Mixing and CPV in Charm: Experiment

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- Overview and definitions
- Recent measurements:
  - LHCb: y_{CP} , A_{Γ}
  - LHCb: $D^0 \to K_S\, \pi^+ \pi^-$
  - LHCb: A_{CP}
  - others
- HFLAV global fit for (x, y, $|q/p|$, ϕ)
- HFLAV global fit for (x_{12} , y_{12} , ϕ_{12}) and (ϕ^M, ϕ^Γ)

Standard parameterization of mixing, CPV

$$|D_1\rangle = p|D^0\rangle + q|\overline{D}{}^0\rangle \qquad |D_1(t)\rangle = |D_1\rangle\, e^{-i\left(m_1-\frac{i}{2}\Gamma_1\right)}$$
$$|D_2\rangle = p|D^0\rangle - q|\overline{D}{}^0\rangle \qquad |D_2(t)\rangle = |D_2\rangle\, e^{-i\left(m_2-\frac{i}{2}\Gamma_2\right)}$$

$$x \equiv \frac{M_2 - M_1}{\Gamma} = \frac{\Delta M}{\Gamma} \qquad y \equiv \frac{\Gamma_2 - \Gamma_1}{2\Gamma} = \frac{\Delta\Gamma}{2\Gamma}$$

$$\frac{q}{p} = \left|\frac{q}{p}\right| e^{i\phi} \qquad \frac{\mathcal{A}_{\overline{D}{}^0\to f}}{\mathcal{A}_{D^0\to f}} = \pm\sqrt{R_D}\, e^{i\delta}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{matrix} x\cos\delta + y\sin\delta \\ y\cos\delta - x\sin\delta \end{matrix}$$

$$\frac{N(D^0\to f)}{dt} \propto e^{-\overline{\Gamma}\,t} \left\{ R_D + \left|\frac{q}{p}\right| \sqrt{R_D}(y'\cos\phi - x'\sin\phi)(\overline{\Gamma}t) + \left|\frac{q}{p}\right|^2 \frac{(x'^2+y'^2)}{4} (\overline{\Gamma}\,t)^2 \right\}$$

$$\frac{N(\overline{D}{}^0\to \bar{f})}{dt} \propto e^{-\overline{\Gamma}\,t} \left\{ \overline{R}_D + \left|\frac{p}{q}\right| \sqrt{\overline{R}_D} y'\cos\phi + x'\sin\phi)(\overline{\Gamma}t) + \left|\frac{p}{q}\right|^2 \frac{(x'^2+y'^2)}{4} (\overline{\Gamma}\,t)^2 \right\}$$

$A_D \equiv \dfrac{(R_D - \overline{R}_D)}{(R_D + \overline{R}_D)}$ — CPV in the decay amplitude (direct CPV)

$\left|\dfrac{q}{p}\right|$ — CPV in mixing

ϕ — CPV in mixed/direct interference

Four recent measurements

Time-dependent $D^0(t) \to K^+K^-, \pi^+\pi^-$
Aaij et al. (LHCb), Phys.Rev.D 104, 072010 (2021)
Aaij et al. (LHCb), arXiv:2202.09106 (2022)

Fit for y_{CP}, A_Γ

$D^0(t) \to K^0{}_S\,\pi^+\pi^-$ Dalitz plot analysis
Aaij et al. (LHCb), PRL 127, 111801 (2021)

Fit for x_{CP}, y_{CP}, Δx, Δy

Time-*integrated* $D^0(t) \to K^+K^-, \pi^+\pi^-$
Aaij et al. (LHCb), PRL 122, 211803 (2019)

Measure $A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) \equiv \Delta A_{CP} \approx 2A_{direct}$

Common features of analyses:

a) *flavor is tagged via $D^{*-} \to \overline{D}{}^0 \pi^-_{slow}$ or $D^{*+} \to D^0 \pi^+_{slow}$*

b) *dominant background is typically "random π_{slow}" – include PDF for this in fits*

c) *decay time t calculated via*

$$t = \frac{\vec{\mathrm{d}} \cdot \hat{\mathrm{p}}}{|\vec{\mathrm{p}}|} \times \left(\frac{m}{c}\right)$$

Observables y_{CP} and A_Γ

Bergmann, Grossman, Ligeti, Nir, Petrov, PLB 486, 418 (2000)

$$\frac{dN}{dt}(D^0\to K^+K^-) \;\propto\; e^{-\Gamma t}\left[1-\left|\frac{q}{p}\right|(y\cos\phi - x\sin\phi)\Gamma t\right]$$

$$\frac{dN}{dt}(\overline{D}{}^0\to K^+K^-) \;\propto\; e^{-\Gamma t}\left[1-\left|\frac{p}{q}\right|(y\cos\phi + x\sin\phi)\Gamma t\right]$$

$$\frac{dN}{dt}(D^0\to K^-\pi^+) \;=\; \frac{dN}{dt}(\overline{D}{}^0\to K^+\pi^-) \;\propto\; e^{-\Gamma t}$$

$$\Rightarrow\quad \hat{\Gamma}(D^0\to K^+K^-) \;=\; \Gamma\left[1+\left|\frac{q}{p}\right|(y\cos\phi - x\sin\phi)\Gamma t\right]$$

$$\Rightarrow\quad \hat{\Gamma}(\overline{D}{}^0\to K^+K^-) \;=\; \Gamma\left[1+\left|\frac{p}{q}\right|(y\cos\phi + x\sin\phi)\Gamma t\right]$$

$$\Rightarrow\quad \hat{\Gamma}(D^0\to K^-\pi^+) \;=\; \Gamma$$

$$\Rightarrow\quad \frac{\hat{\Gamma}(D^0\to K^+K^-)+\hat{\Gamma}(\overline{D}{}^0\to K^+K^-)}{\hat{\Gamma}(D^0\to K^-\pi^+)+\hat{\Gamma}(\overline{D}{}^0\to K^+\pi^-)} - 1 \;=\; \frac{1}{2}\left(\left|\frac{q}{p}\right|+\left|\frac{p}{q}\right|\right) y\cos\phi - \frac{1}{2}\left(\left|\frac{q}{p}\right|-\left|\frac{p}{q}\right|\right) x\sin\phi \;\equiv\; y_{CP}$$

$$\Rightarrow\quad \frac{\hat{\Gamma}(D^0\to K^+K^-)-\hat{\Gamma}(\overline{D}{}^0\to K^+K^-)}{\hat{\Gamma}(D^0\to K^+K^-)+\hat{\Gamma}(\overline{D}{}^0\to K^+K^-)} \;=\; \frac{1}{2}\left(\left|\frac{q}{p}\right|-\left|\frac{p}{q}\right|\right) y\cos\phi - \frac{1}{2}\left(\left|\frac{q}{p}\right|+\left|\frac{p}{q}\right|\right) x\sin\phi \;\equiv\; A_\Gamma$$

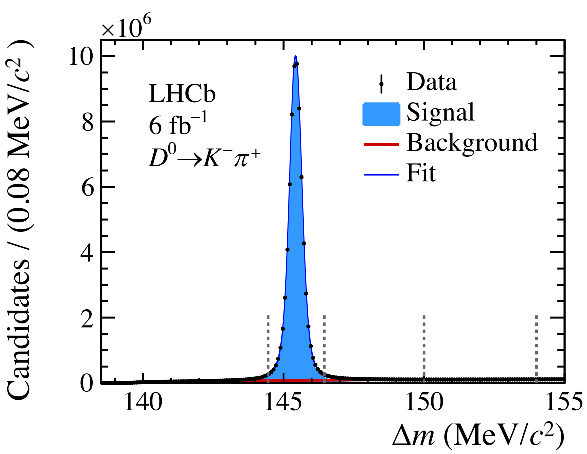
LHCb y_{CP} with $D^0 \to K^+K^-, \pi^+\pi^-$

Aaij et al. (LHCb), arXiv:2202.09106 (2022)

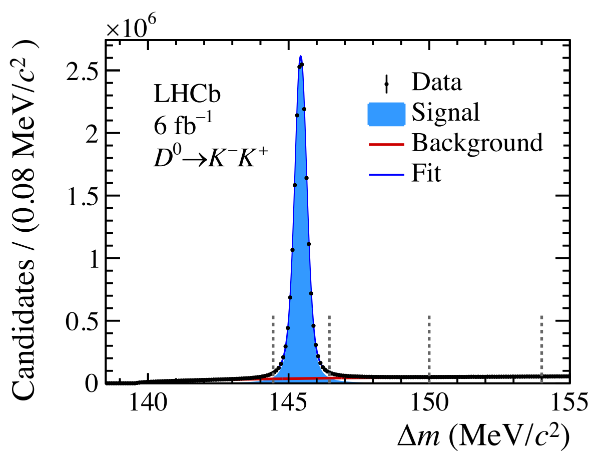
$$R^{KK}(t) \equiv \frac{N_{D^0\to K^+K^-}(t)}{N_{D^0\to K^-\pi^+}(t)} \propto e^{-\Gamma(y_{CP}^{KK}-y_{CP}^{K\pi})\,t} \times \frac{\varepsilon(KK)}{\varepsilon(K\pi)}$$

[and same for $R^{\pi\pi}(t)$]

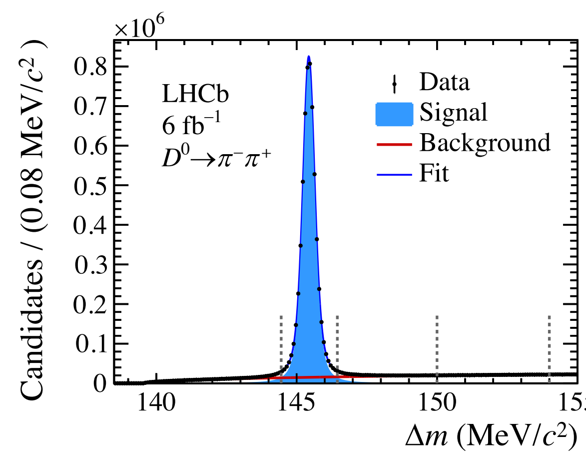
- *ratio or event yields minimizes contributions from B decays*
- *events are re-weighted to get $\varepsilon(KK)/\varepsilon(\pi\pi)$ close to unity*
- *$y_{CP}^{K\pi}$ is a very small correction as $\Gamma(K\pi)$ differs slightly from Γ*



$N_{sig} \approx 70 \times 10^6$ (purity = 98%)



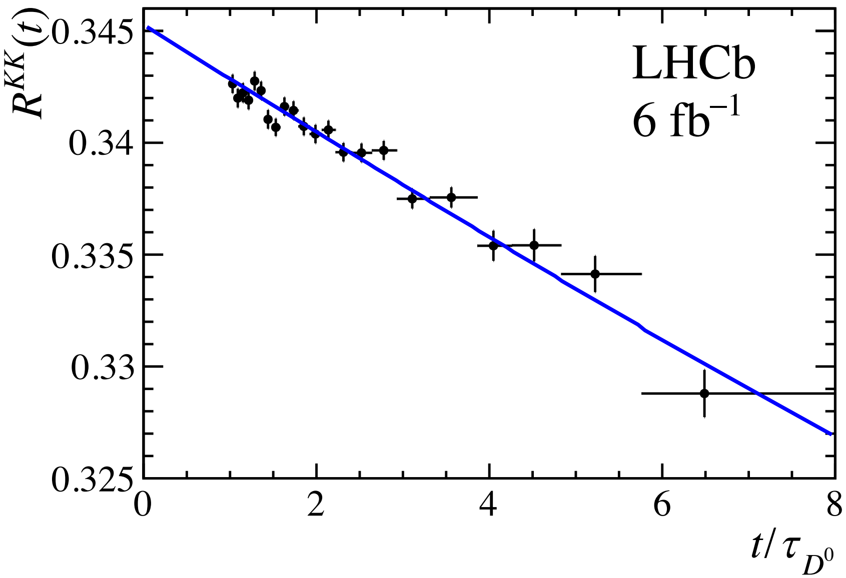
$N_{sig} \approx 18 \times 10^6$ (purity = 96%)

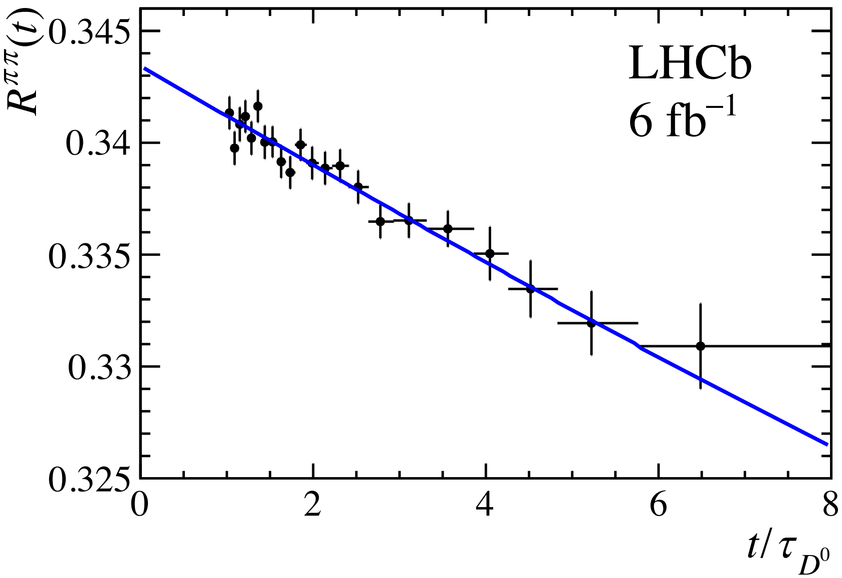


$N_{sig} \approx 6 \times 10^6$ (purity = 95%)

LHCb $D^0 \to K^+K^-, \pi^+\pi^-$

Aaij et al. (LHCb), arXiv:2202.09106 (2022)

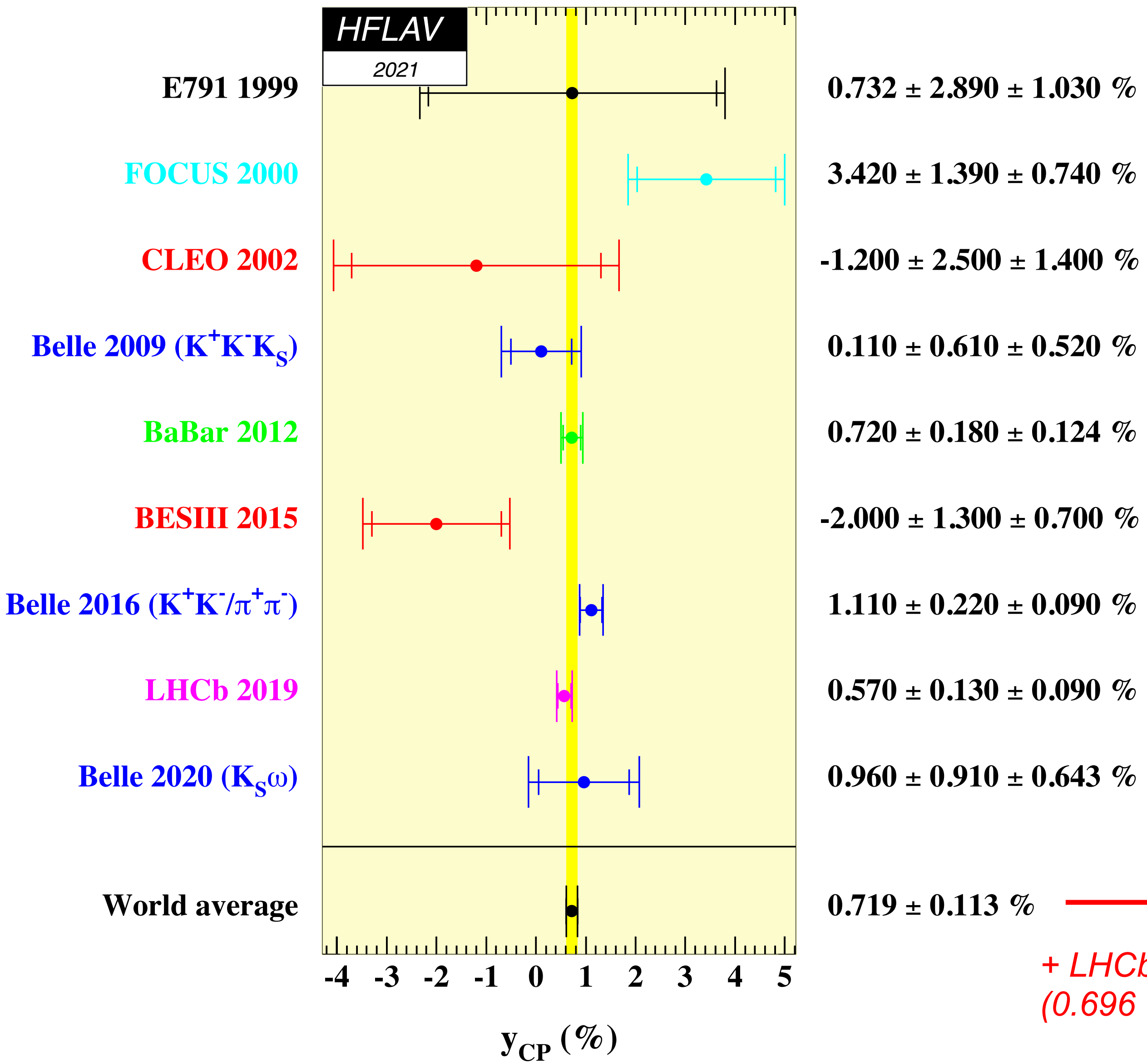




$$y_{CP}^{KK} - y_{CP}^{K\pi} = (0.708 \pm 0.030 \pm 0.014)\,\%$$
$$y_{CP}^{\pi\pi} - y_{CP}^{K\pi} = (0.657 \pm 0.053 \pm 0.016)\,\%$$
$$\Rightarrow \quad y_{CP} - y_{CP}^{K\pi} = (0.696 \pm 0.026 \pm 0.013)\,\%$$

$$y_{CP}^{K\pi} \approx \sqrt{R_D}\left(x_{12}\cos\phi_2^M \sin\delta_{K\pi} + y_{12}\cos\phi_2^\Gamma \cos\delta_{K\pi}\right)$$
$$= 0.036\% \quad \text{(now larger than } \delta y_{CP}^{hh}\text{)}$$

Measurements: y_{CP}



→ *(0.697 + 0.028)%*

+ LHCb 2022
(0.696 + 0.029)%

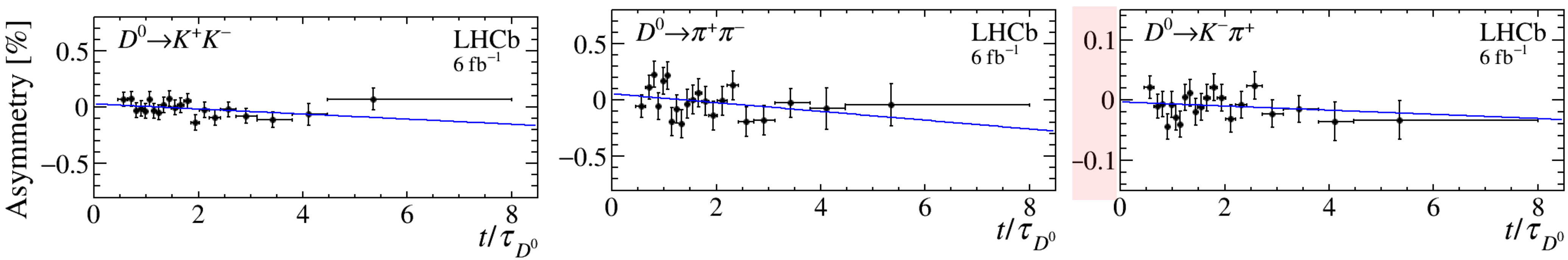
LHCb A_Γ with $D^0 \to K^+K^-, \pi^+\pi^-$

Aaij et al. (LHCb), PRD 104, 072010 (2021)

$$\frac{\hat{\Gamma}(D^0 \to K^+K^-) - \hat{\Gamma}(\overline{D}^0 \to K^+K^-)}{\hat{\Gamma}(D^0 \to K^+K^-) + \hat{\Gamma}(\overline{D}^0 \to K^+K^-)} = \frac{1}{2}\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right) y\cos\phi - \frac{1}{2}\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right) x\sin\phi \equiv A_\Gamma$$

$$A_{CP}(t) \equiv \frac{\frac{dN(D^0 \to f)}{dt} - \frac{dN(\overline{D}^0 \to f)}{dt}}{\frac{dN(D^0 \to f)}{dt} + \frac{dN(\overline{D}^0 \to f)}{dt}} \approx a_f^{\text{direct}} - A_\Gamma\left(\frac{t}{\tau_D}\right)$$

control channel:



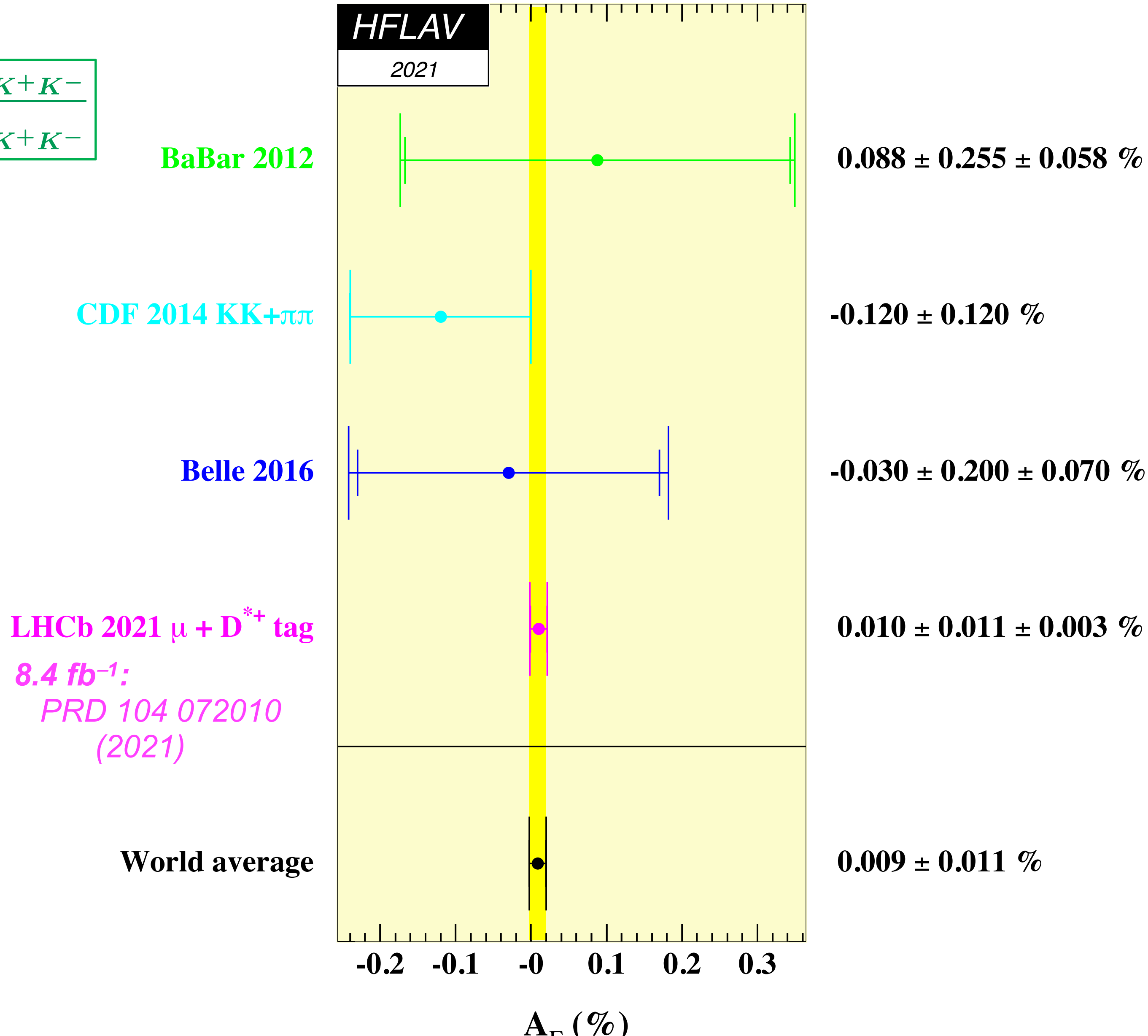
$A_\Gamma(K^+K^-) = (2.3 \pm 1.5 \pm 0.3) \times 10^{-4}$ $A_\Gamma(\pi^+\pi^-) = (4.0 \pm 2.8 \pm 0.4) \times 10^{-4}$

$\Rightarrow$ $A_\Gamma = (2.7 \pm 1.3 \pm 0.3) \times 10^{-4} \quad (6\ \text{fb}^{-1})$

Combining with other LHCb results (2011-2012 data, B-flavor tags): $A_\Gamma = (1.0 \pm 1.1 \pm 0.3) \times 10^{-4} \quad (8.4\ \text{fb}^{-1})$

Measurements: A_Γ $[D^0(t) \to K^+K^- / \pi^+\pi^-]$

$$A_\Gamma = \frac{\tau_{\overline{D}^0 \to K^+K^-} - \tau_{D^0 \to K^+K^-}}{\tau_{\overline{D}^0 \to K^+K^-} + \tau_{D^0 \to K^+K^-}}$$

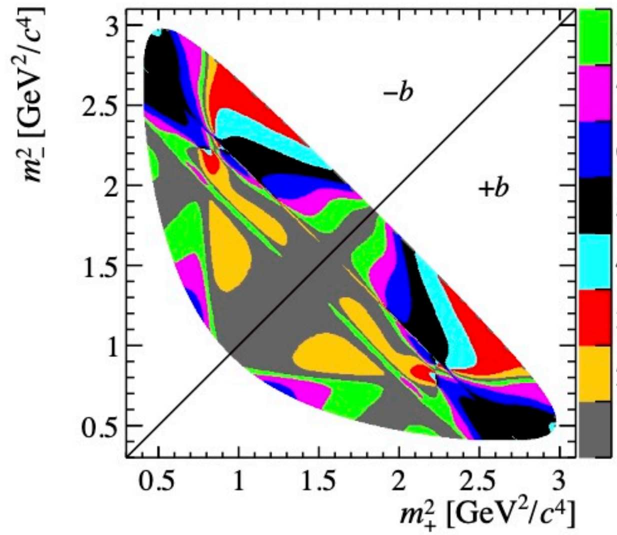


LHCb $D^0(t) \to K^0_S\, \pi^+\pi^-$

Aaij et al. (LHCb), PRL 127, 111801 (2021)

Analysis used "the bin-flip" method [Di Canto et al., PRD 99, 012007 (2019)]:

- *Fit the ratios of event yields as a function of decay time in 8-bin-pairs of the Dalitz plot*
- *For D^0 decays, events in +b bins are ~CF, events in –b bins are ~DCS or mixed; hence ratios of event yields gives sensitivity to mixing. The opposite behavior hold for D^0bar decays. Fitting both Dalitz distributions together gives sensitivity to CPV. For a bin-pair b, the ratios of events are:*



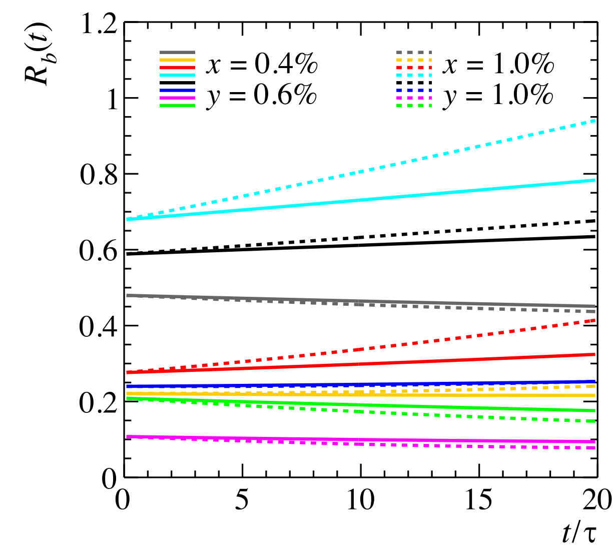
$$R_b \equiv \frac{N_{-b}}{N_b} \approx \frac{R_D\left(1+\frac{1}{4}\mathrm{Re}[z^2]t^2\right) + \sqrt{R_D}\,\mathrm{Re}\left[X_b^*\left(\frac{q}{p}\right)z\right]t + \frac{1}{4}\left|\left(\frac{q}{p}\right)z\right|^2 t^2}{\left(1+\frac{1}{4}\mathrm{Re}[z^2]t^2\right) + \sqrt{R_D}\,\mathrm{Re}\left[X_b\left(\frac{q}{p}\right)z\right]t + \frac{1}{4}R_D\left|\left(\frac{q}{p}\right)z\right|^2 t^2}$$

$$\overline{R}_b \equiv \frac{\overline{N}_{-b}}{\overline{N}_b} \approx \text{same with } \left(\frac{q}{p}\right) \to \left(\frac{p}{q}\right)$$

$$z \equiv -(y+ix)$$

$$X_b \equiv \frac{1}{\sqrt{\int |A_f|^2 \int |\overline{A}_f|^2}} \int A_f^* \overline{A}_f \, dm_+^2 \, dm_-^2$$

(coherence factor)



LHCb $D^0(t) \to K^0_S\, \pi^+\pi^-$

Aaij et al. (LHCb), PRL 127, 111801 (2021)

$$R_b \equiv \frac{N_{-b}}{N_b} \approx \frac{R_D\left(1+\frac{1}{4}\mathrm{Re}[z^2]t^2\right)+\sqrt{R_D}\,\mathrm{Re}\left[X_b^*\left(\frac{q}{p}\right)z\right]t+\frac{1}{4}\left|\left(\frac{q}{p}\right)z\right|^2 t^2}{\left(1+\frac{1}{4}\mathrm{Re}[z^2]t^2\right)+\sqrt{R_D}\,\mathrm{Re}\left[X_b\left(\frac{q}{p}\right)z\right]t+\frac{1}{4}R_D\left|\left(\frac{q}{p}\right)z\right|^2 t^2}$$

$$\overline{R}_b \equiv \frac{\overline{N}_{-b}}{\overline{N}_b} \approx \text{same with } \left(\frac{q}{p}\right) \to \left(\frac{p}{q}\right)$$

$$z \equiv -(y+ix)$$

$$X_b \equiv \frac{1}{\sqrt{\int |A_f|^2 \int |\overline{A}_f|^2}} \int A_f^* \overline{A}_f \, dm_+^2\, dm_-^2$$

(coherence factor)

In practice, the fit is unstable when fitting for the product $(q/p)z$ ⇒ re-define fitted parameters:

$$z_{CP} \equiv \frac{z}{2}\left(\frac{q}{p}+\frac{p}{q}\right) \qquad \Delta z \equiv \frac{z}{2}\left(\frac{q}{p}-\frac{p}{q}\right)$$

$$\Rightarrow \quad \left(\frac{q}{p}\right)z = z_{CP}+\Delta z \qquad \left(\frac{p}{q}\right)z = z_{CP}-\Delta z$$

This re-definition minimizes products of fitted parameters (fit is more stable, errors are Gaussian and better behaved) . Note that z_{CP} and Δz have both real and imaginary parts: 4 degrees of freedom

LHCb $D^0(t) \to K^0_S\,\pi^+\pi^-$

Aaij et al. (LHCb), PRL 127, 111801 (2021)

$$z_{CP} \equiv \frac{z}{2}\left(\frac{q}{p}+\frac{p}{q}\right) \qquad \Delta z \equiv \frac{z}{2}\left(\frac{q}{p}-\frac{p}{q}\right)$$

$$z = -(y+ix)$$

4 degrees of freedom of z_{CP} and Δz are parameterized as:

$$-\mathrm{Re}[z_{CP}] \equiv y_{CP} = \frac{1}{2}\left[y\cos\phi\left(\left|\frac{q}{p}\right|+\left|\frac{p}{q}\right|\right) - x\sin\phi\left(\left|\frac{q}{p}\right|-\left|\frac{p}{q}\right|\right)\right]$$

$$-\mathrm{Im}[z_{CP}] \equiv x_{CP} = \frac{1}{2}\left[x\cos\phi\left(\left|\frac{q}{p}\right|+\left|\frac{p}{q}\right|\right) + y\sin\phi\left(\left|\frac{q}{p}\right|-\left|\frac{p}{q}\right|\right)\right]$$

$$-\mathrm{Re}[\Delta z] \equiv \Delta y = \frac{1}{2}\left[y\cos\phi\left(\left|\frac{q}{p}\right|-\left|\frac{p}{q}\right|\right) - x\sin\phi\left(\left|\frac{q}{p}\right|+\left|\frac{p}{q}\right|\right)\right]$$

$$-\mathrm{Im}[\Delta z] \equiv \Delta x = \frac{1}{2}\left[x\cos\phi\left(\left|\frac{q}{p}\right|-\left|\frac{p}{q}\right|\right) + y\sin\phi\left(\left|\frac{q}{p}\right|+\left|\frac{p}{q}\right|\right)\right]$$

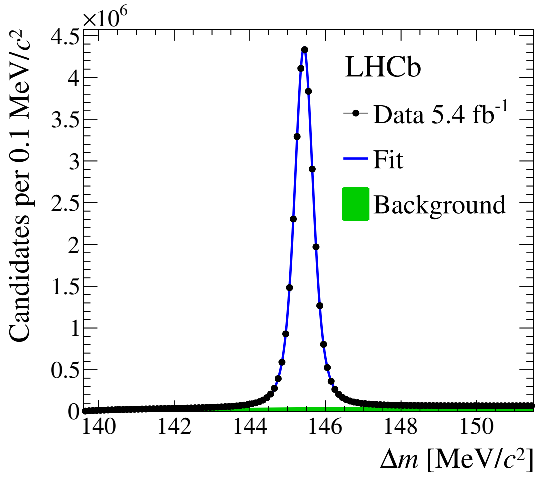
LHCb results:

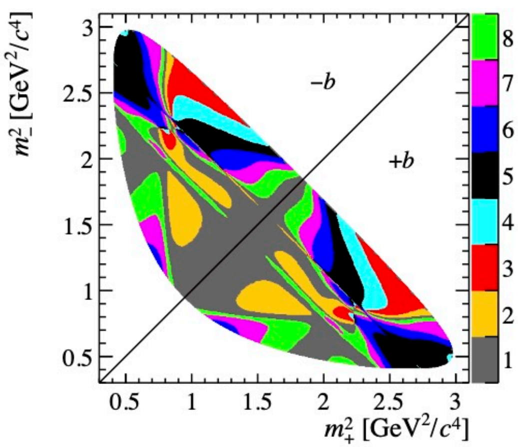
$$\begin{aligned} x_{CP} &= (0.397 \pm 0.046 \pm 0.029)\,\% \\ y_{CP} &= (0.459 \pm 0.120 \pm 0.085)\,\% \\ \Delta x &= (-0.027 \pm 0.018 \pm 0.001)\,\% \\ \Delta y &= (0.020 \pm 0.036 \pm 0.013)\,\% \end{aligned}$$

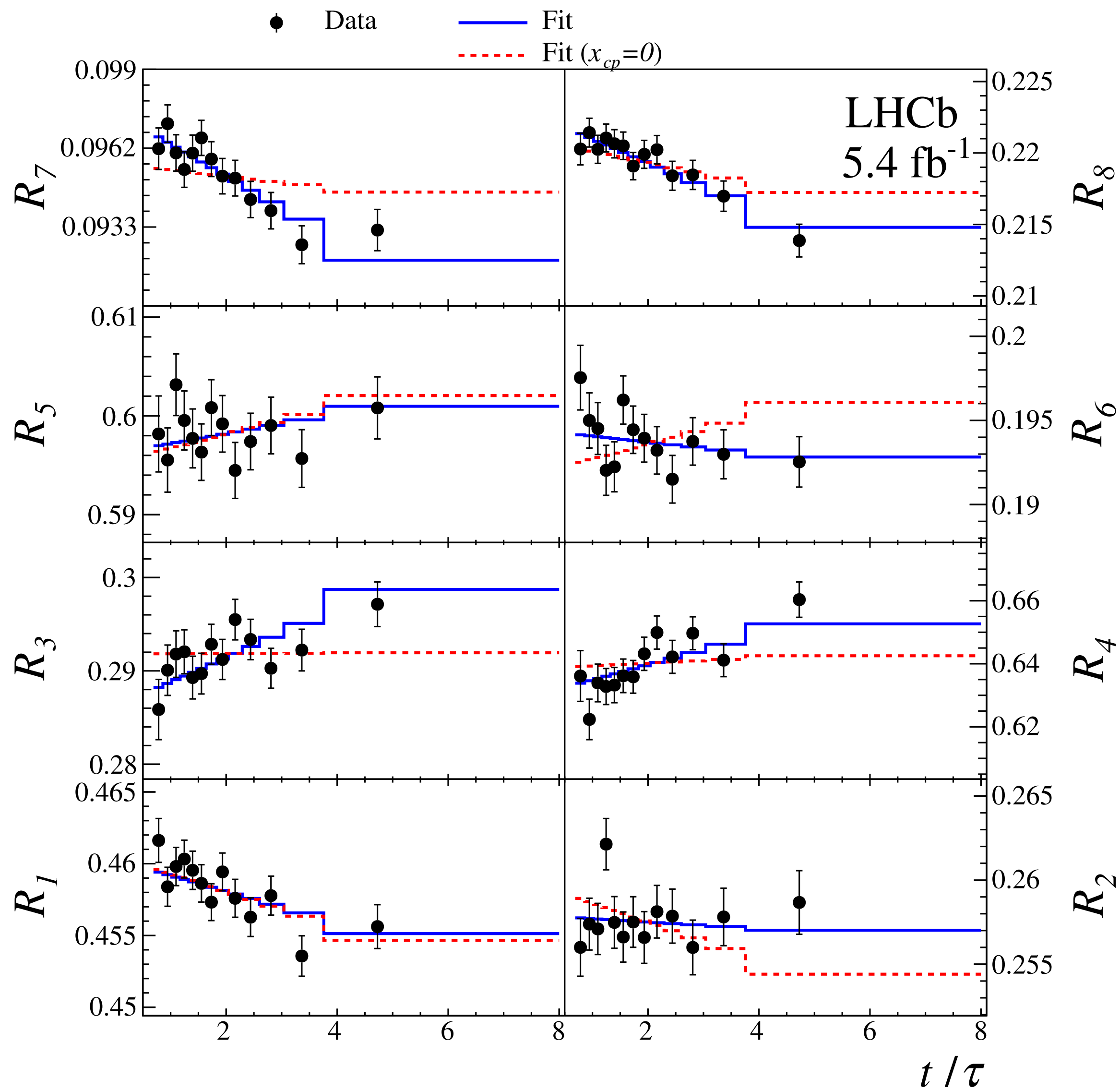
LHCb $D^0(t) \rightarrow K^0_S\, \pi^+\pi^-$

Aaij et al. (LHCb), PRL 127, 111801 (2021)

Fit $\Delta M = M(D^{*+}) - M(D^0)$ for event yields, 31 x 10^6 signal decays (!):







Heavy Flavor Averaging Group Global Fit

19 measured observables (blue)
10 fitted parameters (magenta): x, y, δ, $\delta_{K\pi\pi}$, R_D, A_D, $|q/p|$, ϕ [= Arg(q/p)], A_K, A_π

$$R_M = \frac{1}{2}(x^2 + y^2)$$

$$2\,y_{CP} = \left(|q/p| + |p/q|\right) y\cos\phi - \left(|q/p| - |p/q|\right) x\sin\phi$$
$$2\,A_\Gamma = \left(|q/p| - |p/q|\right) y\cos\phi - \left(|q/p| + |p/q|\right) x\sin\phi$$

$$x_{K^0\pi\pi} = x$$
$$y_{K^0\pi\pi} = y$$
$$|q/p|_{K^0\pi\pi} = |q/p|$$
$$\mathrm{Arg}\,(q/p)_{K^0\pi\pi} = \phi$$

$$\begin{pmatrix} x'' \\ y'' \end{pmatrix}_{K^+\pi^-\pi^0} = \begin{pmatrix} \cos\delta_{K\pi\pi} & \sin\delta_{K\pi\pi} \\ -\sin\delta_{K\pi\pi} & \cos\delta_{K\pi\pi} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$x'^{\pm} = |q/p|^{\pm 1}\,(x'\cos\phi \pm y'\sin\phi)$$
$$y'^{\pm} = |q/p|^{\pm 1}\,(y'\cos\phi \mp x'\sin\phi)$$

$$\frac{\Gamma(D^0\to K^+\pi^-) + \Gamma(\overline{D}{}^0\to K^-\pi^+)}{\Gamma(D^0\to K^-\pi^+) + \Gamma(\overline{D}{}^0\to K^+\pi^-)} = R_D$$

$$\frac{\Gamma(D^0\to K^+\pi^-) - \Gamma(\overline{D}{}^0\to K^-\pi^+)}{\Gamma(D^0\to K^+\pi^-) + \Gamma(\overline{D}{}^0\to K^-\pi^+)} = A_D$$

$$2\mathcal{A}_{CP}^{\mathrm{indirect}} = -\,2\,A_\Gamma$$

$$\frac{\Gamma(D^0\to K^+K^-) - \Gamma(\overline{D}{}^0\to K^+K^-)}{\Gamma(D^0\to K^+K^-) + \Gamma(\overline{D}{}^0\to K^+K^-)} = A_K + \frac{\langle t\rangle}{\tau_D}\mathcal{A}_{CP}^{\mathrm{indirect}}$$

$$\frac{\Gamma(D^0\to \pi^+\pi^-) - \Gamma(\overline{D}{}^0\to \pi^+\pi^-)}{\Gamma(D^0\to \pi^+\pi^-) + \Gamma(\overline{D}{}^0\to \pi^+\pi^-)} = A_\pi + \frac{\langle t\rangle}{\tau_D}\mathcal{A}_{CP}^{\mathrm{indirect}}$$

Observables: $D^0(t) \rightarrow K_S\, \pi^+ \pi^-$

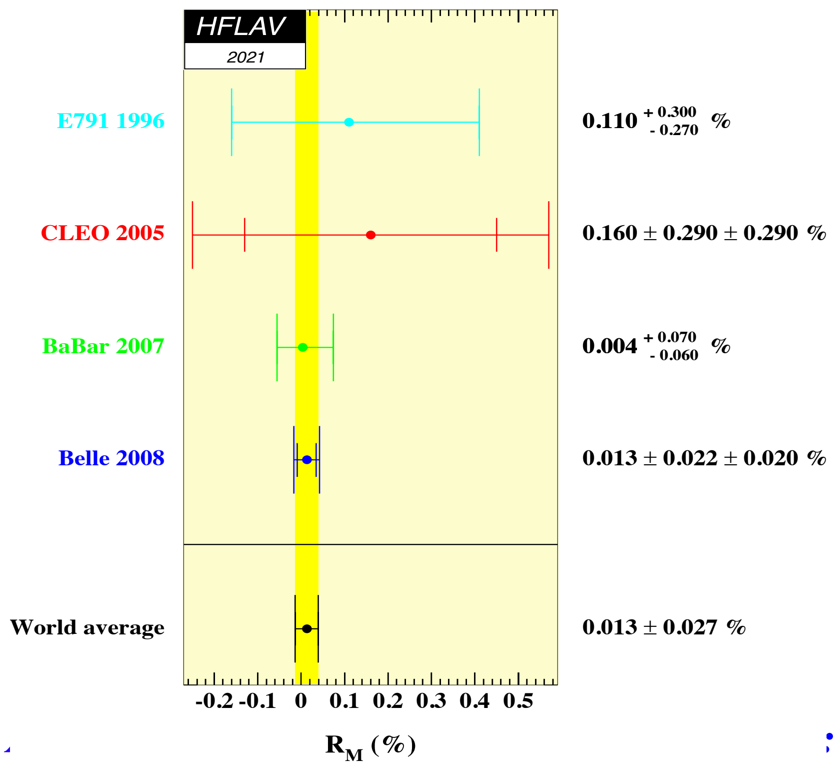
Index	Observable	Value	Source
3-6	x (no CPV) y (no CPV) |q/p| (no dCPV) Arg(q/p)=φ (no dCPV) x y |q/p| φ	$0.56 \pm 0.19\ ^{+0.067}_{-0.127}$ $0.30 \pm 0.15\ ^{+0.050}_{-0.078}$ $0.90\ ^{+0.16}_{-0.15}\ ^{+0.078}_{-0.064}$ $(-6 \pm 11\ ^{+4.2}_{-5})$ degrees $(0.58 \pm 0.19\ ^{+0.0734}_{-0.1177})\%$ $(0.27 \pm 0.16\ ^{+0.0546}_{-0.0854})\%$ $0.82\ ^{+0.20}_{-0.18}\ ^{+0.0807}_{-0.0645}$ $(-13\ ^{+12}_{-13}\ ^{+4.15}_{-4.77})$ degrees	Belle $D^0 \rightarrow K^0{}_S\, \pi^+ \pi^-$ results using 921 fb^{-1}. Correlation coefficient is +0.012 for no-CPV; for CPV-allowed they are: 1 0.054 –0.074 –0.031 0.054 1 0.034 –0.019 –0.074 0.034 1 0.044 –0.031 –0.019 0.044 1
7-10	x (no CPV) y (no CPV) x_{CP} y_{CP} Δx Δy	(–0.86 ± 0.53 ± 0.17)% (0.03 ± 0.46 ± 0.13)% (0.27 ± 0.16 ± 0.04)% (0.74 ± 0.36 ± 0.11)% (–0.053 ± 0.070 ± 0.022)% (0.06 ± 0.16 ± 0.03)%	LHCb $D^0 \rightarrow K^0{}_S\, \pi^+\pi^-$ results using 1 fb^{-1} ($\sqrt{s}$ = 7 TeV) $D^{*+} \rightarrow D^0\pi^+$ flavor tag. Correlation coefficient = +0.37, no CPV. 3 fb^{-1} results ($\sqrt{s}$ = 7, 8 TeV) allowing for CPV. $D^{*+} \rightarrow D^0\pi^+$, $B \rightarrow D^0\mu^- X$ flavor tags. Correlation coefficients (stat. + syst.): 1 (–0.17 + 0.15) (0.04 + 0.01) (–0.02 – 0.02) 1 (–0.03 – 0.05) (0.01 – 0.03) 1 (–0.13 + 0.14)
11-14	x_{CP} y_{CP} Δx Δy	(0.397 ± 0.046 ± 0.029)% (0.459 ± 0.120 ± 0.085)% (–0.027 ± 0.018 ± 0.001)% (0.020 ± 0.036 ± 0.013)%	5.4 fb^{-1} results ($\sqrt{s}$ = 13 TeV) allowing for CPV. $D^{*+} \rightarrow D^0\pi^+$ flavor tags. Correlation coefficients (stat. + syst.): 1 (0.11 + 0.13) (–0.02 + 0.01) (–0.01 + 0.01) 1 (–0.01 – 0.02) (–0.05 + 0.01) 1 (0.08 + 0.31) For (x, y, |q/p|, φ) → (x_{CP}, y_{CP}, Δx, Δy) mapping, see PRD 99, 012007 (2019)
15-16	x y	(0.16 ± 0.23 ± 0.12 ± 0.08)% (0.57 ± 0.20 ± 0.13 ± 0.07)%	BaBar $D^0 \rightarrow K^0{}_S\, \pi^+\pi^-$ and $D^0 \rightarrow K^0{}_S\, K^+ K^-$ combined; Correlation coefficient = +0.0615, no CPV.
17-18	x y	(1.5 ± 1.2 ± 0.6)% (0.2 ± 0.9 ± 0.5)%	BaBar $D^0 \rightarrow \pi^0\, \pi^+\pi^-$ Correlation coefficient = –0.006, no CPV.

Observables:
$D^0 \to K^+\pi^-$

Index	Observable	Value	Source
27-29	R_D x'^{2+} y'^{+}	$(0.303 \pm 0.0189)\%$ $(-0.024 \pm 0.052)\%$ $(0.98 \pm 0.78)\%$	BaBar $K^+\pi^-$ results; correlation coefficients: 1 +0.77 −0.87 +0.77 1 −0.94 −0.87 −0.94 1
30-32	A_D x'^{2-} y'^{-}	$(-2.1 \pm 5.4)\%$ $(-0.020 \pm 0.050)\%$ $(0.96 \pm 0.75)\%$	BaBar $K^+\pi^-$ results; correlation coefficients same as above.
33-35 (no CPV)	R_D x'^{2} y'	$(0.353 \pm 0.013)\%$ $(0.009 \pm 0.022)\%$ $(0.46 \pm 0.34)\%$	Belle $K^+\pi^-$ no-CPV results using 976 fb^{-1}. Correlation coefficients: 1 +0.737 −0.865 +0.737 1 −0.948 −0.865 −0.948 1
33-35	R_D x'^{2+} y'^{+}	$(0.364 \pm 0.018)\%$ $(0.032 \pm 0.037)\%$ $(-0.12 \pm 0.58)\%$	Belle $K^+\pi^-$ CPV-allowed results using 400 fb^{-1}. Correlation coefficients: 1 +0.655 −0.834 +0.655 1 −0.909 −0.834 −0.909 1
36-38	A_D x'^{2-} y'^{-}	$(2.3 \pm 4.7)\%$ $(0.006 \pm 0.034)\%$ $(0.20 \pm 0.54)\%$	Belle $K^+\pi^-$ CPV-allowed results using 400 fb^{-1}; correlation coefficients same as above.
39-41	R_D x'^{2} y'	$(0.351 \pm 0.035)\%$ $(0.008 \pm 0.018)\%$ $(0.43 \pm 0.43)\%$	CDF $K^+\pi^-$ results for 9.6 fb^{-1}. Correlation coefficients: 1 0.90 −0.97 0.90 1 −0.98 −0.97 −0.98 1
42-44	R_D^+ x'^{2+} y'^{+}	$(0.338 \pm 0.0161)\%$ $(-0.0019 \pm 0.0447)\%$ $(0.581 \pm 0.526)\%$	LHCb $K^+\pi^-$ results for 3.0 fb^{-1} ($\sqrt{s} = 7, 8$ TeV) $B \to D^{*+}\mu^- X$, $D^{*+} \to D^0\pi^+$ flavor tags. Correlation coefficients: 1 0.823 −0.920 0.823 1 −0.962 −0.920 −0.962 1
45-47	R_D^- x'^{2-} y'^{-}	$(0.360 \pm 0.0166)\%$ $(0.0079 \pm 0.0433)\%$ $(0.332 \pm 0.523)\%$	LHCb $K^+\pi^-$ results for 3.0 fb^{-1} ($\sqrt{s} = 7, 8$ TeV) $B \to D^{*+}\mu^- X$, $D^{*+} \to D^0\pi^+$ flavor tags. Correlation coefficients: 1 0.812 −0.918 0.812 1 −0.956 −0.918 −0.956 1
48-50	R_D^+ x'^{2+} y'^{+}	$(0.3454 \pm 0.0045)\%$ $(0.0061 \pm 0.0037)\%$ $(0.501 \pm 0.074)\%$	LHCb $K^+\pi^-$ results for 5.0 fb^{-1} ($\sqrt{s} = 7, 8$ TeV) $D^{*+} \to D^0\pi^+$ flavor tag. Correlation coefficients: 1 0.843 −0.935 0.843 1 −0.963 −0.935 −0.963 1
51-53	R_D^- x'^{2-} y'^{-}	$(0.3454 \pm 0.0045)\%$ $(0.0016 \pm 0.0039)\%$ $(0.554 \pm 0.074)\%$	LHCb $K^+\pi^-$ results for 5.0 fb^{-1} ($\sqrt{s} = 7, 8$ TeV) $D^{*+} \to D^0\pi^+$ flavor tag. Correlation coefficients: 1 0.846 −0.935 0.846 1 −0.964 −0.935 −0.964 1

Observables: miscellaneous

Index	Observable	Value	Source
15	$(x^2 + y^2)/2$	$(0.0130 \pm 0.0269)\%$	World average (COMBOS combination) of $D^0 \to K^+ l^- \nu$ results
16-17	x'' y''	$(2.61\ ^{+0.57}_{-0.68} \pm 0.39)\%$ $(-0.06\ ^{+0.55}_{-0.64} \pm 0.34)\%$	BaBar $K^+ \pi^- \pi^0$ result; correlation coefficient = −0.75. Note: $x'' = x \cos \delta_{K\pi\pi} + y \sin \delta_{K\pi\pi}$, $y'' = y \cos \delta_{K\pi\pi} - x \sin \delta_{K\pi\pi}$.
18	R_D x^2 y $\cos \delta$ $\sin \delta$	$(0.533 \pm 0.107 \pm 0.045)\%$ $(0.06 \pm 0.23 \pm 0.11)\%$ $(4.2 \pm 2.0 \pm 1.0)\%$ $0.81\ ^{+0.22}_{-0.18}\ ^{+0.07}_{-0.05}$ $-0.01 \pm 0.41 \pm 0.04$	CLEO-c $\Psi(3770)$ results; correlation coefficients: 1 0 0 −0.42 0.01 1 −0.73 0.39 0.02 1 −0.53 −0.03 1 0.04 1
31	$(x^2 + y^2)/4$	$(0.0048 \pm 0.0018)\%$	LHCb 3.0 fb^{-1} pp collisions at $\sqrt{s}$ = 7, 8 TeV $D^0 \to K^+ \pi^- \pi^+ \pi^-$



Fit Results: all CPV allowed

- Code written in Fortran (Fortran was convenient for this type of calculation)
- Fitter is MINUIT; FCN function calculates global χ^2, where each measurement contributes according to:

$$\chi^2(\boldsymbol{x}) = \sum_i (\boldsymbol{x}_i - \boldsymbol{x})^{\mathrm{T}} \boldsymbol{V}_i^{-1} (\boldsymbol{x}_i - \boldsymbol{x})$$

$\Rightarrow$ most of the code is spent calculating error matrices and inverting

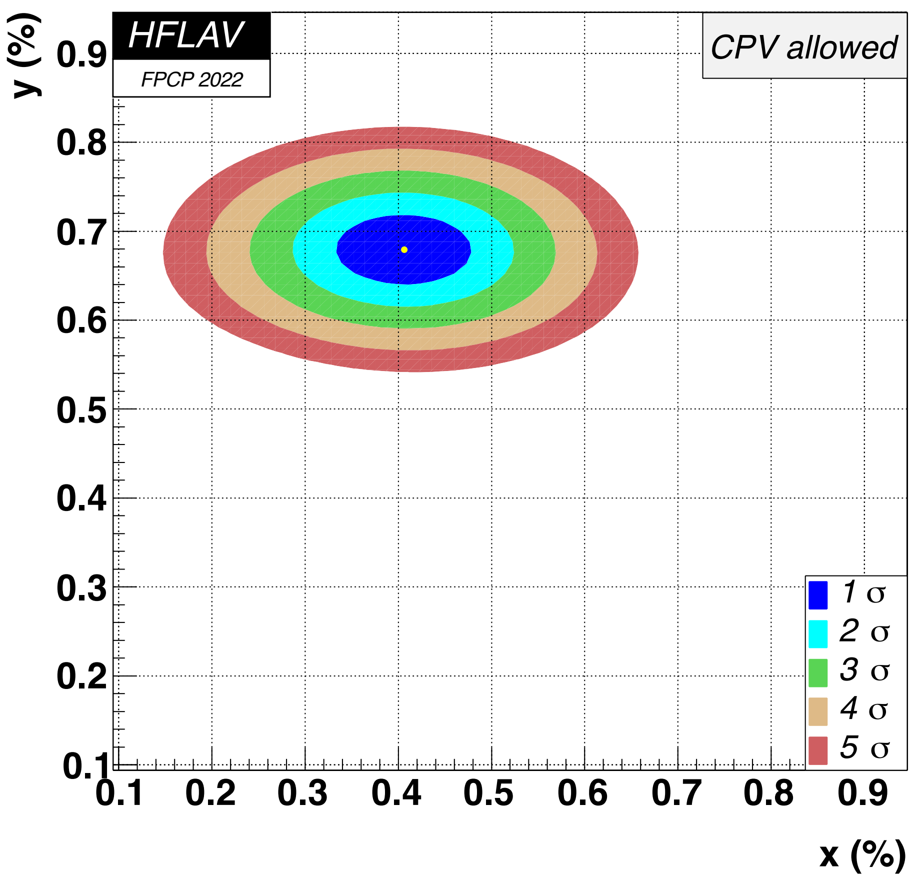
- world averages for y_{CP}, A_Γ, R_M are calculated via separate COMBOS code
- all errors are calculated via MINOS
- plotting routine is done separately in ROOT

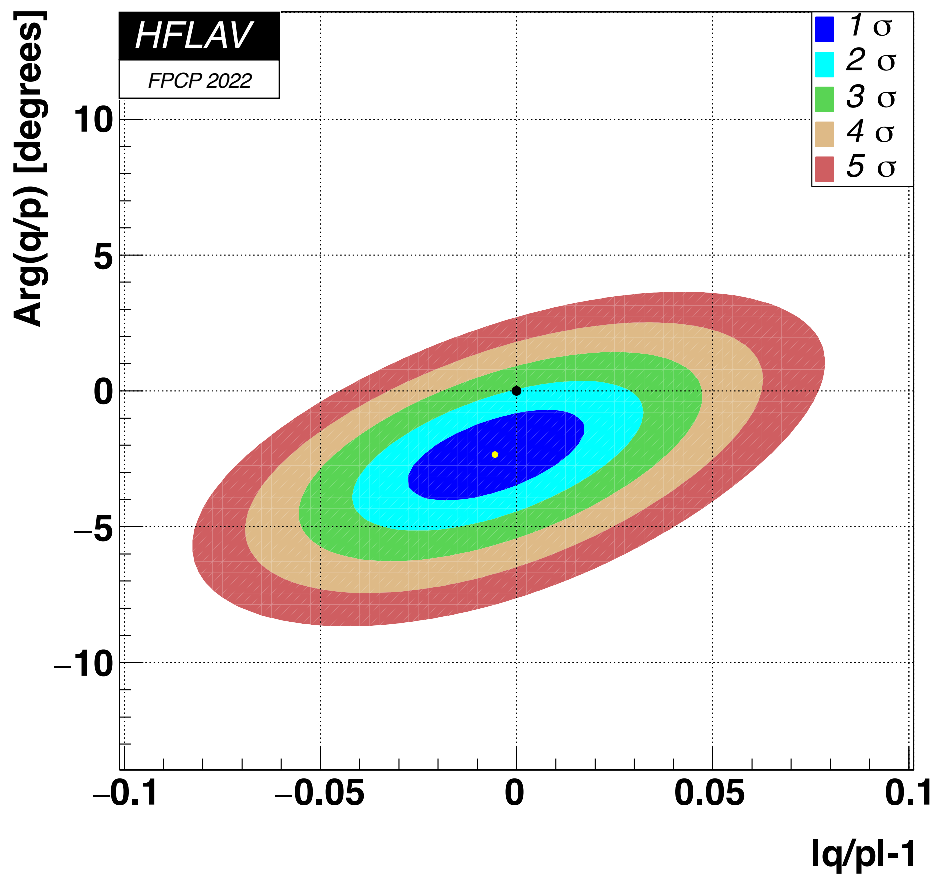
```
FCN=   65.27469    FROM MINOS     STATUS=SUCCESSFUL  1297 CALLS
11167 TOTAL    EDM=  0.47E-13    ERROR MATRIX UNCERTAINTY=  0.0%
```

EXT NO.	PARAMETER NAME	VALUE	PARABOLIC ERROR	MINOS ERRORS NEGATIVE	MINOS ERRORS POSITIVE
1	x	0.40612	0.047802	-0.047901	0.047640
2	y	0.67944	0.025813	-0.025821	0.025821
3	delta	0.27834	0.063654	-0.065794	0.062182
4	rd	0.34418	0.0018137	-0.0018147	0.0018129
5	ad	-0.77508	0.35340	-0.35348	0.35330
6	qovp	0.99448	0.014910	-0.014801	0.015038
7	phi	-0.041031	0.019178	-0.019317	0.019114
8	delta2	0.45839	0.37990	-0.38907	0.36997
9	ap	0.042559	0.13719	-0.13718	0.13718
10	ak	-0.11627	0.13659	-0.13658	0.13658

Observable	χ^2	$\sum\chi^2$
y_{CP} $(K^+K^-, \pi^+\pi^-)$ World Average	0.44	0.44
A_Γ World Average	0.13	0.57
$x_{K^0\pi^+\pi^-}$ Belle	0.61	1.17
$y_{K^0\pi^+\pi^-}$ Belle	5.09	6.27
$\lvert q/p\rvert_{K^0\pi^+\pi^-}$ Belle	0.65	6.92
$\phi_{K^0\pi^+\pi^-}$ Belle	0.68	7.60
x_{CP} $(K^0\pi^+\pi^-)$ LHCb 3 fb^{-1}	0.68	8.28
y_{CP} $(K^0\pi^+\pi^-)$ LHCb 3 fb^{-1}	0.03	8.31
Δx $(K^0\pi^+\pi^-)$ LHCb 3 fb^{-1}	0.10	8.40
Δy $(K^0\pi^+\pi^-)$ LHCb 3 fb^{-1}	0.03	8.44
x_{CP} $(K^0\pi^+\pi^-)$ LHCb 5 fb^{-1}	0.03	8.47
y_{CP} $(K^0\pi^+\pi^-)$ LHCb 5 fb^{-1}	2.23	10.70
Δx $(K^0\pi^+\pi^-)$ LHCb 5 fb^{-1}	0.03	10.73
Δy $(K^0\pi^+\pi^-)$ LHCb 5 fb^{-1}	-0.02	10.71
$x_{K^0h^+h^-}$ Babar	0.82	11.53
$y_{K^0h^+h^-}$ Babar	0.15	11.68
$x_{\pi^0\pi^+\pi^-}$ Babar	0.67	12.35
$y_{\pi^0\pi^+\pi^-}$ Babar	0.21	12.56
$(x^2+y^2)_{K^+\ell^-\nu}$	0.13	12.70
$x_{K^+\pi^-\pi^0}$ Babar	6.89	19.59
$y_{K^+\pi^-\pi^0}$ Babar	3.62	23.21
CLEOc		
$(x/y/R_D/\cos\delta/\sin\delta)$	10.81	34.02
$R_D^+/x'^{2+}/y'^+$ Babar	8.55	42.57
$R_D^-/x'^{2-}/y'^-$ Babar	4.35	46.91
$R_D^+/x'^{2+}/y'^+$ Belle	1.88	48.79
$R_D^-/x'^{2-}/y'^-$ Belle	2.33	51.12
$R_D/x'^2/y'$ CDF	1.52	52.64
$R_D^+/x'^{2+}/y'^+$ LHCb (D^* tag)	0.90	53.54
$R_D^-/x'^{2-}/y'^-$ LHCb (D^* tag)	0.33	53.87
$R_D^+/x'^{2+}/y'^+$ LHCb (B tag)	0.64	54.51
$R_D^-/x'^{2-}/y'^-$ LHCb (B tag)	1.57	56.08
$A_{KK}/A_{\pi\pi}$ Babar	0.35	56.43
$A_{KK}/A_{\pi\pi}$ Belle	1.45	57.88
$A_{KK}/A_{\pi\pi}$ CDF	4.09	61.97
$A_{KK} - A_{\pi\pi}$ LHCb ($D^* + B^0 \to D^0\mu X$ tags)	0.08	62.05
$(x^2+y^2)_{K^+\pi^-\pi^+\pi^-}$ LHCb	3.23	65.27

Fit Results: all CPV allowed

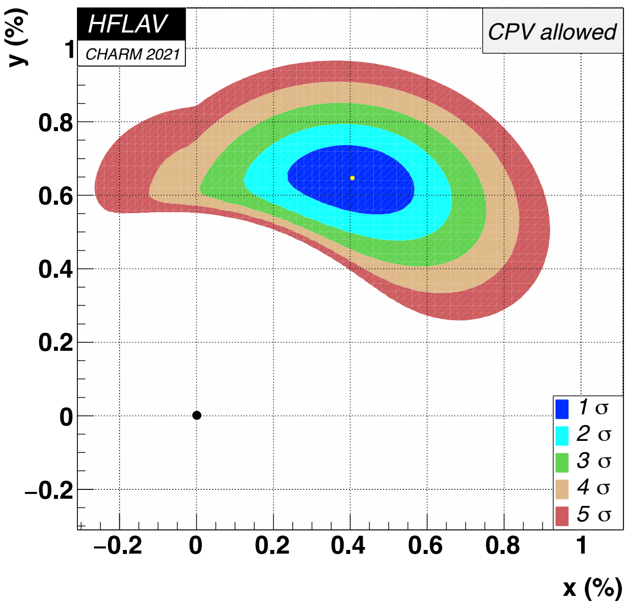


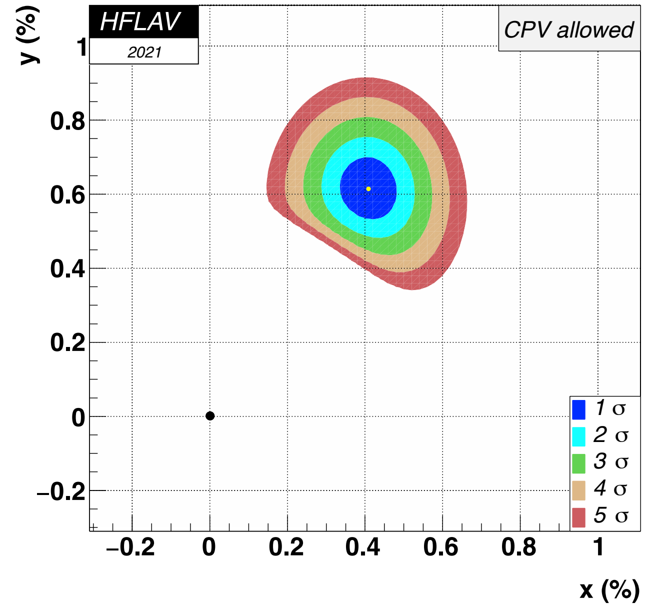


No mixing point (x,y) = (0,0):
$\Delta\chi^2$ = 2671, excluded at ≫ 11.5σ

No CPV point (|q/p|, φ) = (1,0):
$\Delta\chi^2$ = 5.815, consistent at 1.6σ

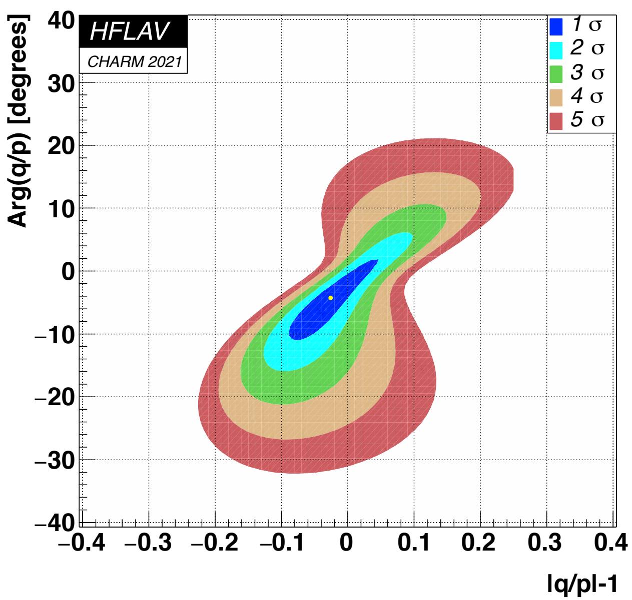
Fit Results: some history

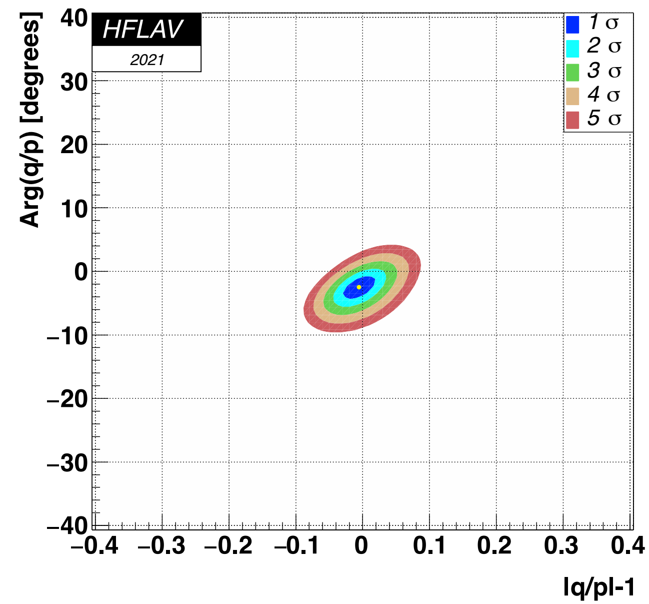




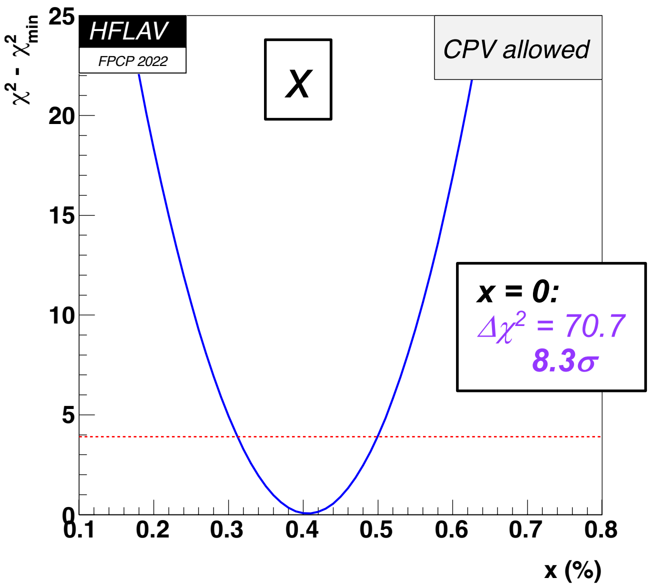
include LHCb
$D^0 \to K^0_S\, \pi^+\pi^-$
$(x_{CP}\,,\, y_{CP}\,,\, \Delta x,\, \Delta y)$

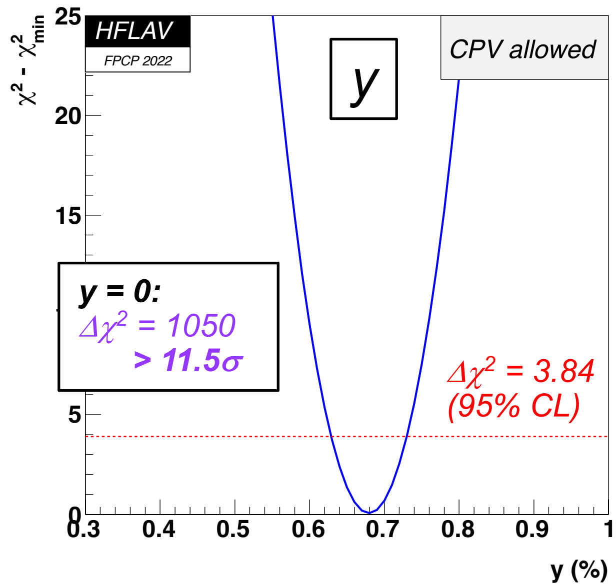
(summer 2021)

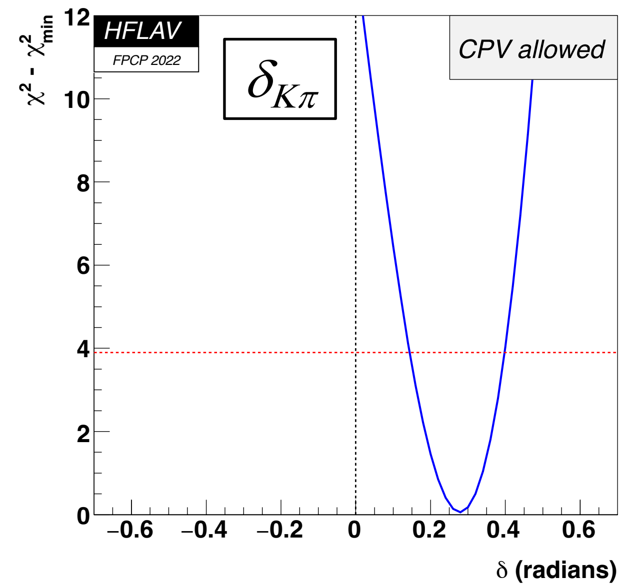


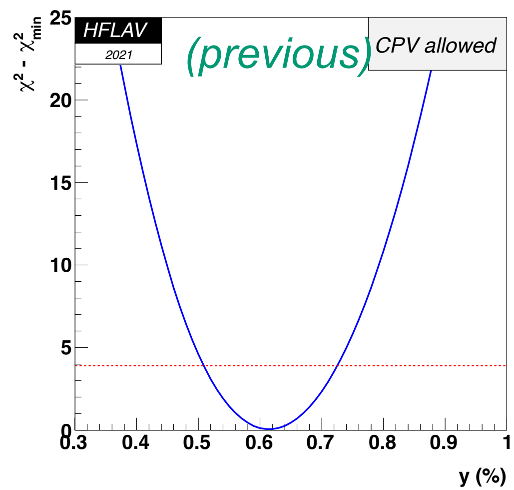


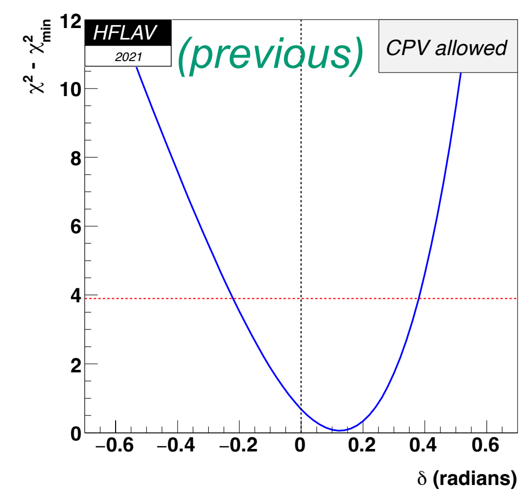
Fit Results: 1-d likelihoods (all CPV allowed)



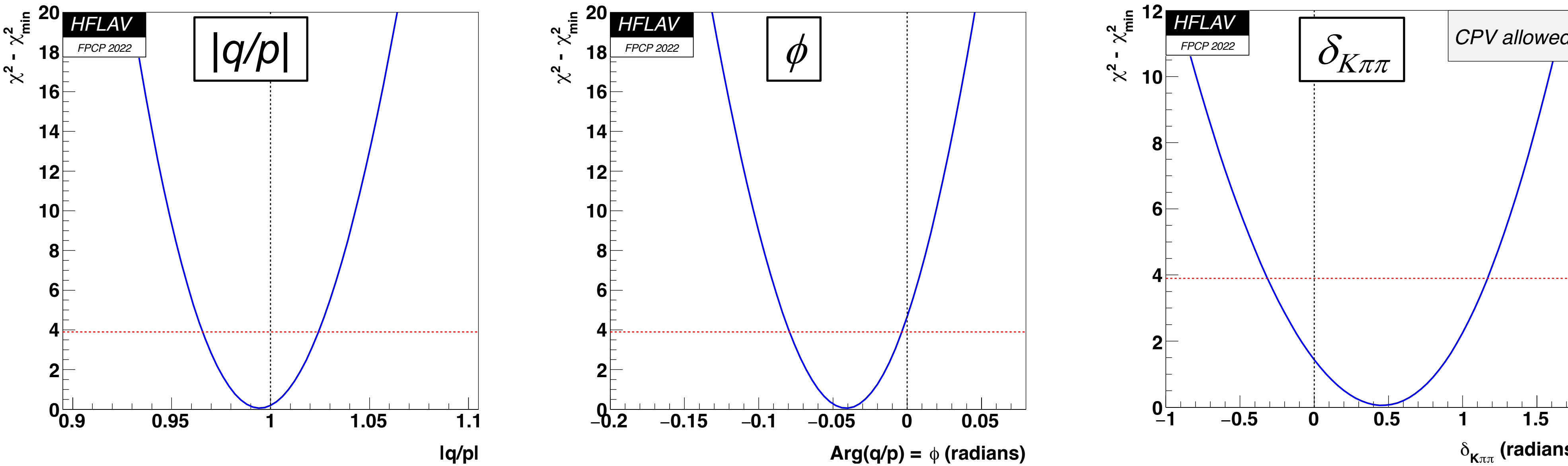








Fit Results: 1-d likelihoods (all CPV allowed)



"Superweak" limit: no sub-leading phases

Kagan and Sokoloff, PRD80, 076008 (2009)

Given the 4 decay amplitudes $A_f(D^0\to f)$ $\quad A_{\bar f}(D^0\to \bar f)$ $\quad \overline{A}_f(\overline{D}^0\to f)$ $\quad \overline{A}_{\bar f}(\overline{D}^0\to \bar f)$

If no sub-leading or NP weak phases, the general decay amplitudes simplify:

$$
\begin{aligned}
A_f &= A_f^t\, e^{i\phi_f^t}\left[1 + r_f e^{i(\phi_f+\delta_f)}\right] &\rightarrow&\quad A_f^t\, e^{i\phi_f^t} \\
A_{\bar f} &= A_{\bar f}^t\, e^{i(\varphi_{\bar f}^t+\Delta_{\bar f})}\left[1 + r_{\bar f} e^{i(\varphi_{\bar f}+\delta_{\bar f})}\right] &\rightarrow&\quad A_{\bar f}^t\, e^{i(\varphi_{\bar f}^t+\Delta_{\bar f})} \\
\overline{A}_{\bar f} &= A_f^t\, e^{-i\phi_f^t}\left[1 + r_f e^{i(-\phi_f+\delta_f)}\right] &\rightarrow&\quad A_f^t\, e^{-i\phi_f^t} \\
\overline{A}_f &= A_{\bar f}^t\, e^{i(-\varphi_{\bar f}^t+\Delta_{\bar f})}\left[1 + r_{\bar f} e^{i(-\varphi_{\bar f}+\delta_{\bar f})}\right] &\rightarrow&\quad A_{\bar f}^t\, e^{i(-\varphi_{\bar f}^t+\Delta_{\bar f})}
\end{aligned}
$$

NOTE:
no direct CPV in DCS decays

Defining 3 new parameters $\quad x_{12} \equiv \frac{2|M_{12}|}{\Gamma} \qquad y_{12} \equiv \frac{|\Gamma_{12}|}{\Gamma} \qquad \phi_{12} \equiv \mathrm{Arg}\left(\frac{M_{12}}{\Gamma_{12}}\right)$

One can derive

$$
\begin{aligned}
x &= \left[x_{12}^2 - y_{12}^2 + \sqrt{(x_{12}^2+y_{12}^2)^2 - 4x_{12}^2 y_{12}^2 \sin^2\phi_{12}}\right]^{1/2} \\
y &= \left[y_{12}^2 - x_{12}^2 + \sqrt{(x_{12}^2+y_{12}^2)^2 - 4x_{12}^2 y_{12}^2 \sin^2\phi_{12}}\right]^{1/2} \\
\left|\frac{q}{p}\right|^4 &= \frac{x_{12}^2 + y_{12}^2 + 2x_{12}y_{12}\sin\phi_{12}}{x_{12}^2 + y_{12}^2 - 2x_{12}y_{12}\sin\phi_{12}} \\
\tan 2\phi &= \frac{-\sin 2\phi_{12}}{\cos 2\phi_{12} + (y_{12}/x_{12})^2}
\end{aligned}
$$

NOTE:
CPV in mixing |q/p|, and CPV due to interference (ϕ) both depend on (x_{12}, y_{12}, ϕ_{12})

⇒ they can be related:

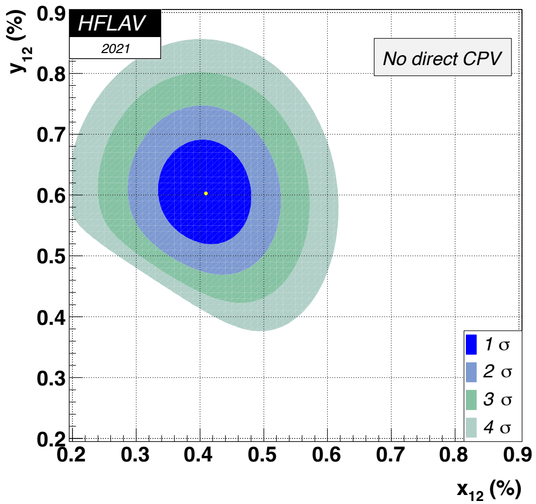
$$\tan\phi = \left(1 - \left|\frac{q}{p}\right|\right)\frac{x}{y}$$

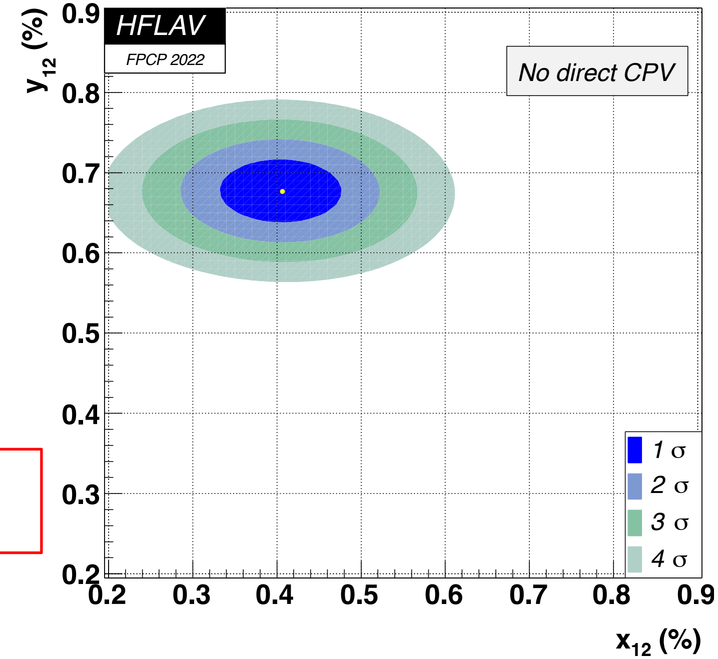
Ciuchini et al., PLB 655, 162 (2007)
Kagan and Sokoloff, PRD80, 076008 (2009)

Fit Results II: no sub-leading phases

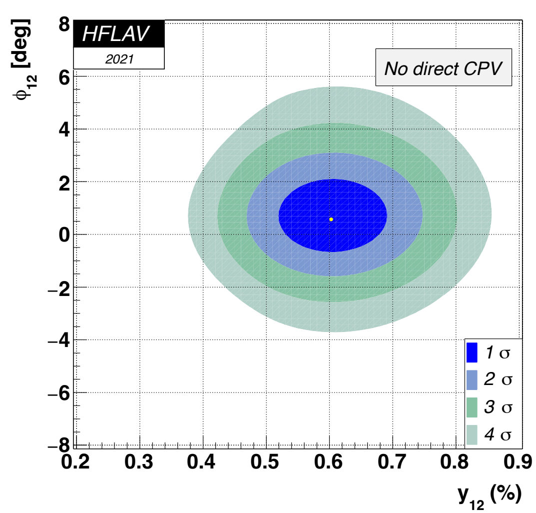
Fit 2c:
fit for x_{12}, y_{12}, ϕ_{12}:

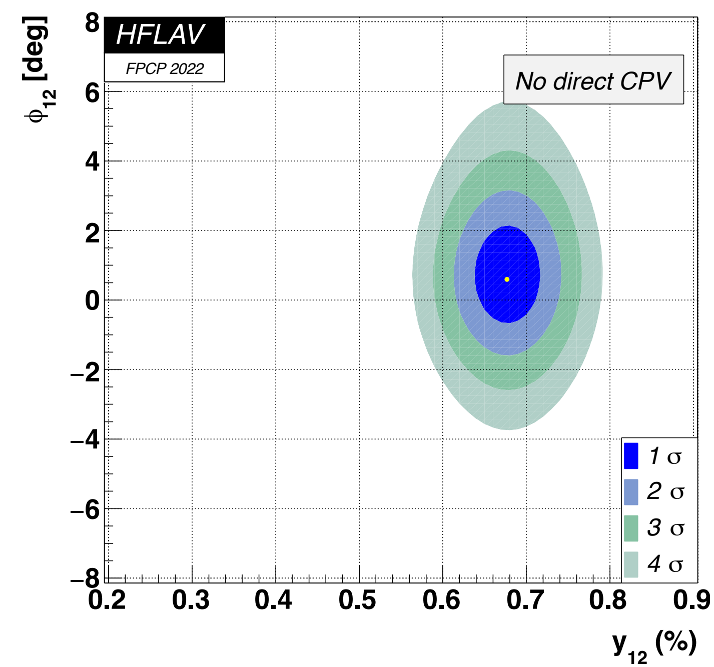
$\Rightarrow$ mixing is established, but no sign of indirect CPV





include LHCb 6 fb^{-1} y_{CP}





General case: dispersive and absorptive CPV

Kagan and Silvestrini, PRD 103, 053008 (2021):

In the general case, there could be sub-leading amplitudes that give arg(Γ_{12})$\neq$ 0. In this case we must add an additional phase ϕ_2^{Γ}:

$$x = \left[x_{12}^2 - y_{12}^2 + \sqrt{(x_{12}^2 + y_{12}^2)^2 - 4x_{12}^2 y_{12}^2 \sin^2(\phi_2^M - \phi_2^\Gamma)}\right]^{1/2}$$

$$y = \left[y_{12}^2 - x_{12}^2 + \sqrt{(x_{12}^2 + y_{12}^2)^2 - 4x_{12}^2 y_{12}^2 \sin^2(\phi_2^M - \phi_2^\Gamma)}\right]^{1/2}$$

$$\left|\frac{q}{p}\right|^4 = \frac{x_{12}^2 + y_{12}^2 + 2x_{12}y_{12}\sin(\phi_2^M - \phi_2^\Gamma)}{x_{12}^2 + y_{12}^2 - 2x_{12}y_{12}\sin(\phi_2^M - \phi_2^\Gamma)}$$

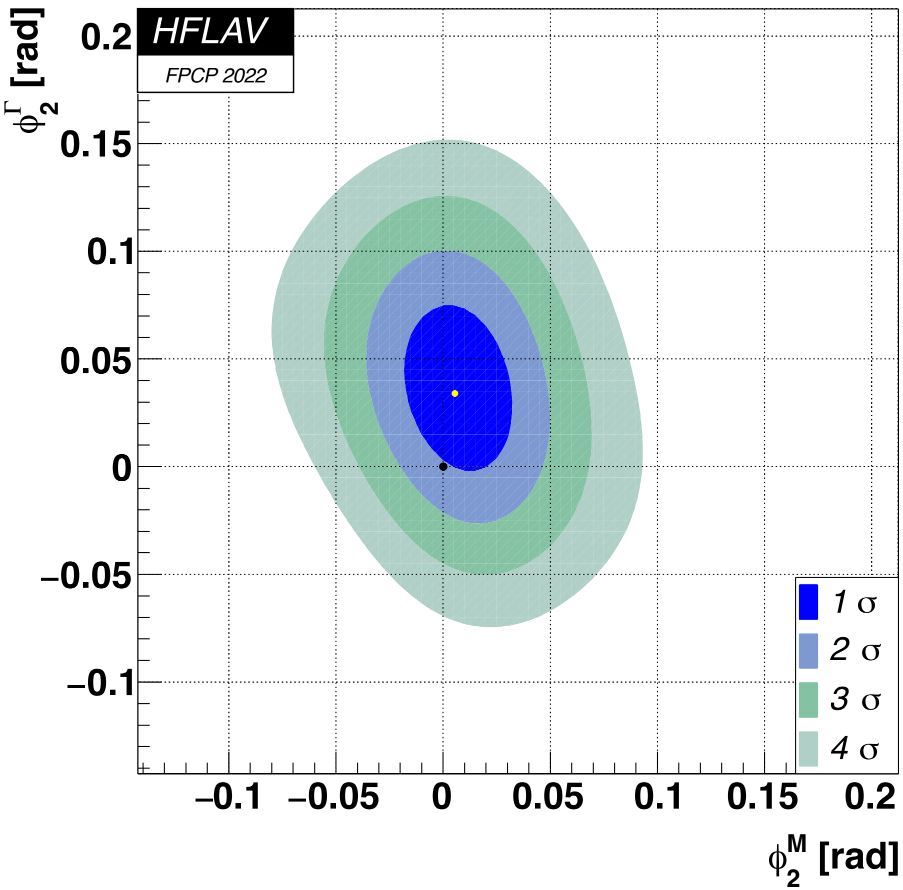
$$\tan 2\phi = -\frac{x_{12}^2 \sin 2\phi_2^M + y_{12}^2 \sin 2\phi_2^\Gamma}{x_{12}^2 \cos 2\phi_2^M + y_{12}^2 \cos 2\phi_2^\Gamma}$$

- *2 types of indirect CPV: that due to a dispersive phase, that due to an absorptive phase (traditional way: CPV in mixing, CPV in interference between a mixed an unmixed amplitude)*
- *The subscript "2" refers to the phase reference: the phase of the ΔU = 2 component of the <D^0|M(Γ)|D^0bar> amplitude*
- *Still fitting for 4 fundamental parameters: (x, y, |q/p|, ϕ) $\rightarrow$ (x_{12} , y_{12} , ϕ_2^M, ϕ_2^Γ)*
- *The fitter discriminates between ϕ_2^M and ϕ_2^Γ because CPV in CP-eigenstate final states depend only on ϕ_2^M (due to there being no strong phase difference)*

Fit for dispersive and absorptive CPV

Kagan and Silvestrini, PRD 103, 053008 (2021)

***Fit 4**:*
fit for x_{12}, y_{12}, ϕ^M, ϕ^Γ:



$$x_{12} = (0.407 \pm 0.047)\,\%$$
$$y_{12} = (0.675 \pm 0.026)\,\%$$

[essentially the same as for super-weak fit]

$$\phi_2^M = (0.31 \pm 0.93)^\circ$$
$$\phi_2^\Gamma = (1.95 \pm 1.45)^\circ$$

Observables: A_{CP} $[D^0 \to K^+K^-/\pi^+\pi^-]$

Index	Observable	Value	Source
54-55	$A_{CP}{}^K$ $A_{CP}{}^\pi$	$(0.00 \pm 0.34 \pm 0.13)\%$ $(-0.24 \pm 0.52 \pm 0.22)\%$	BaBar 385.8 fb^{-1} near $\Upsilon(4S)$ resonance
56-57	$A_{CP}{}^K$ $A_{CP}{}^\pi$	$(-0.43 \pm 0.30 \pm 0.11)\%$ $(0.43 \pm 0.52 \pm 0.12)\%$	Belle 540 fb^{-1} near $\Upsilon(4S)$ resonance
58-59	$A_{CP}{}^K$ $A_{CP}{}^\pi$	$(-0.32 \pm 0.21)\%$ $(0.31 \pm 0.22)\%$	CDF 9.7 fb^{-1} $\bar{p}p$ collisions at $\sqrt{s} = 1.96$ TeV $(\langle t\rangle_K - \langle t\rangle_\pi)/\tau_D = 0.27 \pm 0.01$
60	$A_{CP}{}^K - A_{CP}{}^\pi$	$(-0.154 \pm 0.029)\%$	LHCb 8.9 fb^{-1} pp collisions at $\sqrt{s} = 7, 8, 13$ TeV $D^{*+} \to D^0\pi^+$ and $B \to D^0\mu^- X$ flavor tags $(\langle t\rangle_K - \langle t\rangle_\pi)/\tau_D = 0.115 \pm 0.002$; $\overline{\langle t\rangle}/\tau_D = 1.71 \pm 0.10$

First observation of dCPV: *PRL 122, 211803 (2019)*

$$A_{CP} \equiv \frac{\Gamma(D^0(t) \to f) - \Gamma(\overline{D}^0(t) \to f)}{\Gamma(D^0(t) \to f) + \Gamma(\overline{D}^0(t) \to f)}$$

$$\approx a^f_{\rm direct} - A_\Gamma \frac{\langle t\rangle}{\tau_D}$$

U-spin (d↔ s) symmetry: *[Grossman, Kagan, Nir, PRD 75, 036008 (2007)]*

$$A_{CP}(K^+K^-) \approx -A_{CP}(\pi^+\pi^-)$$

$$\Rightarrow \Delta A_{CP} \equiv A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) \approx 2a^{\rm dir}_{CP}$$

Summary

Parameter	No direct CPV in DCS decays	CPV-allowed	95% CL Interval
x (%)	–	0.406 ± 0.048	[0.312, 0.499]
y (%)	–	0.679 ± 0.026	[0.629, 0.730]
$\delta_{K\pi}$ (°)	–	15.9 ± 3.6	[8.26, 22.8]
R_D (%)	–	0.344 ± 0.002	[0.341, 0.348]
A_D (%)	–	-0.78 ± 0.35	[−1.47, −0.08]
$\lvert q/p \rvert$	–	0.994 ± 0.015	[0.97, 1.02]
ϕ (°)	–	-2.4 ± 1.1	[−4.54, −0.21]
$\delta_{K\pi\pi}$ (°)	–	26.3 ± 21.8	[−18.1, 66.9]
A_π(%)	–	0.043 ± 0.137	[−0.23, 0.31]
A_K(%)	–	-0.116 ± 0.137	[−0.38, 0.15]
x_{12} (%)	0.407 ± 0.047		[0.313, 0.500]
y_{12} (%)	0.675 ± 0.025		[0.624, 0.726]
ϕ_{12}(°)	0.60 ± 0.91		[−1.20, 2.46]
ϕ_2^M(°)	0.31 ± 0.93		[−1.59, 2.19]
ϕ_2^Γ(°)	1.95 ± 1.45		[−0.87, 4.84]

- *D^0's definitely mix*
- *there is direct CPV in singly Cabibbo-suppressed decays $D^0 \to K^+ K^-$, $D^0 \to \pi^+ \pi^-$*
- *there is no evidence for indirect CPV: (|q/p|, ϕ) consistent with (1,0) at 1.6σ level. This slight disagreement is in ϕ_2^Γ , the absorptive phase. Phase difference ϕ_{12} is consistent with 0.*