Review of BSM models for muon g-2

2022 Conference on Flavor Physics and CP Violation (FPCP2022) - 25.05.2022

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1222·2022 800 ANNI

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Plan of the Talk

1. Status of the muon g-2 as of mid 2022

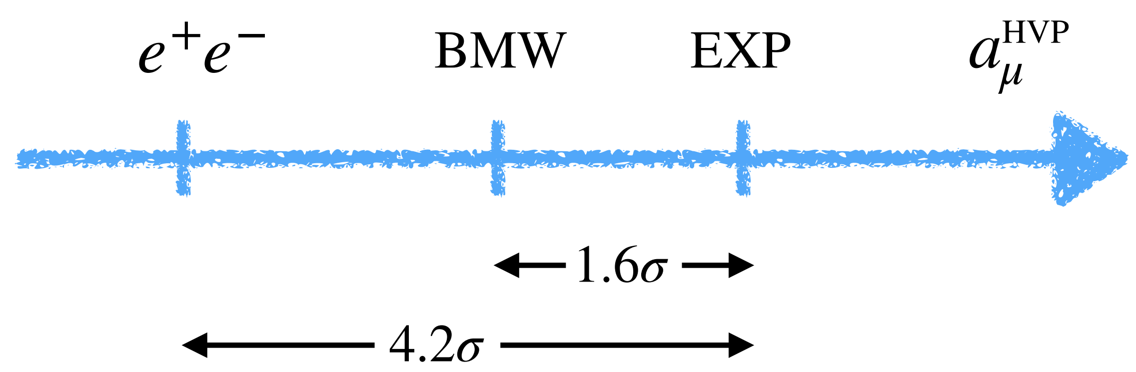
[exp + SM theory review:
see talks by E. Barlas Yucel + M. Golterman]

2. New physics & the "old" muon g-2 puzzle

around for ~ 20 years: one observable and many BSM possibilities

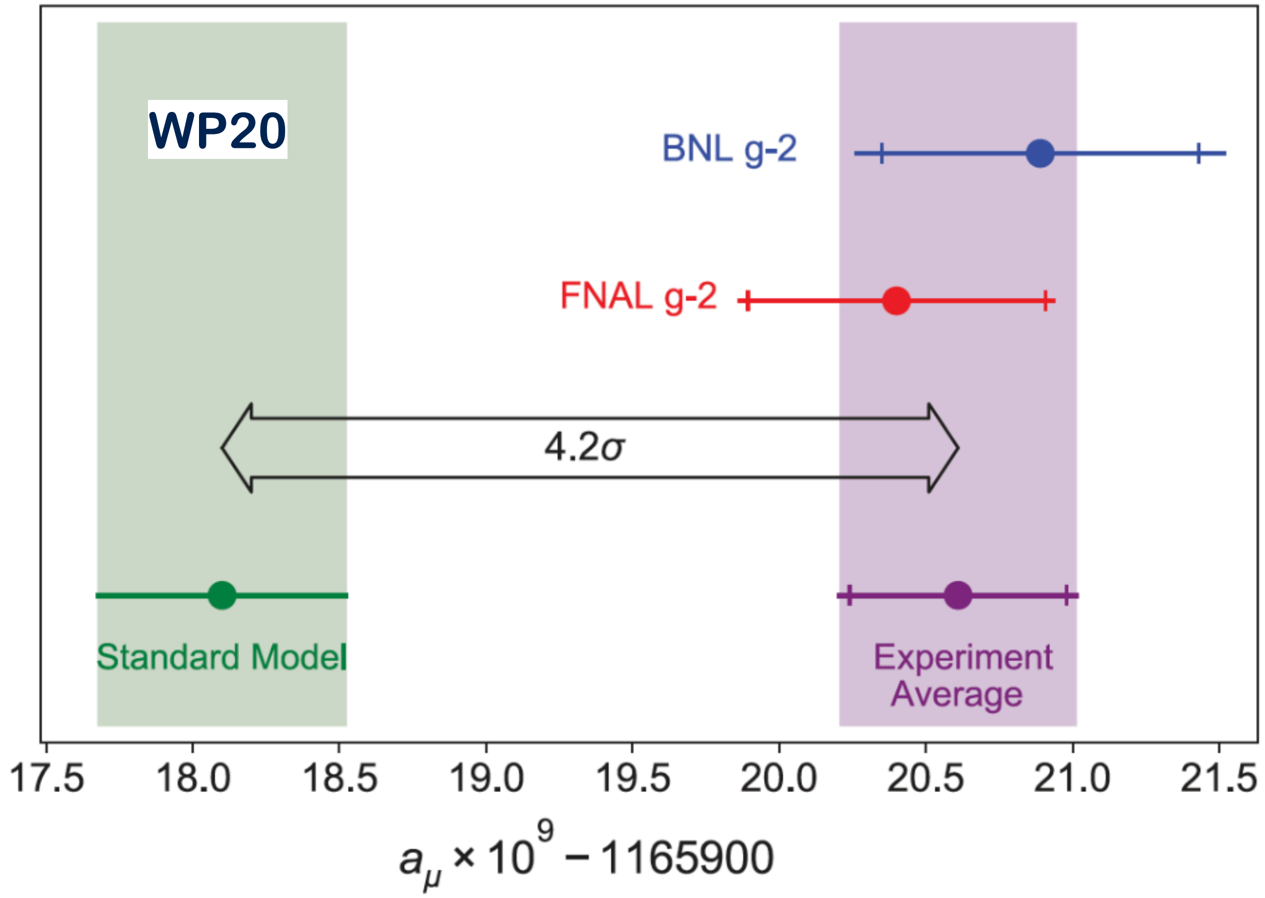
3. New physics & the "new" muon g-2 puzzle

Assuming that the BMW lattice determination of HVP contribution is correct,
what's going on with e^+e^- data ?



FNAL confirms BNL

WP20 = White Paper of the Muon g-2 Theory Initiative: arXiv:2006.04822



3.7 σ

3.3 σ

4.2 σ

$a_\mu^{EXP} = (116592089 \pm 63) \times 10^{-11}$ [0.54ppm] BNL E821

$a_\mu^{EXP} = (116592040 \pm 54) \times 10^{-11}$ [0.46ppm] FNAL E989 Run 1

$a_\mu^{EXP} = (116592061 \pm 41) \times 10^{-11}$ [0.35ppm] WA

- *FNAL aims at* 16×10^{-11}*. First 4 runs completed, 5th in progress*
- *Muon g-2 proposal at J-PARC: Phase-1 with similar BNL precision*

Breakdown of SM contributions

- a_μ from WP20 (w/o BMW lattice result)

[updated from Colangelo EPS-HEP2021 proceeding]

Contribution	Value $\times 10^{11}$	References
Experiment (E821)	116 592 089(63)	Ref. [3]
Experiment (FNAL)	116 592 040(54)	Ref. [1]
Experiment (World-Average)	116 592 061(41)	
HVP LO (e^+e^-)	6931(40)	Refs. [6–11]
HVP NLO (e^+e^-)	−98.3(7)	Ref. [11]
HVP NNLO (e^+e^-)	12.4(1)	Ref. [12]
HVP LO (lattice, *udsc*)	7116(184)	Refs. [13–21]
HLbL (phenomenology)	92(19)	Refs. [22–34]
HLbL NLO (phenomenology)	2(1)	Ref. [35]
HLbL (lattice, *uds*)	79(35)	Ref. [36]
HLbL (phenomenology + lattice)	90(17)	
QED	116 584 718.931(104)	Refs. [37, 38]
Electroweak	153.6(1.0)	Refs. [39, 40]
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)	
HLbL (phenomenology + lattice + NLO)	92(18)	
Total SM Value	116 591 810(43)	
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	251(59)	

HVP LO is the bottle-neck of the SM prediction

Breakdown of SM contributions

- a_μ from WP20 (w/o BMW lattice result)

$$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} \equiv a_\mu^{\rm NP} = 251\,(59) \times 10^{-11} \qquad \text{(4.2}\sigma\text{ discrepancy!)}$$

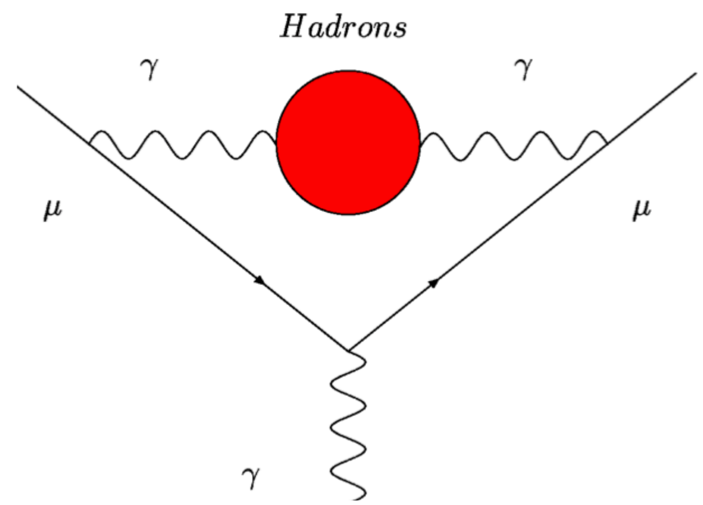
$$\underbrace{(0.1)_{\rm QED}\,, \quad (1)_{\rm EW}\,, \quad (18)_{\rm HLbL}\,, \quad (40)_{\rm HVP}\,,}_{(43)_{\rm TH}} \quad (41)_{\delta a_\mu^{\rm EXP}}\,.$$

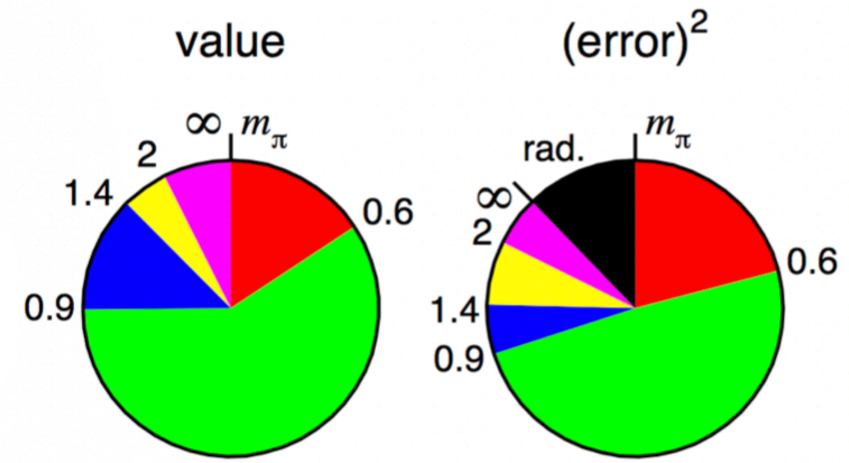
($\delta a_\mu^{\rm EXP} \approx 16 \times 10^{-11}$ by the E989 Muon g-2 exp. in a few years)

→ HVP LO is the bottle-neck of the SM prediction

Hadronic LO contribution

- Hadron Vacuum Polarization (HVP)





Keshavarzi, Nomura, Teubner 2018

$\sim \quad \sigma(e^+e^- \to \gamma^* \to \text{hadrons})$

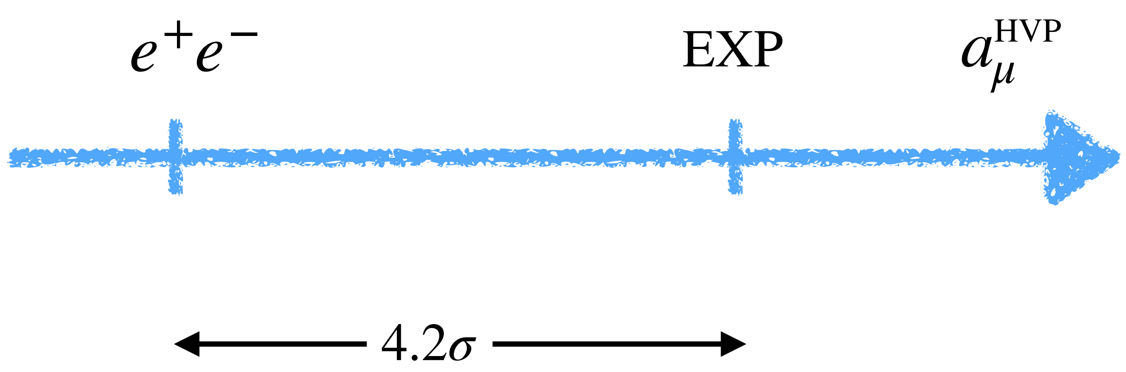
$$(a_\mu^{\rm HVP})_{e^+e^-} = \frac{1}{4\pi^3}\int_{m_\pi^2}^{\infty} ds\, K(s)\, \sigma_{\rm had}(s) \qquad K(s) \approx m_\mu^2/3s \quad \text{for} \quad \sqrt{s} \gg m_\mu$$

$a_\mu^{HLO} = 6895\ (33) \times 10^{-11}$	F. Jegerlehner, arXiv:1711.06089
$= 6939\ (40) \times 10^{-11}$	Davier, Hoecker, Malaescu, Zhang, arXiv:1908.00921
$= 6928\ (24) \times 10^{-11}$	Keshavarzi, Nomura, Teubner, arXiv:1911.00367
$= 6931\ (40) \times 10^{-11}\ (0.6\%)$	WP20 value

“old” muon g-2 puzzle

$$(a_\mu^{\rm HVP})_{\rm EXP} = a_\mu^{\rm EXP} - a_\mu^{\rm SM,\,rest}$$

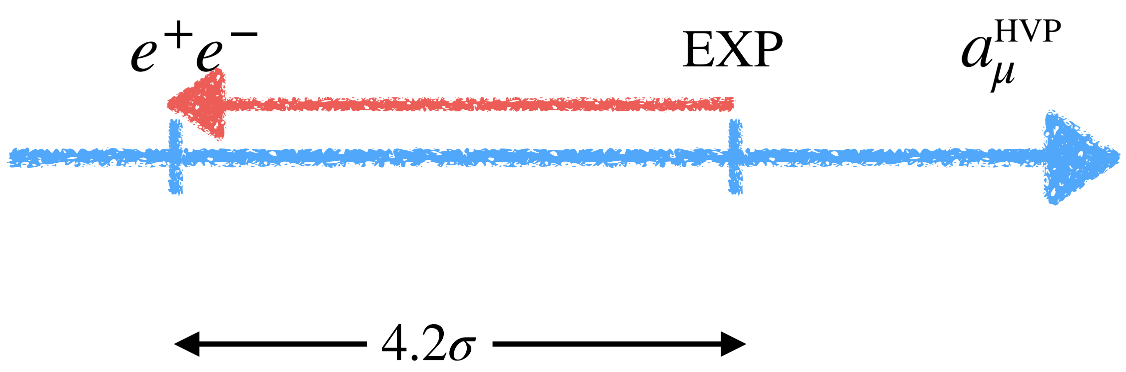
$$(a_\mu^{\rm HVP})^{\rm WP20}_{e^+e^-} = 6931(40) \times 10^{-11}$$



“old” muon g-2 puzzle

$$(a_\mu^{\rm HVP})_{\rm EXP} = a_\mu^{\rm EXP} - a_\mu^{\rm SM,\,rest} - a_\mu^{\rm NP}$$

$$(a_\mu^{\rm HVP})_{e^+e^-}^{\rm WP20} = 6931(40) \times 10^{-11}$$



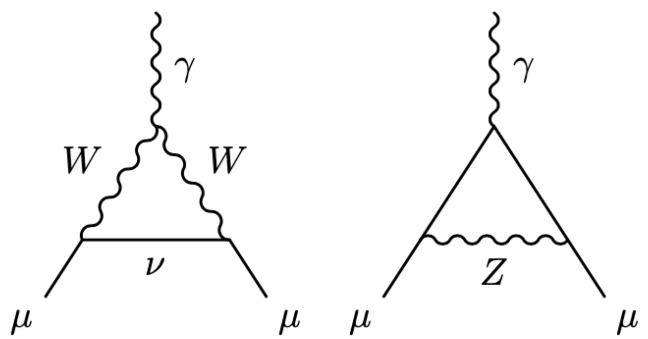
“old puzzle”: a direct NP contribution to a_μ *is required in order to match the EXP value*

$$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} \equiv a_\mu^{\rm NP} \approx (a_\mu^{\rm SM})_{\rm EW} \approx \frac{m_\mu^2}{16\pi^2 v^2} \approx 200 \times 10^{-11}$$

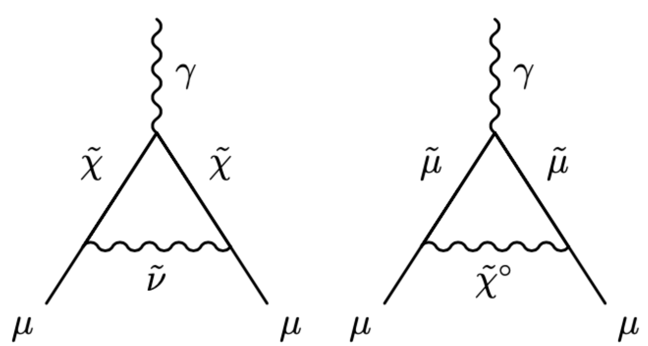
- NP is at the weak scale ($\Lambda \approx v$) and weakly coupled to SM particles.
- NP is very heavy ($\Lambda \gg v$) and strongly coupled to SM particles.
- NP is very light ($\Lambda \lesssim 1$ GeV) and feebly coupled to SM particles.

$\Lambda \approx v$: SUSY

- Motivated by hierarchy problem and WIMP DM but disfavoured by LHC bounds

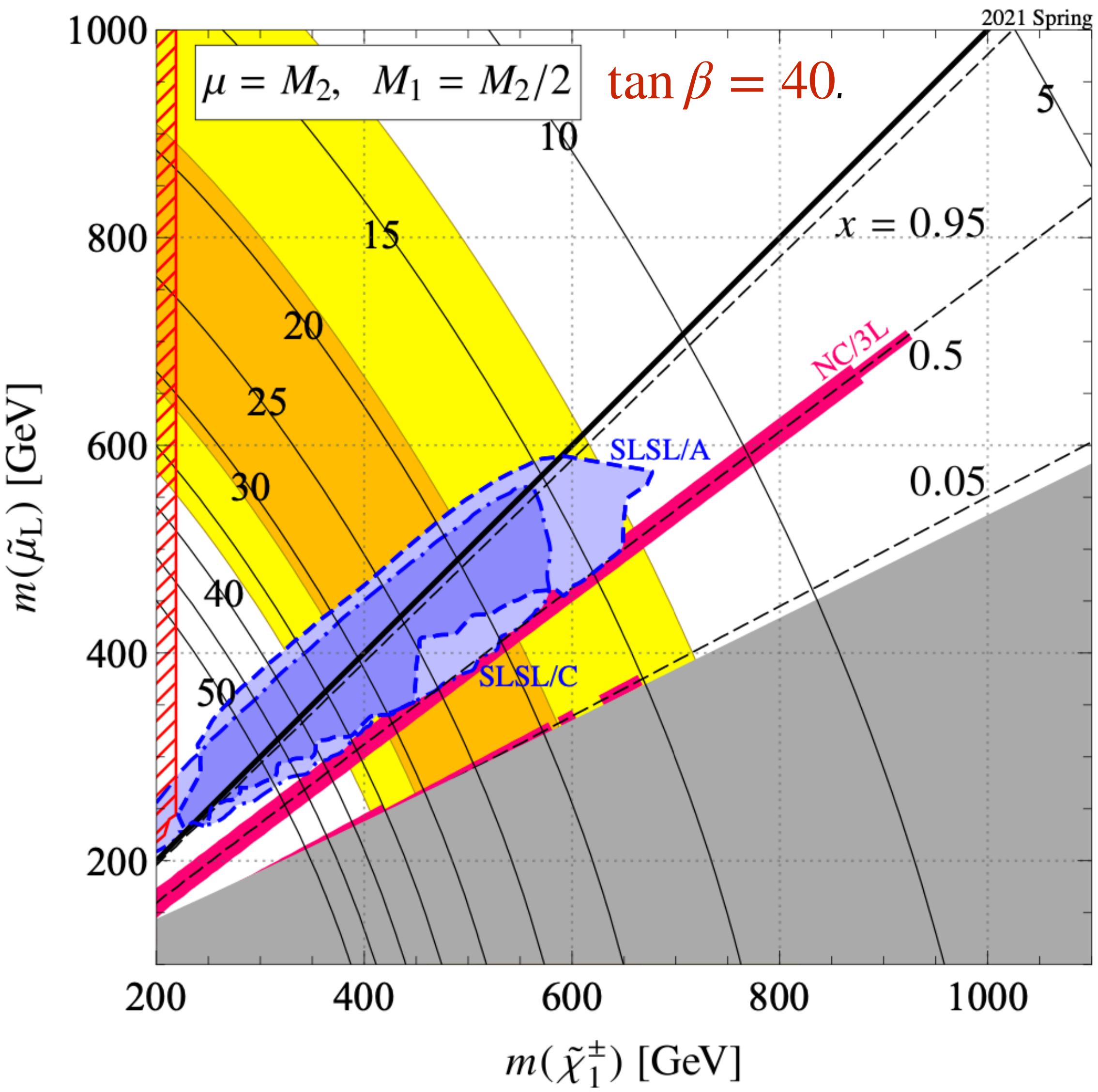


$$(a_\mu^{\rm SM})_{weak} \approx \frac{g^2 m_\mu^2}{32\pi^2 M_W^2} \approx 2 \times 10^{-9}$$



$$a_\mu^{\rm SUSY} \approx \frac{g^2 m_\mu^2 \tan\beta}{32\pi^2 \tilde{m}^2} \underbrace{\approx 2 \times 10^{-9}}_{\tilde{m} = 500{\rm GeV}\ \&\ \tan\beta = 40}$$

[Endo, Hamaguchi, Iwamoto, Kitahara 2104.03217]



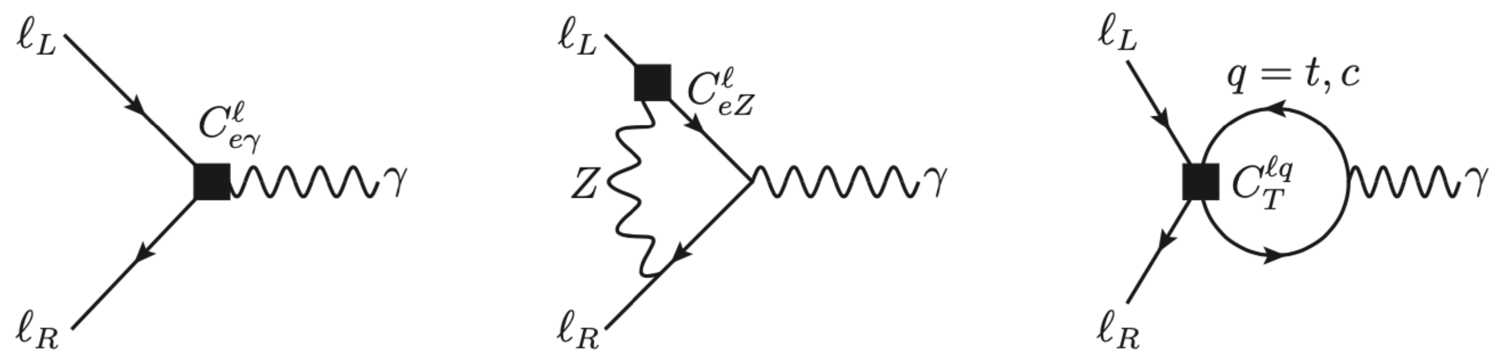
Orange (yellow) regions satisfy the muon g-2 anomaly at the 1σ (2σ) level

$\Lambda \gg v$: SMEFT

- The correct language is that of the SM EFT

[Buttazzo, Paradisi 2012.02769
Aebischer et al 2102.08954]

$$\mathcal{L} = \sum_{V=B,W} \frac{C^{\ell}_{eV}}{\Lambda^2} (\bar{\ell}_L \sigma^{\mu\nu} e_R) H V_{\mu\nu} + \sum_{q=c,t} \frac{C^{\ell q}_T}{\Lambda^2} (\bar{\ell}_L \sigma_{\mu\nu} e_R)(\overline{Q}_L \sigma^{\mu\nu} q_R) + h.c.$$



$$\Delta a_\ell \simeq \frac{4 m_\ell^2}{e\Lambda^2} \frac{v}{m_\ell} \left(C^{\ell}_{e\gamma} - \frac{3\alpha}{2\pi} \frac{c_W^2 - s_W^2}{s_W c_W} C^{\ell}_{eZ} \log \frac{\Lambda}{m_Z} \right) - \sum_{q=c,t} \frac{4 m_\ell^2}{\pi^2} \frac{m_q}{m_\ell} \frac{C^{\ell q}_T}{\Lambda^2} \log \frac{\Lambda}{m_q}$$

$$\frac{|\Delta a_\mu|}{3 \times 10^{-9}} \approx \left(\frac{250 \text{ TeV}}{\Lambda} \right)^2 |C^{\mu}_{e\gamma}|$$

strongly-coupled NP:
upper limit set by partial-wave unitarity $\sqrt{s} \lesssim 900$ *TeV*

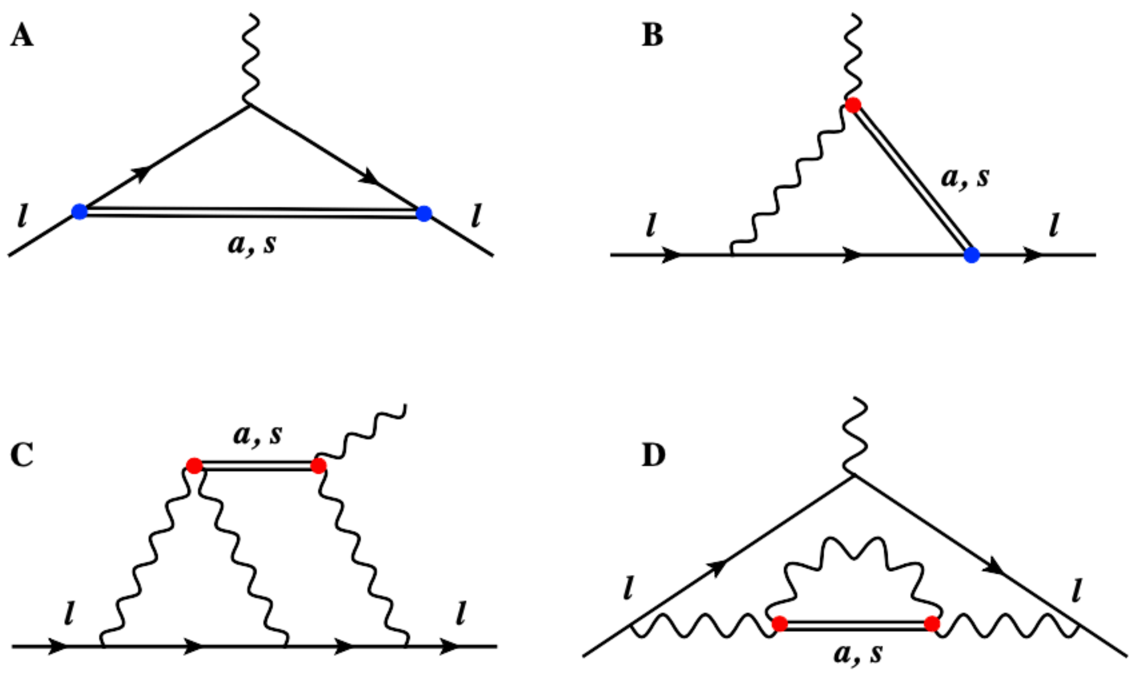
[Allwicher, LDL, Fedele, Mescia, Nardecchia 2105.13981]

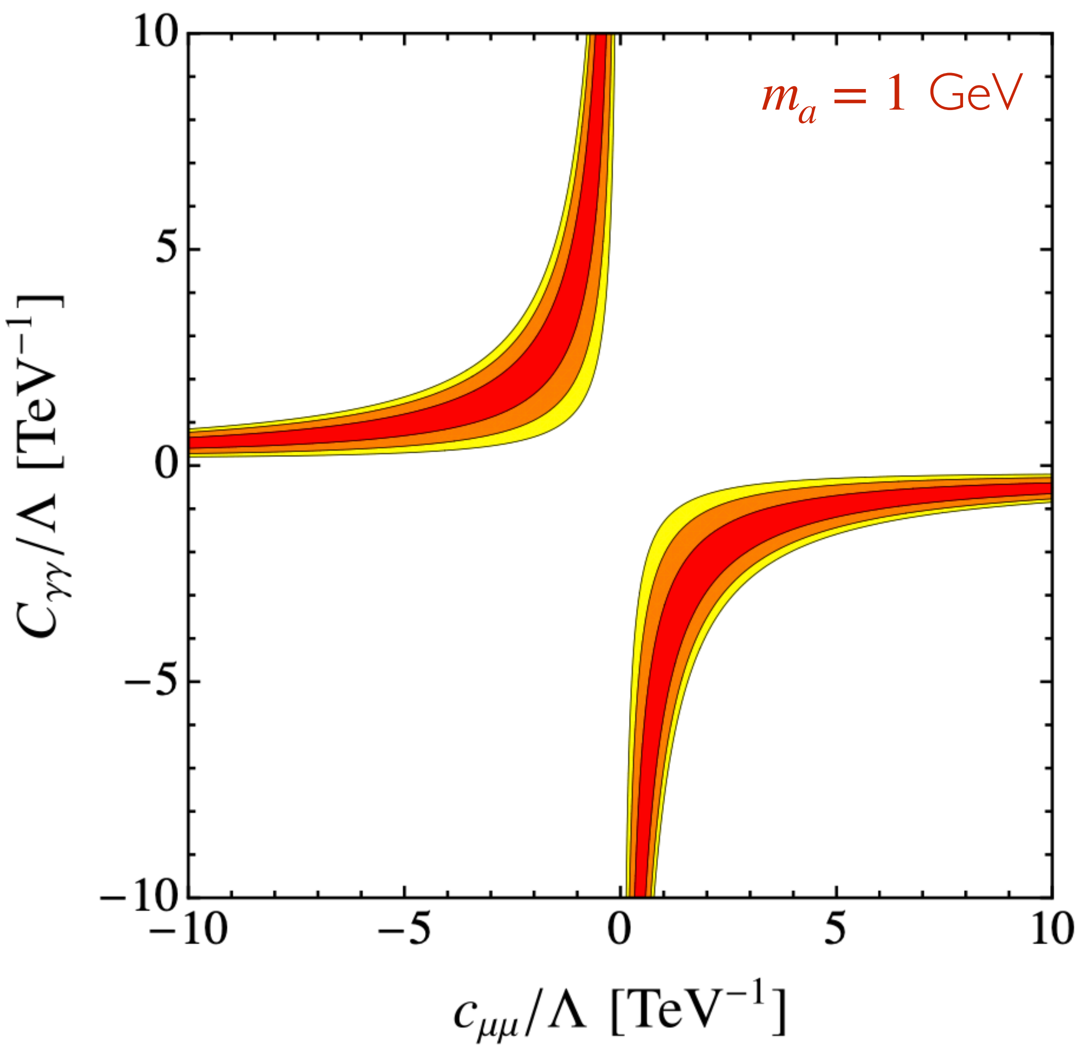
$m_a \lesssim 1$ GeV : axion-like particles

- Axion-like particle EFT

[Marciano, Masiero, Paradisi, Passera 1607.01022
Bauer, Neubert, Thamm 1704.08207 …]

$$\mathcal{L} = e^2 C_{\gamma\gamma} \frac{a}{\Lambda} F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{c_{\mu\mu}}{2} \frac{\partial^\nu a}{\Lambda} \bar{\mu} \gamma_\nu \gamma_5 \mu$$



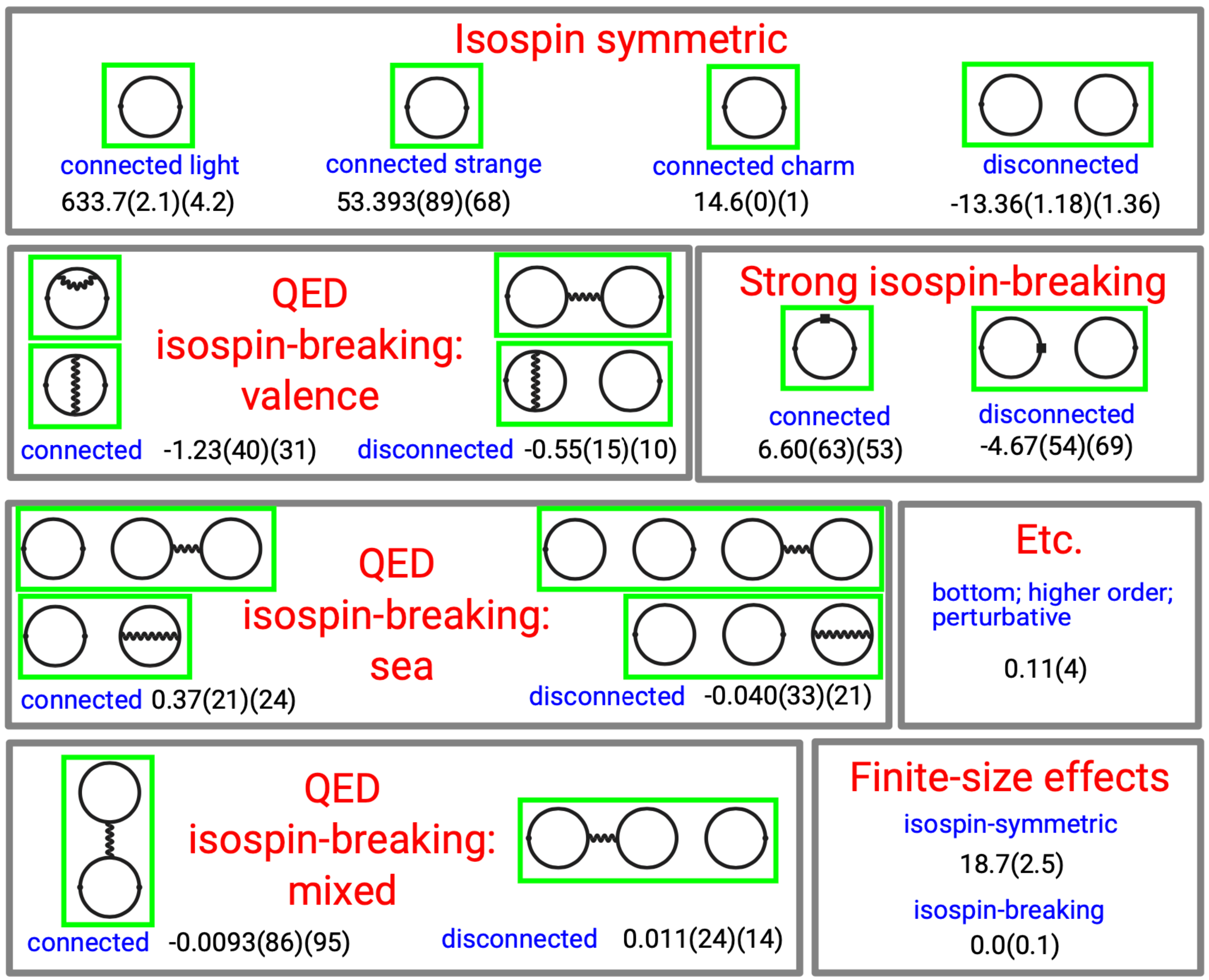


$$\Delta a_\mu = \frac{m_\mu^2}{\Lambda^2} \left[\frac{12\alpha^3}{\pi} C_{\gamma\gamma}^2 \ln^2 \frac{\Lambda^2}{m_\mu^2} - \frac{(c_{\mu\mu})^2}{16\pi^2} h_1\left(\frac{m_a^2}{m_\mu^2}\right) - \frac{2\alpha}{\pi} c_{\mu\mu} C_{\gamma\gamma} \ln \frac{\Lambda^2}{m_\mu^2} \right]$$

LO HVP from lattice QCD

Great progress in lattice QCD results. The BMW collaboration reached 0.8% precision:

$a_\mu^{HLO} = 7075\underbrace{(23)_{stat}(50)_{syst}}_{55} \times 10^{-11}$.

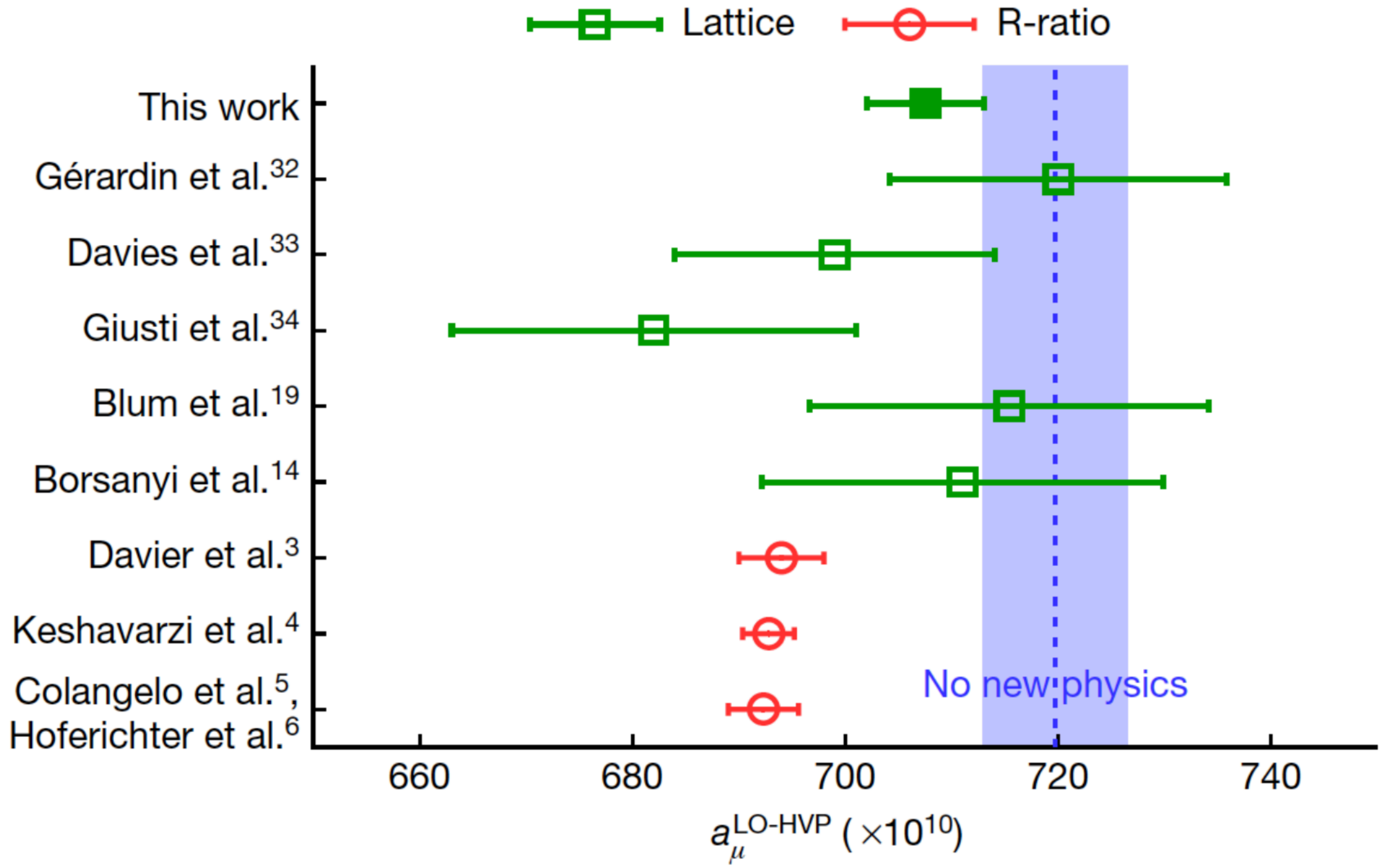


$10^{10} \times a_\mu^{LO\text{-}HVP} = 707.5(2.3)_{stat}(5.0)_{sys}[5.5]_{tot}$

Borsanyi et al (BMWc), Nature 2021

LO HVP from lattice QCD

Great progress in lattice QCD results. The BMW collaboration reached 0.8% precision:
$a_\mu^{HLO} = 7075\underbrace{(23)_{stat}(50)_{syst}}_{55} \times 10^{-11}$. Some tension with dispersive evaluations. BMWc 2021



Borsanyi et al (BMWc), Nature 2021

“new” muon g-2 puzzle

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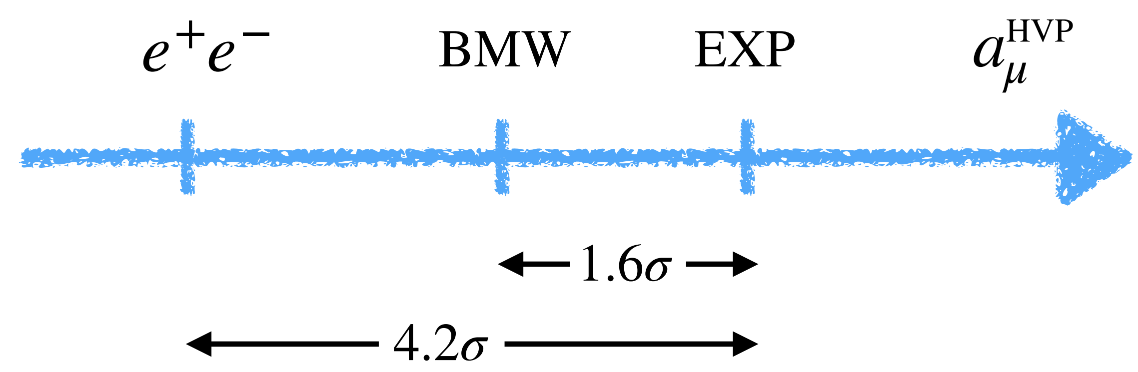
$$(a_\mu^{\rm HVP})_{e^+e^-}^{\rm WP20} = 6931(40) \times 10^{-11}$$

$$(a_\mu^{\rm HVP})_{\rm BMW} = 7075(55) \times 10^{-11}$$



“new puzzle”: if BMW is correct, the “old” g-2 discrepancy (4.2σ) would be basically gone

→ *however, this brings in a new tension with e^+e^- data (2.2σ)*

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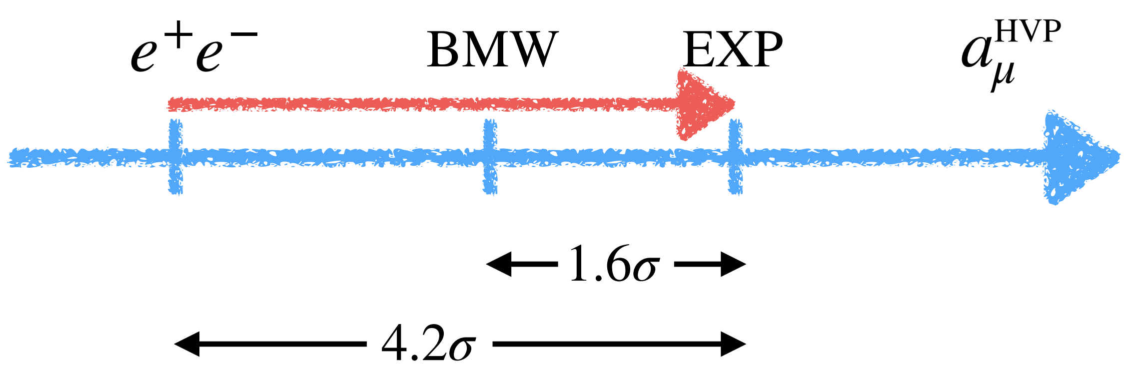

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NP in $\sigma_{\rm had}(e^+e^- \to \text{hadrons})$ such that [LDL, Masiero, Paradisi, Passera 2112.08312]

1. $(a_\mu^{\rm HVP})^{\rm WP20}_{e^+e^-} \approx (a_\mu^{\rm HVP})_{\rm EXP}$
2. the approximate agreement between BMW and EXP is not spoiled
3. w/o a direct contribution $a_\mu^{\rm NP}$ (i.e. NP not in muons)

Muon g-2 $\rightleftarrows$ $\Delta\alpha$ connection

- Can Δa_μ be due to a missing contribution in $\sigma_{\rm had}$? [Marciano, Passera, Sirlin 0804.1142 & 1001.4528; Keshavarzi, Marciano, Passera, Sirlin 2006.12666]

→ *an upward shift of $\sigma_{\rm had}$ induces an increase of $\Delta\alpha^{(5)}_{\rm had}(M_Z)$*

$$\alpha(M_Z) = \frac{\alpha}{1 - \Delta\alpha_{\rm lep}(M_Z) - \Delta\alpha^{(5)}_{\rm had}(M_Z) - \Delta\alpha_{\rm top}(M_Z)}$$

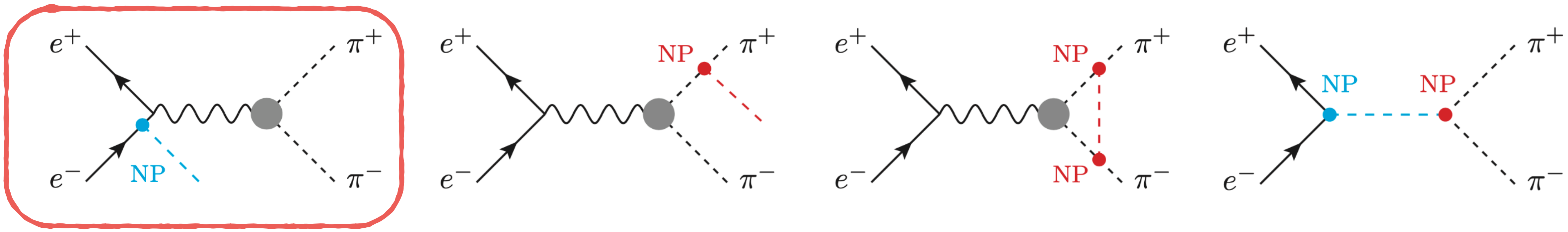
$$a_\mu^{\rm HLO} \simeq \frac{m_\mu^2}{12\pi^3}\int_{4m_\pi^2}^{\infty} ds\, \frac{\sigma(s)}{s}\,, \qquad \Delta\alpha^{(5)}_{\rm had} = \frac{M_Z^2}{4\pi\alpha^2}\int_{4m_\pi^2}^{\infty} ds\, \frac{\sigma(s)}{M_Z^2 - s}$$

- disfavoured by the EW fit (at about 2σ), if the shift happens at $\sqrt{s} \gtrsim 1\,{\rm GeV}$

→ *selects <u>light NP</u> inducing a sub-GeV modification of $\sigma_{\rm had}$ (basically, $e^+e^- \to \pi^+\pi^-$ channel)*

[See also Crivellin, Hoferichter, Manzari, Montull 2003.04886; Malaescu, Schott 2008.08107; Colangelo, Hoferichter, Stoffer 2010.07943]

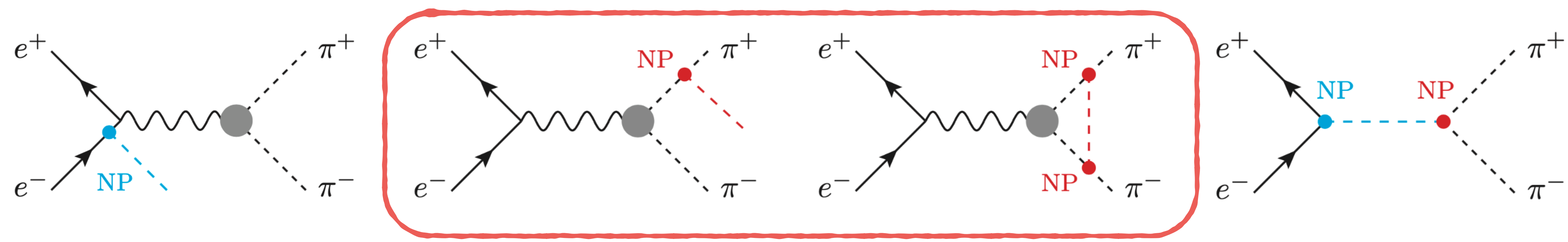
Light NP in σ_{had}



1. NP coupled only to electrons → severe bounds

[See however
Darmé, Grilli di Cortona, Nardi 2112.09139
NP in Bhabha scattering can affect extraction of KLOE luminosity ? → backup slides]

Light NP in σ_{had}



2. NP coupled only to hadrons

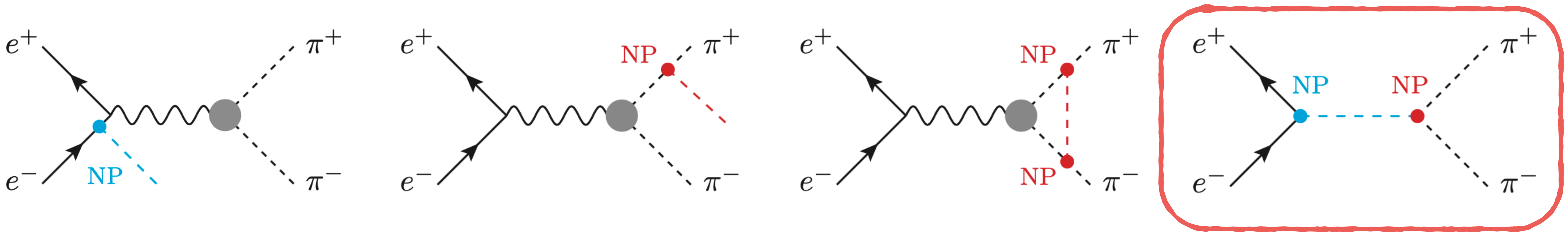
FSR effects due to NP should be included into $\sigma_{\text{had}}(s)$*, not easy to be accounted for…*
(depend on exp. cuts and mass of NP)

however, we know that in the QED case

$$(a_\mu^{\text{HVP}})^{\text{FSR}}_{e^+e^-} \approx 50 \times 10^{-11} \longleftrightarrow |(a_\mu^{\text{HVP}})_{\text{BMW}} - (a_\mu^{\text{HVP}})^{\text{WP20}}_{e^+e^-}| \approx 150 \times 10^{-11}$$

moreover, we expect an extra $(g_{\text{NP}}/e)^2 \ll 1$ *suppression*

Light NP in σ_{had}



3. NP coupled both to hadrons and electrons

$$(a_\mu^{\text{HVP}})_{e^+e^-} = \frac{\alpha}{\pi^2}\int_{m_{\pi^0}^2}^{\infty} \frac{\mathrm{d}s}{s} K(s)\,\mathrm{Im}\,\Pi_{\text{had}}(s) = \frac{1}{4\pi^3}\int_{m_{\pi^0}^2}^{\infty} \mathrm{d}s\, K(s)\sigma_{\text{had}}(s)$$

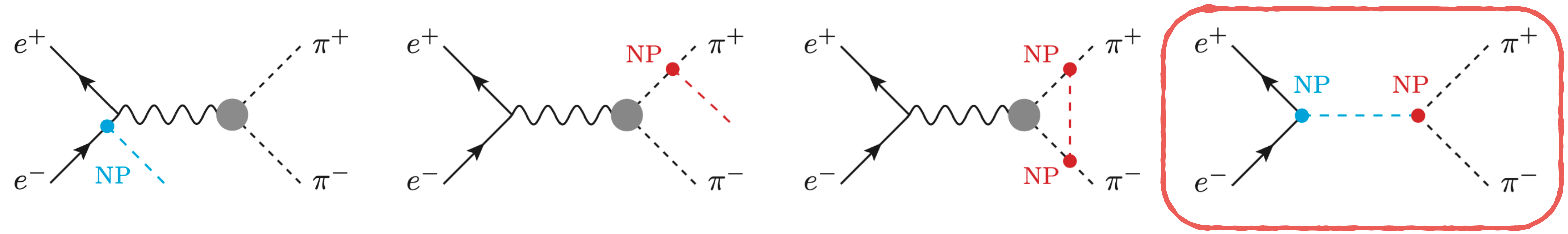
$$\sigma_{\text{had}} = \sigma_{\text{had}}^{\text{SM}} + \Delta\sigma_{\text{had}}^{\text{NP}}$$

optical theorem

experimental x-section

$\sim \quad \sigma(e^+e^- \to \gamma^* \to \text{hadrons})$

Light NP in $\sigma_{\rm had}$



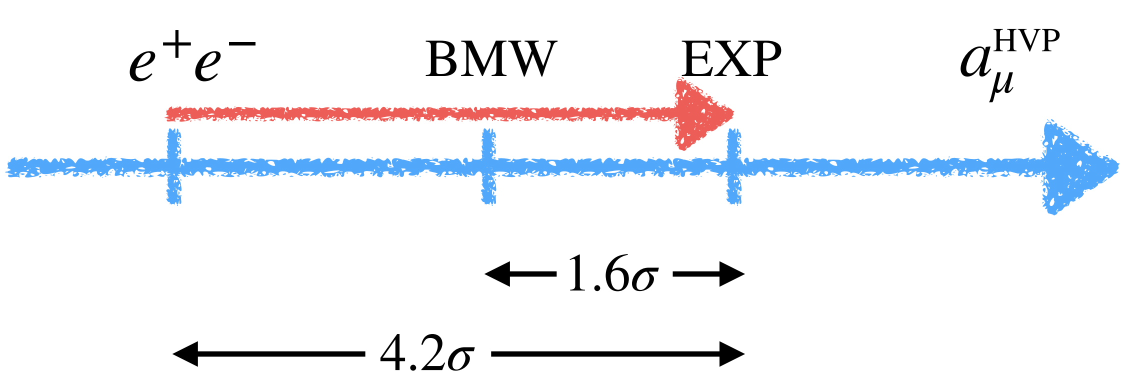
3. NP coupled both to hadrons and electrons

$$(a_\mu^{\rm HVP})_{e^+e^-} = \frac{\alpha}{\pi^2}\int_{m^2_{\pi^0}}^{\infty}\frac{{\rm d}s}{s}K(s)\,{\rm Im}\,\Pi_{\rm had}(s) = \frac{1}{4\pi^3}\int_{m^2_{\pi^0}}^{\infty}{\rm d}s\,K(s)\sigma_{\rm had}(s)$$

$$\sigma_{\rm had} = \sigma^{\rm SM}_{\rm had} + \Delta\sigma^{\rm NP}_{\rm had}$$

$$\downarrow$$

$$\sigma_{\rm had} - \Delta\sigma^{\rm NP}_{\rm had}$$



should be "subtracted" by NP,
since NP does not contribute to HVP at the LO
(but it contributes at the LO to the x-section)

a positive sift on $(a_\mu^{\rm HVP})_{e^+e^-}$ requires $\Delta\sigma^{\rm NP}_{\rm had} < 0$ (negative interference)

Basically, a unique scenario

- Requirements:

1. σ_{had} modified at $\sqrt{s} \lesssim 1\,\text{GeV}$ (mostly $\pi^+\pi^-$ channel)
2. A sizeable negative interference with the SM

sub-GeV mediator

couples to u, d quarks (and electrons)

tree-level mediator

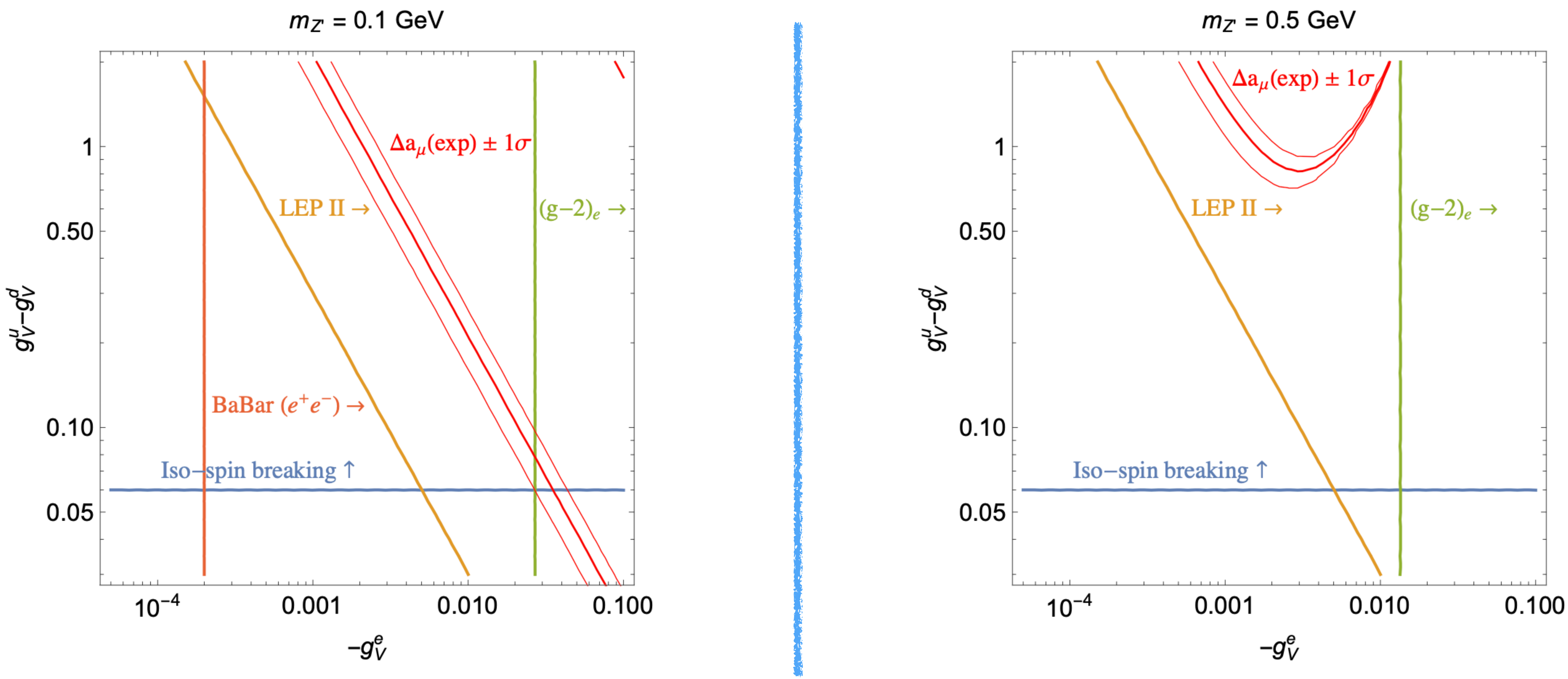
couples via a vector current

→ *a light spin-1 mediator with vector couplings to first generation SM fermions*

$$\mathcal{L}_{Z'} \supset (g_V^e\, \bar{e}\gamma^\mu e + g_V^q\, \bar{q}\gamma^\mu q) Z'_\mu \qquad q = u, d \qquad m_{Z'} \lesssim 1\ \text{GeV}$$

$$\frac{\sigma_{\pi\pi}^{\text{SM+NP}}}{\sigma_{\pi\pi}^{\text{SM}}} = \left| 1 + \frac{g_V^e (g_V^u - g_V^d)}{e^2} \frac{s}{s - m_{Z'}^2 + i m_{Z'} \Gamma_{Z'}} \right|^2$$

Z' constraints



→ *at least two independent bounds preventing to solve the "new muon g-2 puzzle"*

[LDL, Masiero, Paradisi, Passera 2112.08312]

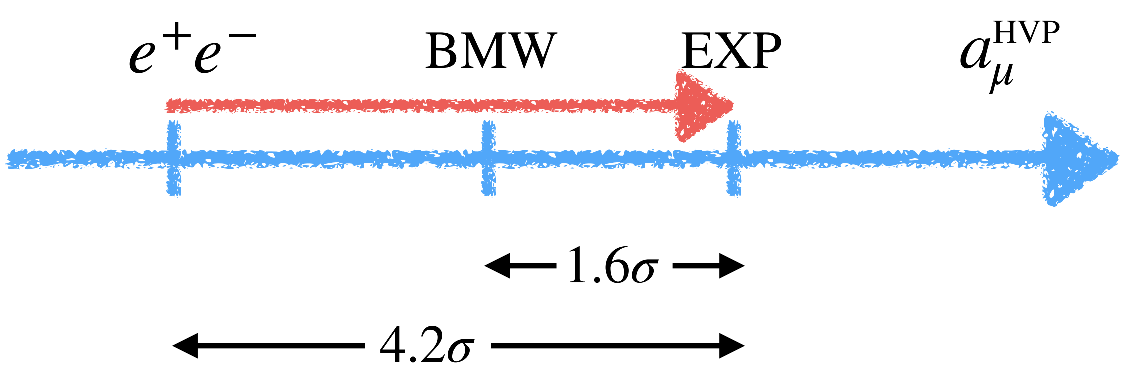
Conclusions

- Fermilab's Muon g-2 experiment confirms BNL's result

→ *muon g-2 is the longstanding hint of NP growing up to 4.2σ*

- The BMW lattice result weakens the exp-SM discrepancy, but brings in a tension with e^+e^- data

→ *"new muon g-2 puzzle"*



- Can this be due to NP (not in muons) that modifies $\sigma_{\rm had}$?

→ *strongly disfavoured in explicit BSM scenarios by a number of exp. constraints*

- Alternative determinations of HVP contribution will be crucial (lattice, MUonE, …)

Backup slides

Thank you for your attention !

σ_{had} data

[WP20, 2006.04822]

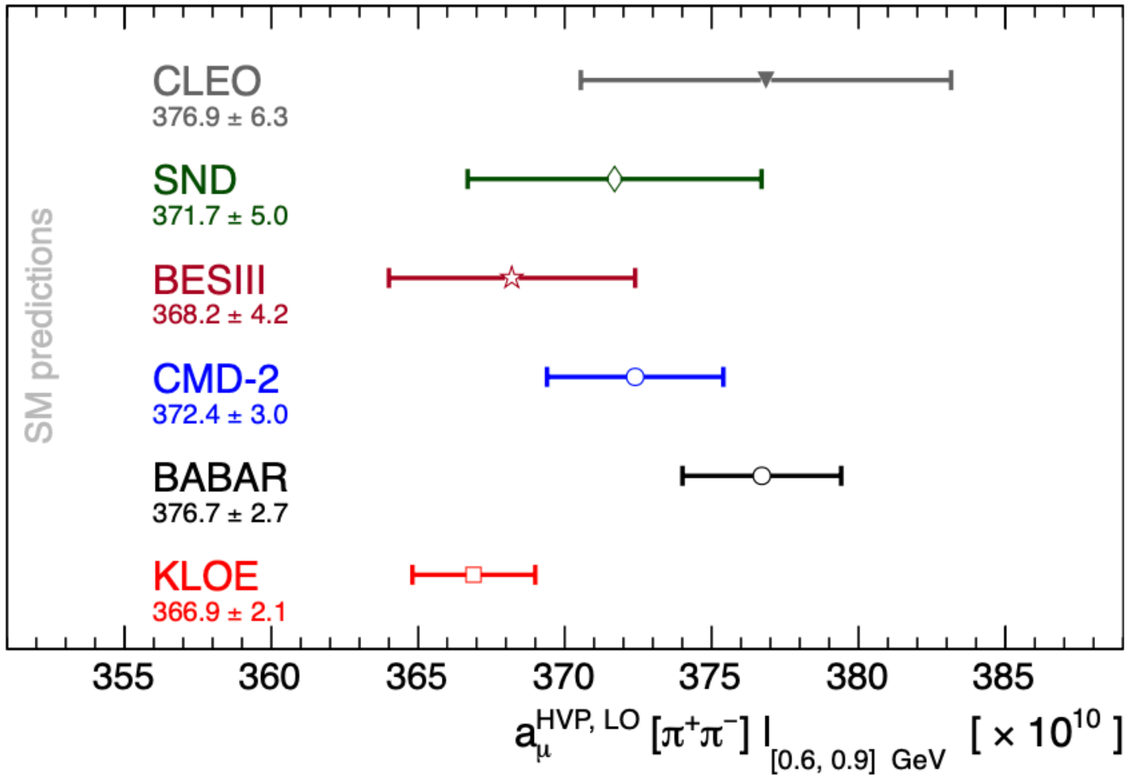


Figure 15: Comparison of results for $a_\mu^{\mathrm{HVP,\,LO}}[\pi\pi]$, evaluated between 0.6 GeV and 0.9 GeV for the various experiments.

σ_{had} data

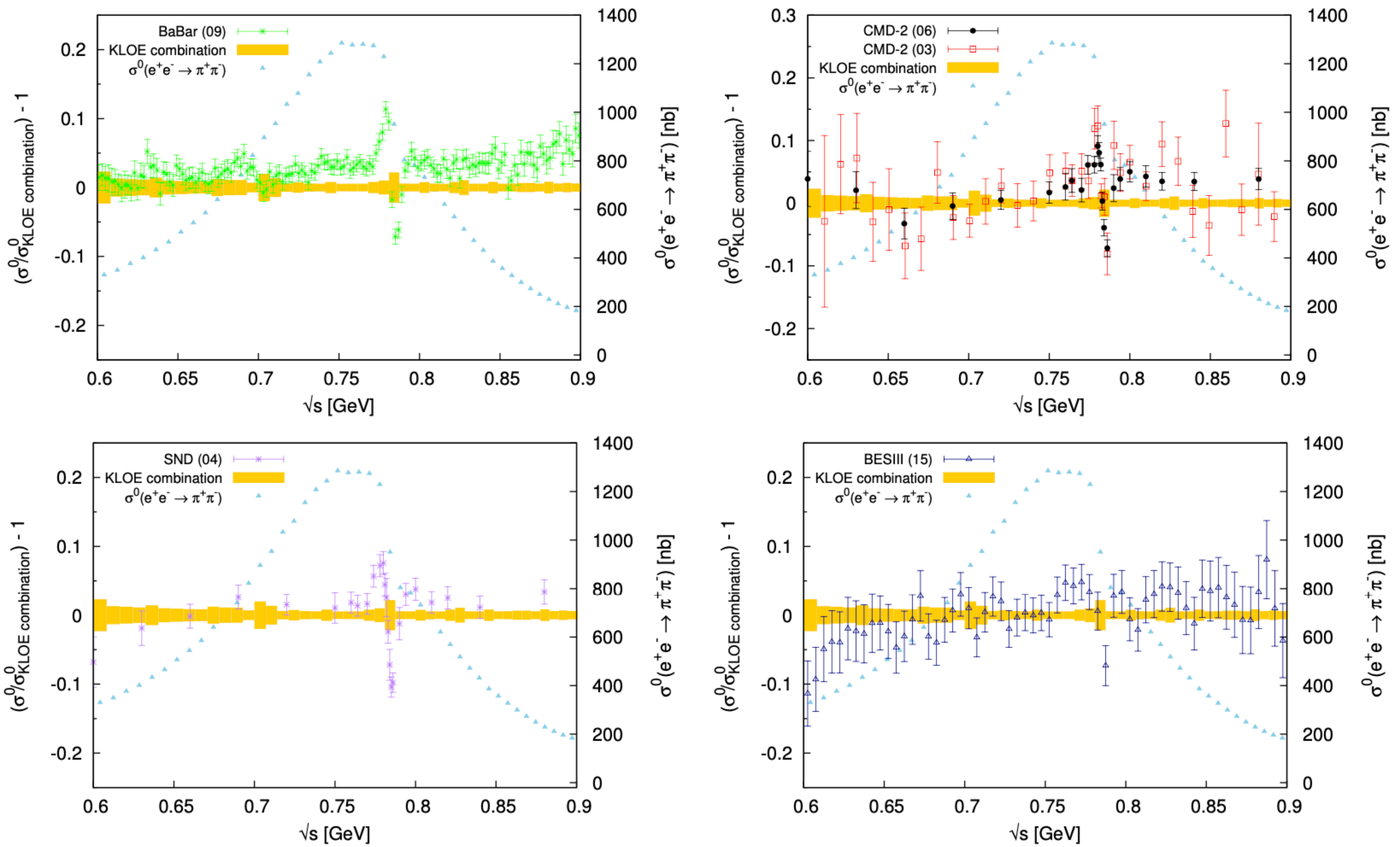


Figure 13: The $\pi^+\pi^-$ cross section from the KLOE combination compared to the BABAR, CMD-2, SND, and BESIII data points in the 0.6–0.9 GeV range [82]. The KLOE combination is represented by the yellow band. The uncertainties shown are the diagonal statistical and systematic uncertainties summed in quadrature. Reprinted from Ref. [82].

NP in Bhabha scattering ?

- What if the measurement of the KLOE luminosity is affected by NP ?

[Darmé, Grilli di Cortona, Nardi 2112.09139]

$$\mathcal{L}^{\rm SM}_{e^+e^-} = \frac{N_{\rm Bha}}{\sigma^{\rm SM}_{\rm eff}} \longrightarrow \mathcal{L}_{e^+e^-} = \mathcal{L}^{\rm SM}_{e^+e^-}\frac{\sigma^{\rm SM}_{\rm eff}}{\sigma_{\rm eff}}$$

$$\sigma_{\rm eff} = \sigma^{\rm SM}_{\rm eff}(1+\delta_R)$$

$$\sigma_{\rm had} \propto N_{\rm had}/\mathcal{L}_{e^+e^-} \longrightarrow \sigma_{\rm had} \to \sigma_{\rm had}(1+\delta_R)$$

$$a_\mu^{\rm LO,HVP} \to a_\mu^{\rm LO,HVP}(1+\delta_R)$$

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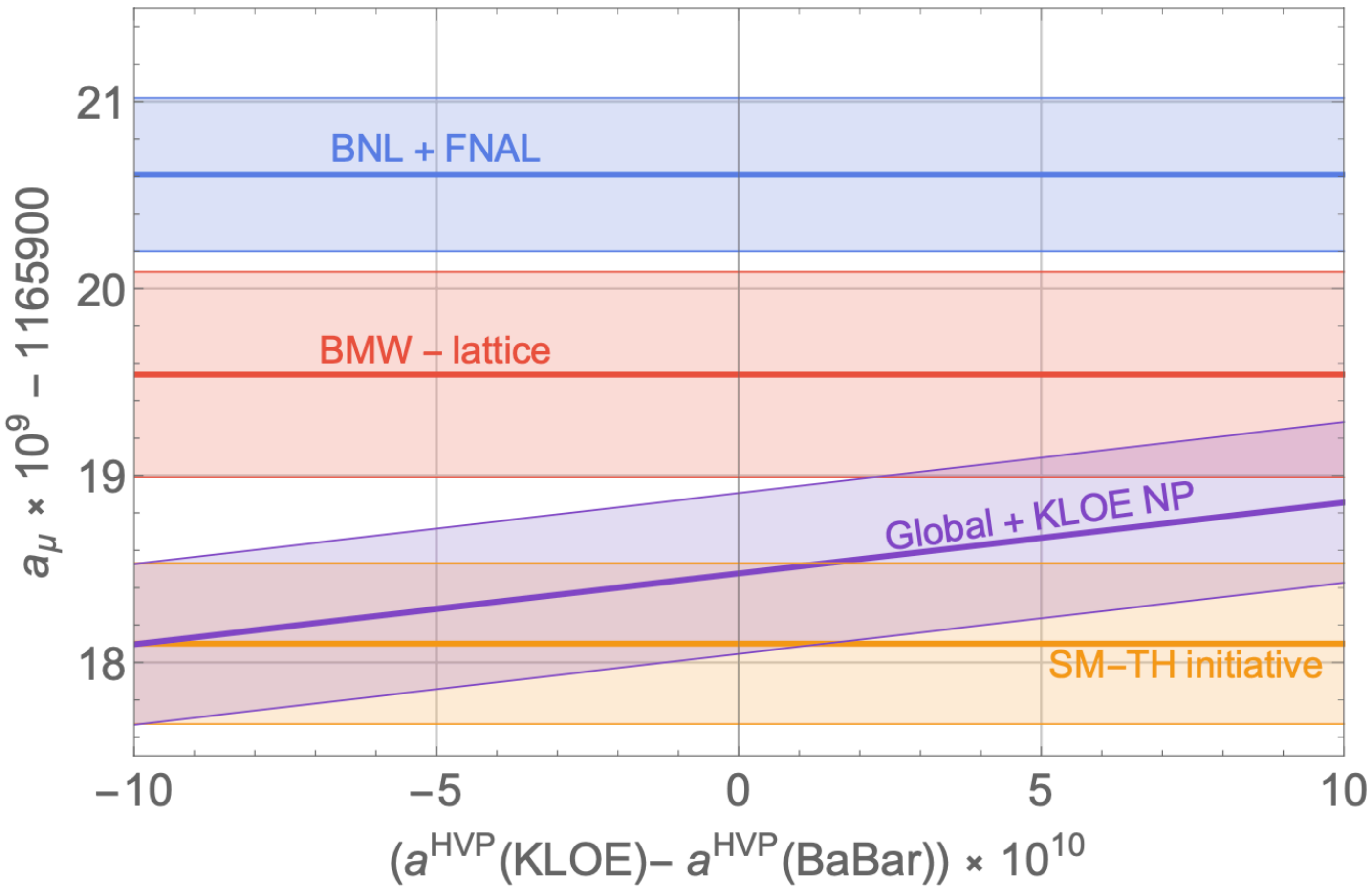

Figure 1. Theoretical prediction for a_μ obtained by modifying the KLOE result in the data driven global fit to $a_\mu^{\mathrm{LO,HVP}}$ (oblique violet band). The blue band corresponds to the combined BNL and FNAL experimental results, the red band to the prediction obtained with the BMW lattice estimate of $a_\mu^{\mathrm{LO,HVP}}$, and the orange band to the one obtained from σ_{had} without modifications of the KLOE results. The width of the bands represents $1\,\sigma$ uncertainties.

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- What if the measurement of the KLOE luminosity is affected by NP ?

[Darmé, Grilli di Cortona, Nardi 2112.09139]

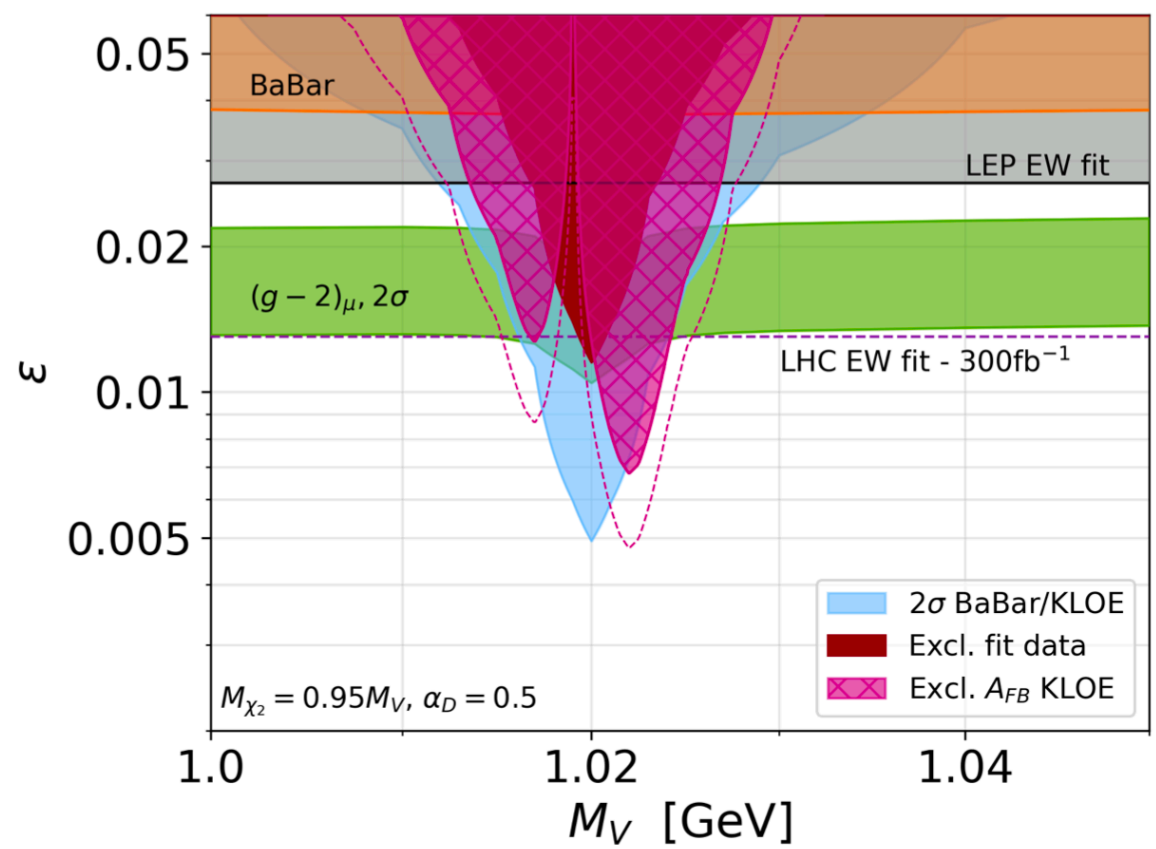

Figure 3. Parameter range compatible at 2σ with the experimental measurement of Δa_μ (green region) resulting from a redetermination of the KLOE luminosity, for $\alpha_D = 0.5, m_{\chi_2} = 0.95 m_V$ and $m_{\chi_1} = 25$ MeV. In the blue region the KLOE and BaBar results for $\sigma_{\rm had}$ are brought into agreement at 2σ. The red region corresponds to a shift of the KLOE measurement in tension with BaBar (and with the other experiments) at more than 2σ. The limit from the electroweak fit at LEP (gray band), the projection for LHC run-3 [73] (violet dashed line), and the recasting of the BaBar limit [51] (orange band) are also shown (see text). The hatched magenta region corresponds to the conservative 2σ exclusion from ΔA_{FB}, while the magenta dashed line corresponds to the more aggressive exclusion limit

NP in Bhabha scattering ?

- What if the measurement of the KLOE luminosity is affected by NP ?

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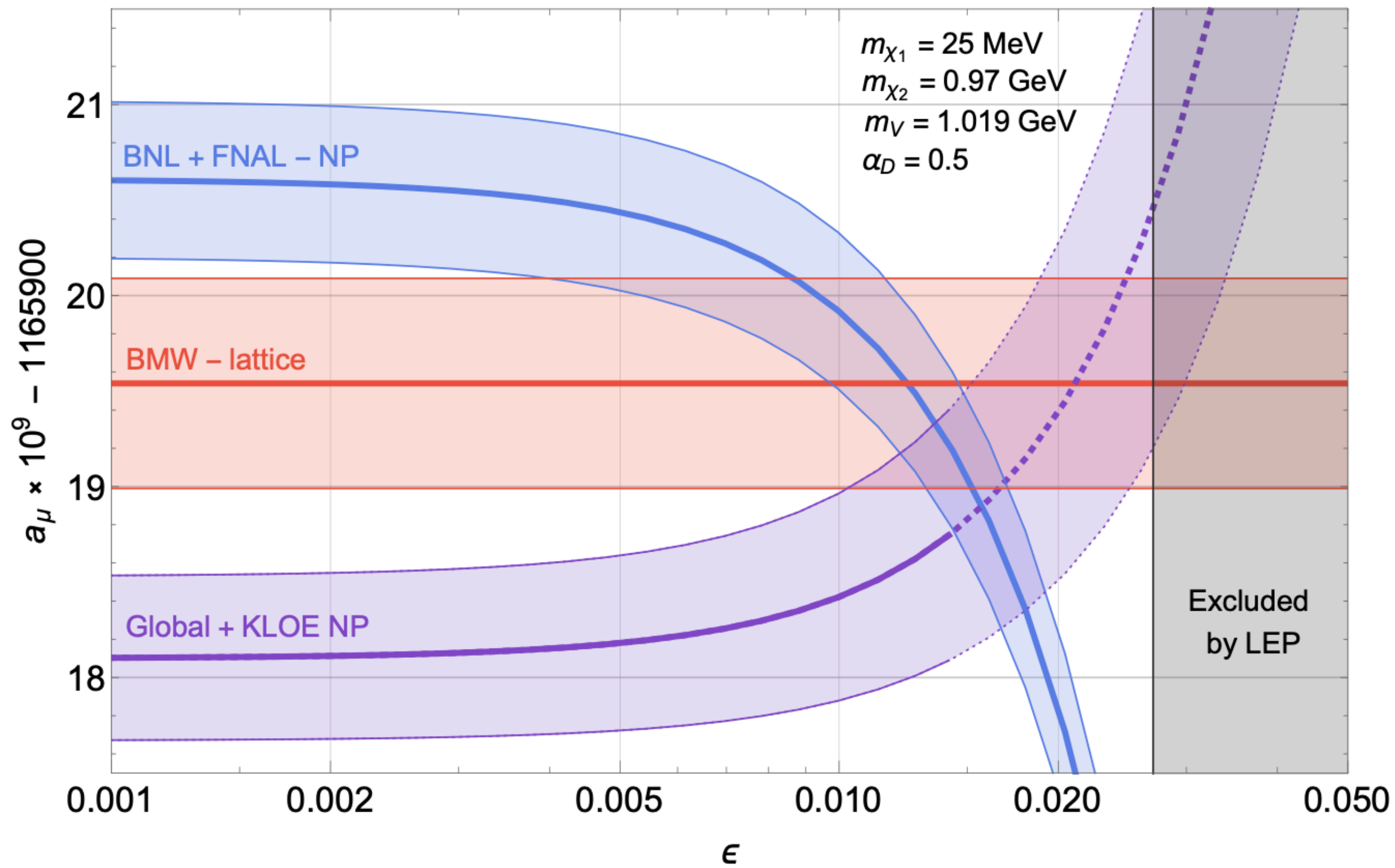
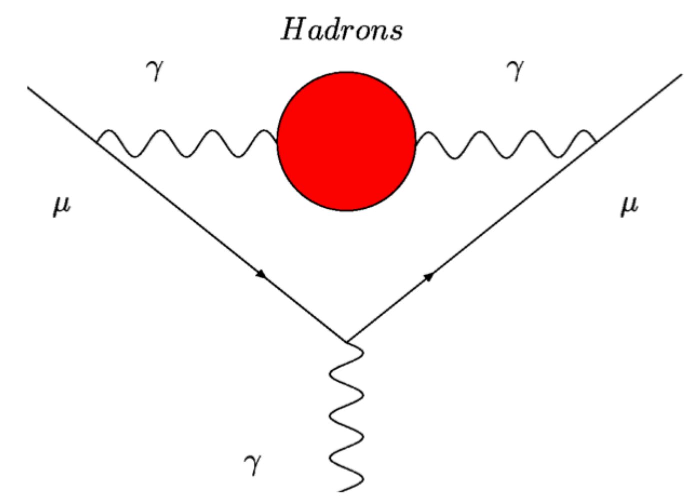


Figure 4. Theoretical prediction (purple) for a_μ as a function of ε for a dark photon model with $m_{\chi_1} = 25$ MeV, $m_{\chi_2} = 0.9$ GeV, $m_V = 1.019$ GeV and $\alpha_D = 0.5$. The dashed purple curve denotes the region where the KLOE and BaBar results are more than 2σ away. The blue band corresponds to the combined BNL and FNAL experimental results after subtracting the direct NP contribution from the dark photon. The red band shows the prediction obtained with the BMW lattice estimate of $a_\mu^{\rm LO,HVP}$. The width of the bands represents $1\,\sigma$ uncertainties. The grey region is excluded by LEP.

A closer look at HVP LO

- dominated by $e^+e^- \to \pi^+\pi^-$ channel (70% of the full hadronic)



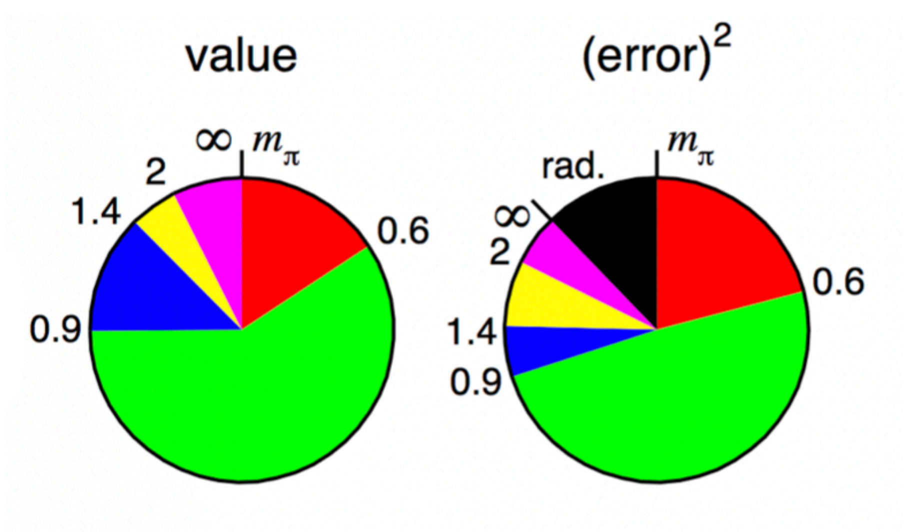
$$(a_\mu^{\rm HVP})_{e^+e^-} = \frac{\alpha}{\pi^2}\int_{m_{\pi^0}^2}^{\infty} \frac{\mathrm{d}s}{s}\, K(s)\, \mathrm{Im}\,\Pi_{\rm had}(s) = \frac{1}{4\pi^3}\int_{m_{\pi^0}^2}^{\infty} \mathrm{d}s\, K(s)\sigma_{\rm had}(s)$$

dispersion relations

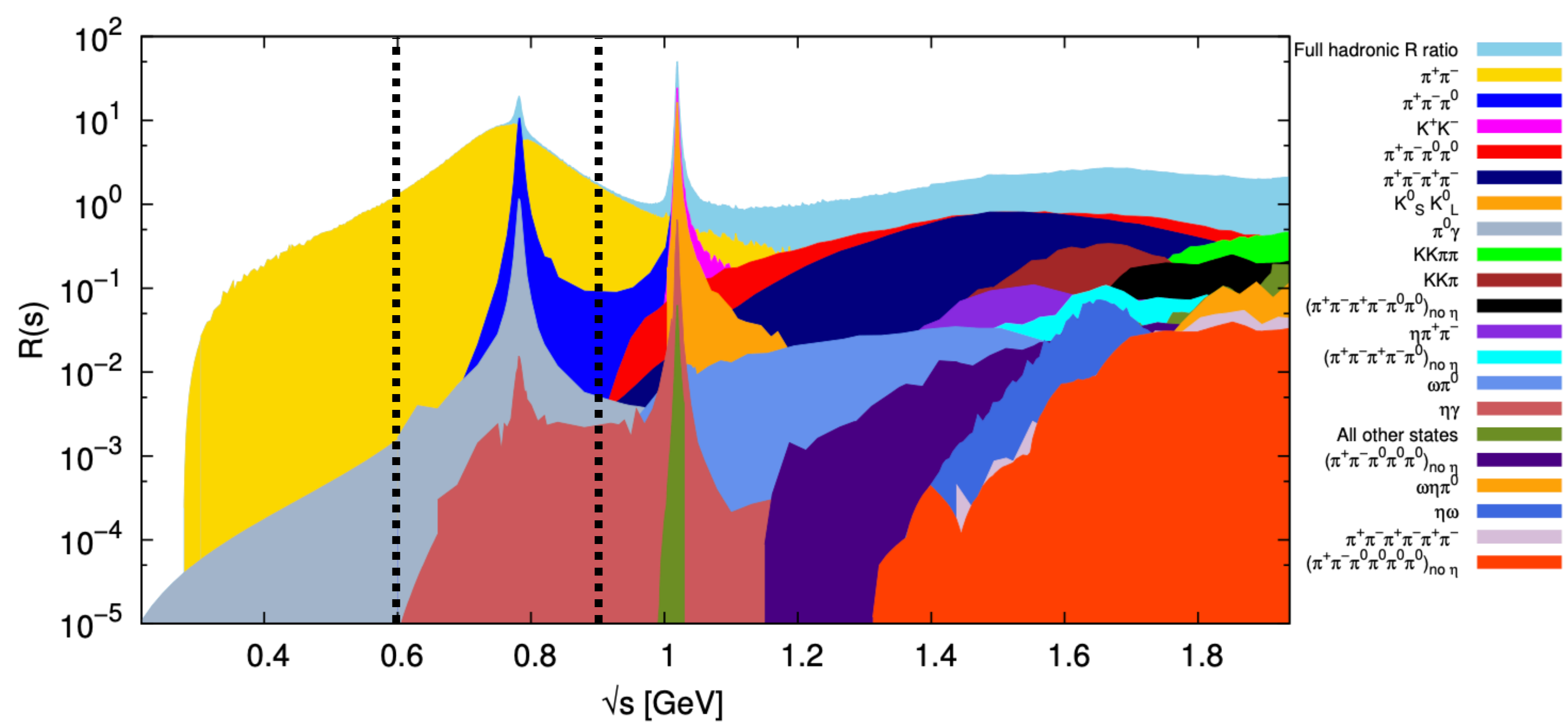
optical theorem

kernel function

$K(s) \approx m_\mu^2/3s$ **for** $\sqrt{s} \gg m_\mu$



Keshavarzi, Nomura, Teubner 2018



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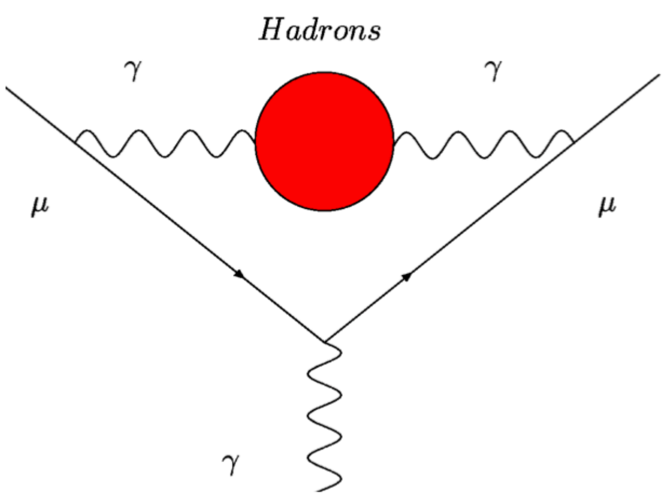

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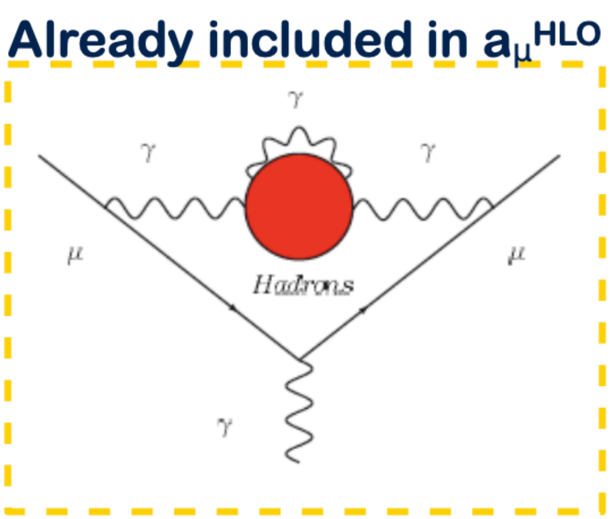
- what is $\sigma_{\rm had}(s)$?

  - Includes Final State Radiation (FSR)



$$(a_\mu^{\rm HVP})^{\rm FSR}_{e^+e^-} \approx 50 \times 10^{-11}$$

A closer look at HVP LO

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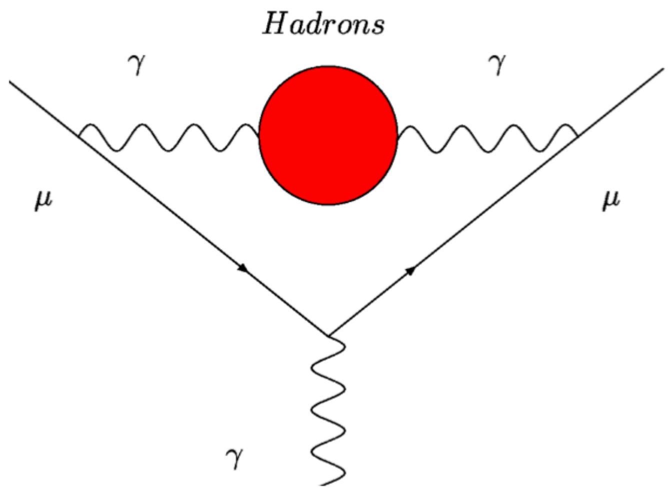


$$(a_\mu^{\mathrm{HVP}})_{e^+e^-} = \frac{\alpha}{\pi^2}\int_{m_{\pi^0}^2}^{\infty} \frac{\mathrm{d}s}{s} K(s)\,\mathrm{Im}\,\Pi_{\mathrm{had}}(s) = \frac{1}{4\pi^3}\int_{m_{\pi^0}^2}^{\infty} \mathrm{d}s\,K(s)\sigma_{\mathrm{had}}(s)$$

- what is $\sigma_{\mathrm{had}}(s)$?

- Includes Final State Radiation (FSR)

- Initial State Radiation (ISR) and FSR/ISR interference are subtracted

- Vacuum polarization also subtracted (by rescaling exp. cross-section by $|\alpha/\alpha(s)|^2$)

→ part of higher-order HVP

Z' shift on $(a_\mu^{\mathrm{HVP}})_{e^+e^-}$

- It can be shown that (neglecting iso-spin breaking corrections due to NP)

$$\frac{\sigma_{\pi\pi}^{\mathrm{SM+NP}}}{\sigma_{\pi\pi}^{\mathrm{SM}}} = \left| 1 + \frac{g_V^e (g_V^u - g_V^d)}{e^2} \frac{s}{s - m_{Z'}^2 + i m_{Z'} \Gamma_{Z'}} \right|^2$$

- Hence, requiring that the shift in the x-section saturates the g-2 discrepancy

$$\Delta a_\mu = \frac{1}{4\pi^3} \int_{s_{\mathrm{exp}}}^{\infty} \mathrm{d}s\, K(s) (-\Delta\sigma_{\mathrm{had}}^{\mathrm{NP}}(s))$$

$\sqrt{s_{\mathrm{exp}}} \approx 0.3$ GeV
for $\pi^+\pi^-$ channel

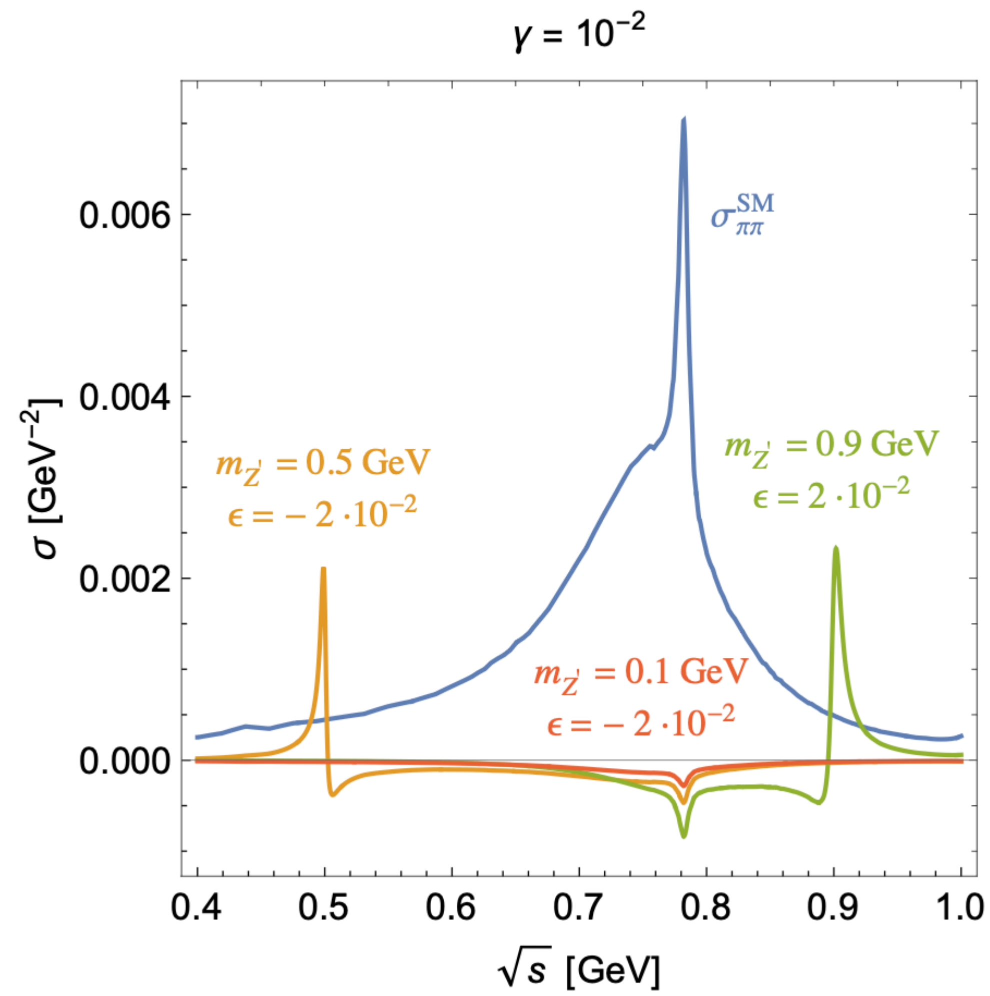
$$\Delta\sigma_{\mathrm{had}}^{\mathrm{NP}}(s) \approx \sigma_{\pi\pi}^{\mathrm{SM}}(s) \times \frac{2\epsilon s (s - m_{Z'}^2) + \epsilon^2 s^2}{(s - m_{Z'}^2)^2 + m_{Z'}^4 \gamma^2}$$

$$\epsilon \equiv g_V^e (g_V^u - g_V^d)/e^2$$

$$\gamma \equiv \Gamma_{Z'}/m_{Z'}$$

Z' shift on $(a_\mu^{\mathrm{HVP}})_{e^+e^-}$

- Typical benchmarks solving the g-2 discrepancy



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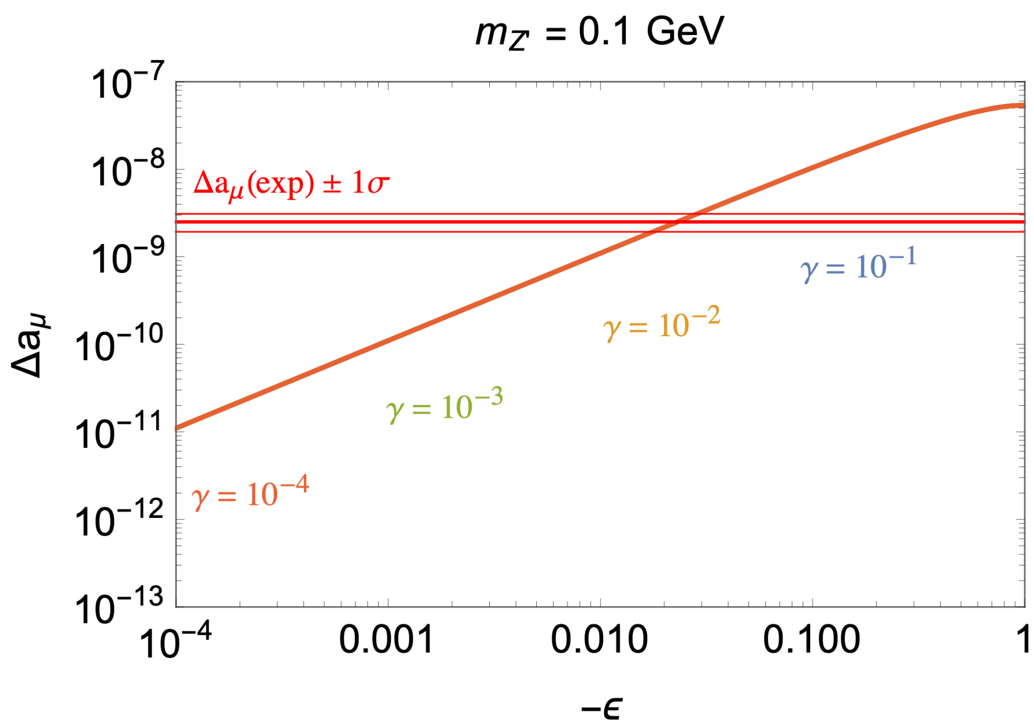
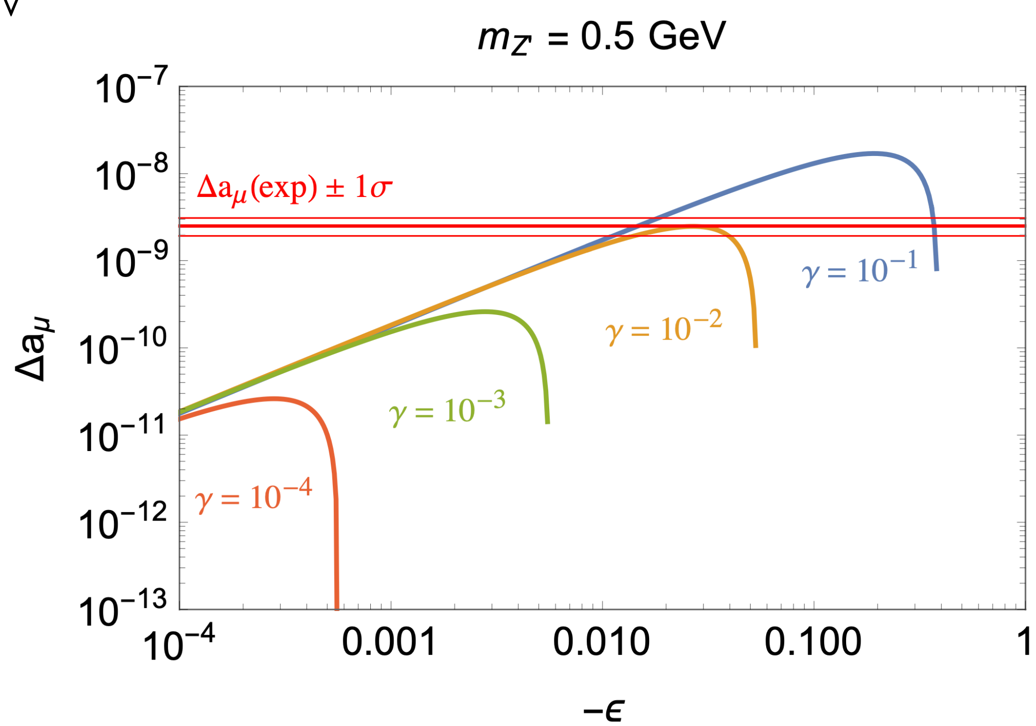


$|\epsilon| \gtrsim 10^{-2}$



$|\epsilon| \gtrsim 10^{-2} \quad |\gamma| \gtrsim 10^{-3}$

$$\epsilon \equiv g_V^e (g_V^u - g_V^d)/e^2$$

$$\gamma \equiv \Gamma_{Z'}/m_{Z'}$$

$$\gamma_{ee} \approx \frac{(g_V^e)^2}{12\pi} = 2.7 \times 10^{-10} \left(\frac{g_V^e}{10^{-4}}\right)^2 \qquad \gamma_{\pi\pi} = \frac{(g_V^u - g_V^d)^2}{48\pi} |F_\pi^V(m_{Z'}^2)|^2 \left(1 - \frac{4m_\pi^2}{m_{Z'}^2}\right)^{3/2}$$

Z' constraints

1. Semi-leptonic processes

$e^+e^- \to q\bar{q}$ *has been measured with per-cent accuracy at LEP-II*

$$\frac{\sigma_{qq}^{\mathrm{SM+NP}}}{\sigma_{qq}^{\mathrm{SM}}} \approx 1 + 2\frac{g_V^e g_V^q}{e^2 Q_q} \longrightarrow |g_V^e g_V^q| \lesssim 4.6 \cdot 10^{-4} |Q_q| \qquad (\epsilon \lesssim 3.3 \cdot 10^{-3})$$

Z' constraints

1. Semi-leptonic processes

2. Leptonic processes

- for $m_{Z'} \lesssim 0.3$ GeV ($Z' \to e^+e^-$ is the main decay mode)

$e^+e^- \to \gamma Z'$ @ *BaBar* $\longrightarrow$ $g_V^e \lesssim 2 \cdot 10^{-4}$

- for $m_{Z'} \gtrsim$ MeV

electron g-2 $\longrightarrow$ $|g_V^e| \lesssim 10^{-2}\, (m_{Z'}/0.5\ \mathrm{GeV})$

Z' constraints

1. Semi-leptonic processes

2. Leptonic processes

3. Iso-spin breaking observables

charged vs. neutral pion mass^2 difference $\Delta m^2 = m^2_{\pi^+} - m^2_{\pi^0}$

$$(\Delta m^2)_{Z'} \sim \frac{(g_V^u - g_V^d)^2}{(4\pi)^2}\Lambda_\chi^2 \qquad (\Lambda_\chi \approx 1 \text{ GeV})$$

$\longrightarrow$ $\left|g_V^u - g_V^d\right| \lesssim 0.06$ [Rescaling lattice QCD calculation of Frezzotti et al 2112.01066]