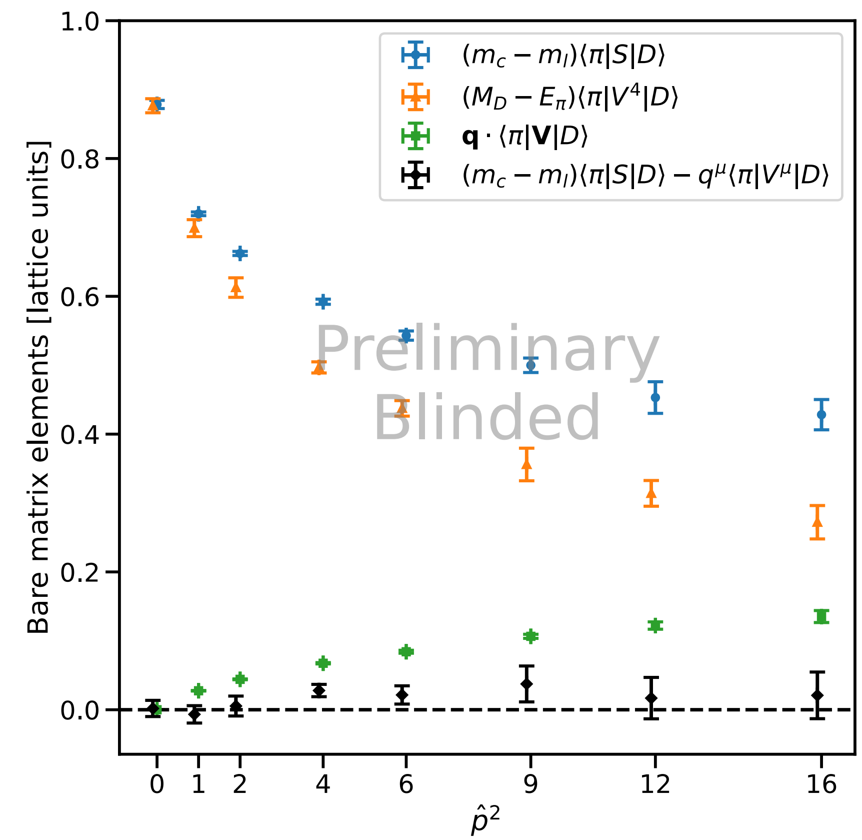
$E^2/(p^2+m^2)$ vs. p^2



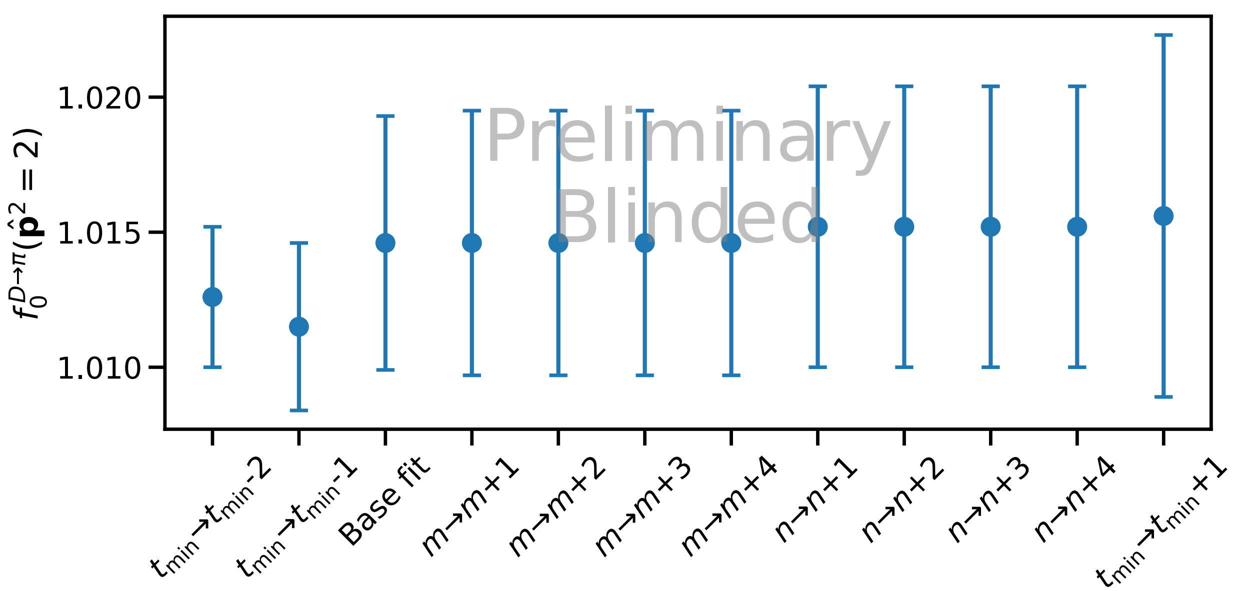
$\langle 0|\mathcal{O}_\pi|\pi\rangle$ vs. p^2



Test of Ward identity: $\langle \pi|J|D\rangle$



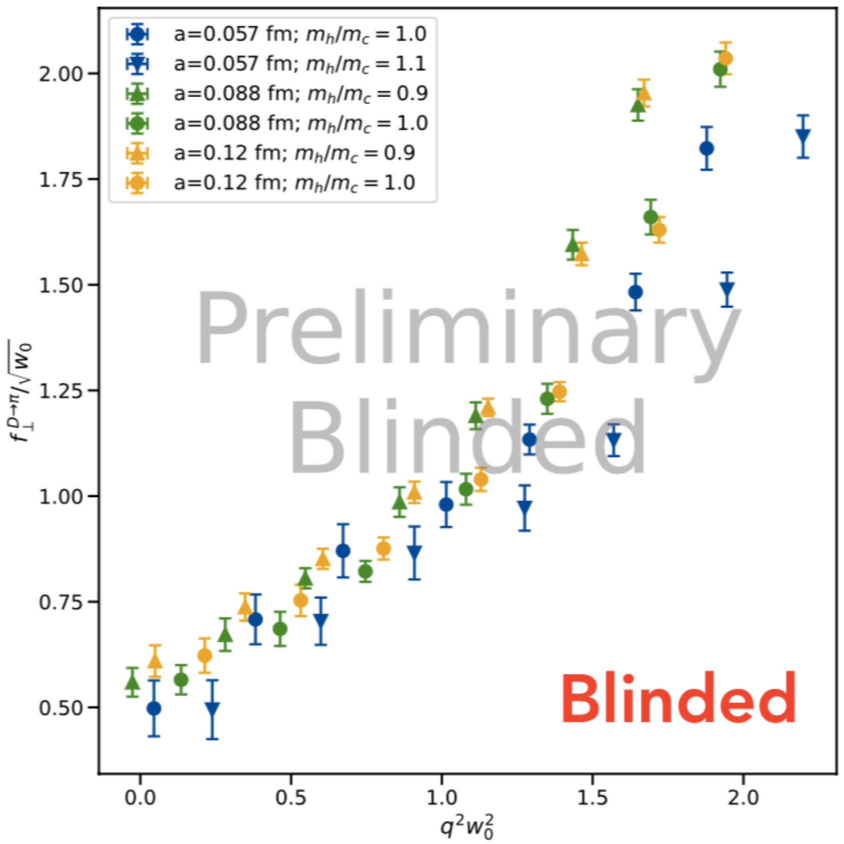
Stability of fit for $f_0^{D\to\pi}$ to 2pt+3pt functions, varying fit parameters (# excited states, t_{min})



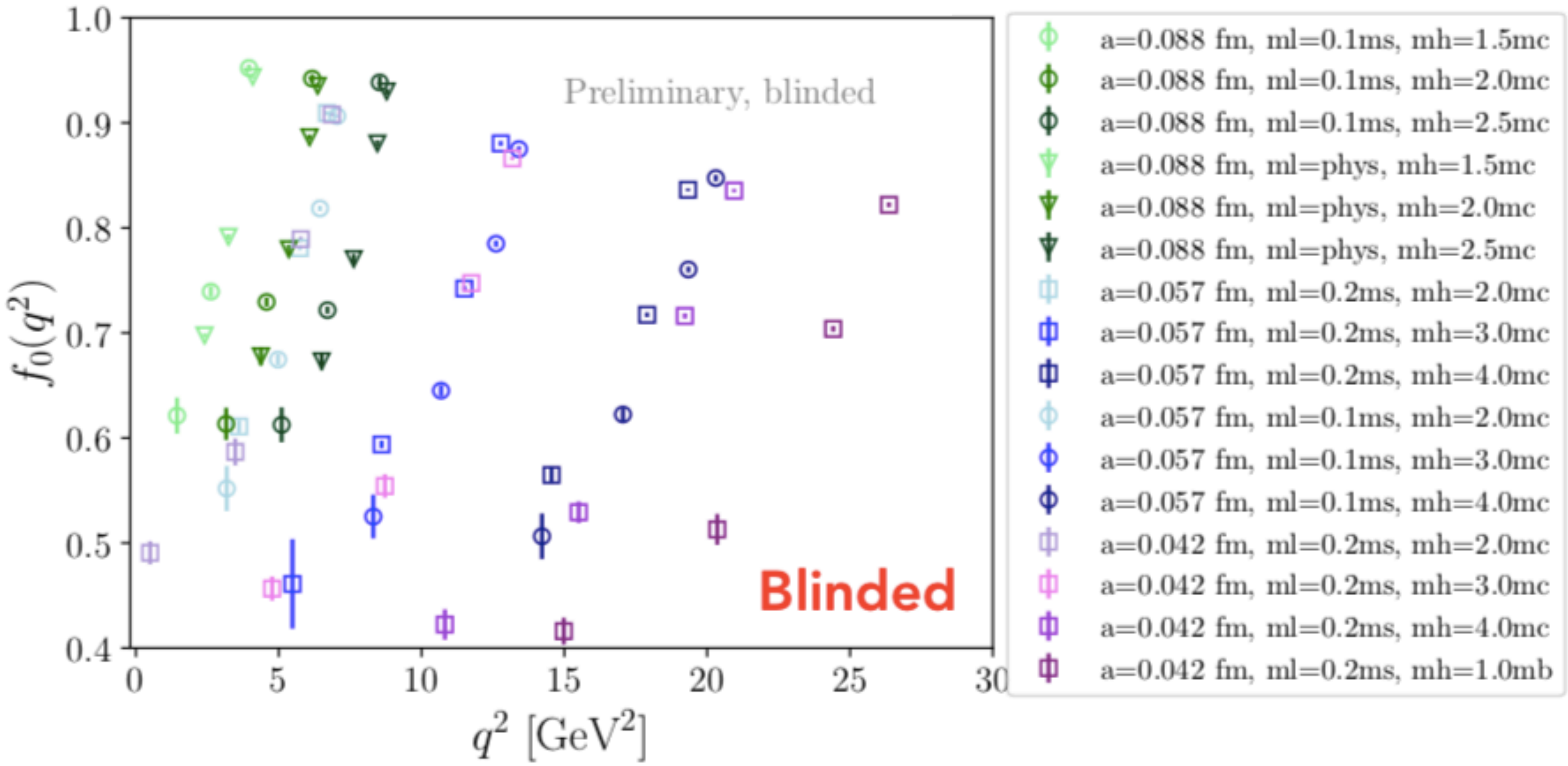
- **Small statistical uncertainties**
- **Small discretization effects**
- **Renormalization $Z_V \approx 1$**

FNAL/MILC all-HISQ campaign

$f_\perp^{D\to\pi}(q^2)$



$f_0^{H_s\to K}(q^2)$



In progress:

Combined chiral interpolation + continuum extrapolation

Summary

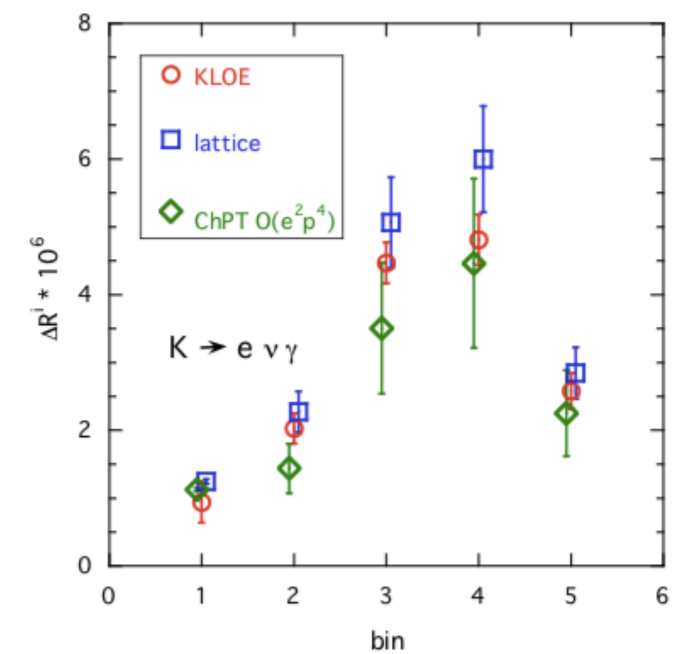
- Interesting tensions are driving scientific efforts in B-physics
- Recent/upcoming experiments are expected to deliver ~1% precision for decays rates like $B \rightarrow D^{(*)}$, $B \rightarrow \pi$
- Lattice QCD calculations of form factors for semileptonic decays of B-mesons are reaching a high level of maturity and precision
- The lattice QCD community is on track to match the expected improvements from experiments

Backup

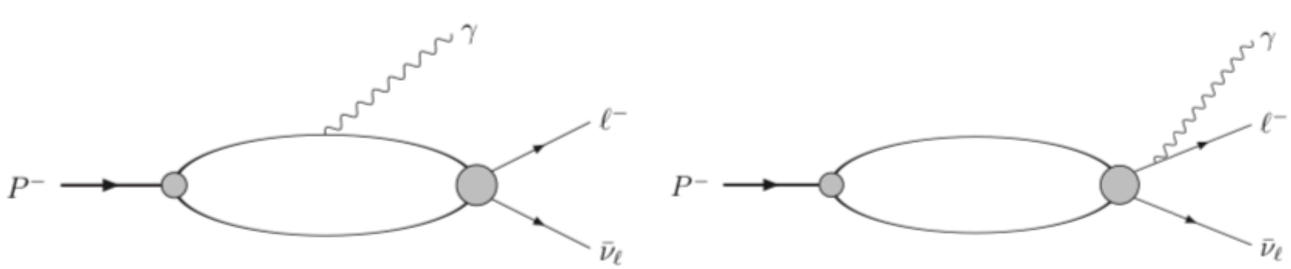
Radiative Decays: P→ℓνγ

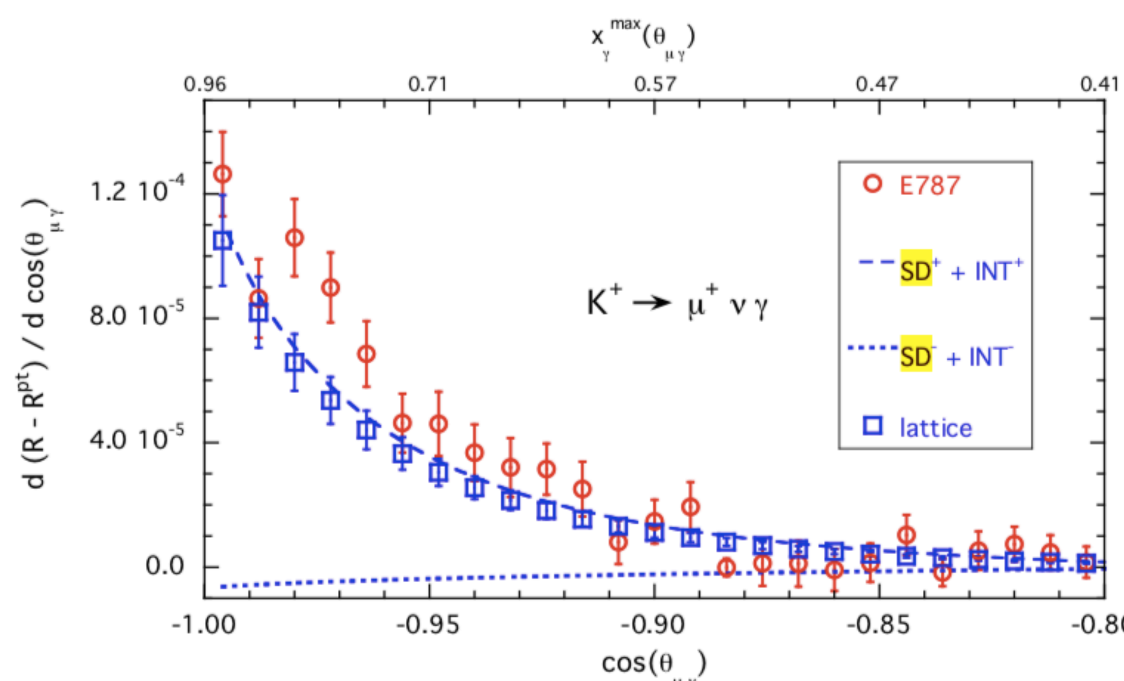
arXiv:2006.05358
PRD 103 (2021) 1, 014502
arXiv:2012.02120
PRD 103 (2021) 5, 053005

- Rome-Southampton
- Ensembles: 12x (N_f=2+1+1) ETMC
- a ≈ 0.6, 0.8, 0.9 fm, m_π = 230 - 450 MeV
- Finite-volume QED_L prescription for photons
- Computed structure-dependent V, A form factors
- Compared to KLOE, PIBETA, E787, ISTRA+, OKA
- Agreement with KLOE data (K →eνγ)
- Tension, e.g., with FNAL E787 (K →$\mu\nu\gamma$)



"We are able to separate unambiguously and non-pertubatively the point-like contribution, from the structure-dependent, IR-safe, terms in the amplitude."



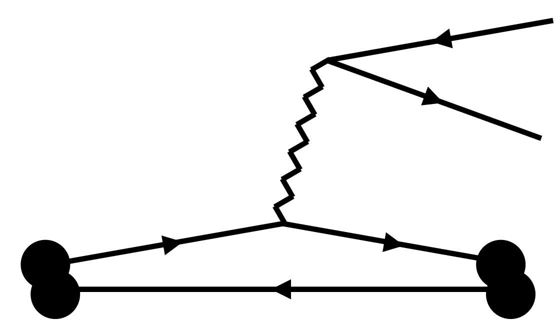


The Role of Lattice QCD

- Compute hadronic matrix elements needed by experiment
- Focus on **gold-plated processes**
    - Single-hadron in initial state
    - Zero or one hadron in final state
    - All hadrons stable under QCD
- Unstable particles (like D*) are possible in narrow-width approximation with EFT

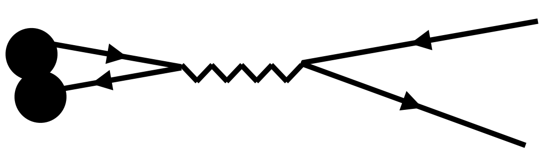
$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \ell\nu \\ & K \to \pi\ell\nu & B \to \pi\ell\nu \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ \langle B_d|\bar{B}_d\rangle & \langle B_s|\bar{B}_s\rangle & \end{pmatrix}$$

Semileptonic decays



(Form factors)

Leptonic decays



(Decay constants)

$$\langle 0|\, A^\mu \,|H(P)\rangle = i f_H\, p^\mu$$

$$f_J(p) \propto \langle \text{final}|\, J(p) \,|\text{initial}\rangle$$